CRACK GROWTH IN INELASTIC FLUID-SATURATED POROUS MEDIA

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Sommario

Viene ricavata una soluzione asintotica dei campi di tensione e della pressione interstiziale in prossimità dell'apice di una frattura che si propaga in un materiale elastoplastico poroso saturo. In particolare, viene analizzato il problema piano di deformazione in condizioni quasistatiche di Modo I, sia per superfici di frattura permeabili che impermeabili. Il modello costitutivo considerato si basa sulla condizione di snervamento di Drucker-Prager in termini di sforzo efficace, con legge di flusso associata e non, per incrudimento lineare ed isotropo.

Abstract

An asymptotic solution is obtained for stress and pore pressure fields near the tip of a crack, steadily running in elastic-plastic fluid-saturated porous solids. Reference is made to the case of quasi-static crack growth under plane strain and Mode I loading conditions. In particular, the effective stress is assumed to obey the Drucker-Prager yield condition with associative or non-associative flow-rule, displaying linear isotropic hardening. Both conditions of permeable and impermeable crack faces are considered.

1. Introduction

Fracture initiation and growth in porous, fluid saturated rocks is an important issue in a class of geophysical problems. These are characterized by the interaction between the pore fluid and the volumetric elastic and inelastic deformation of the solid skeleton and are believed to be important in fault creep and in the propagation of hydraulic fracturing for oil resources exploration [1, 9-11]. Implications of inelastic deformation in the mechanics of crack growth in elastic-plastic porous and fluid-saturated materials may be relevant in view of the extensive utilization of hydraulic fracturing process.

The analysis of steady-state crack propagation has been systematically investigated by the authors for inelastic constitutive behaviors with pressure-sensitive yield conditions [2], non-associative flow-laws [6], and porosity [7, 8]. However, for a better understanding of fracture mechanisms in elastic-plastic fluid-saturated solids, the effect of the coupling between the volumetric dilatation of the elastic-plastic skeleton, and the pressure in diffusing pore fluid, should be investigated in the proximity of the crack tip.

In the present article, an asymptotic solution is provided for stress and pore pressure fields near the tip of a crack, steadily running in elastic-plastic fluid-saturated porous solids, under Mode I plane strain conditions. Both conditions of permeable and impermeable crack faces are considered. The mechanical behavior of the saturated porous medium is described using coupled constitutive equations developed on the basis of the theory of mixtures [5]. In particular, when compressible, the fluid constituent has a reversible behavior, while the fluid phase has not. The solid phase is assumed to obey the Drucker-Prager yield condition with associative or volumetric-non-associative flow rule, displaying linear isotropic hardening. The Darcy's law is used to model the fluid diffusion process. The results show that the explicit coupling between plastic dilatancy and fluid compressibility results in a peculiar behavior of the pore pressure near the crack-tip. Moreover, the flux of fluid together with plasticity effects may dissipate the amount of supplied energy, leading to a reduction of the energy available to fracture the material.

2. Constitutive equations

The adopted constitutive model refers to the elastic-plastic model for saturated porous media proposed by Loret and Harireche [5]. Within the context of small deformations incremental theory, the total strain rate $\dot{\boldsymbol{\varepsilon}}$ is the sum of elastic $\dot{\boldsymbol{\varepsilon}}^{e}$ and plastic $\dot{\boldsymbol{\varepsilon}}^{p}$ parts. Similarly, the rate of the variation of fluid content per unit volume $\dot{\zeta}$ is the sum of elastic $\dot{\zeta}^{e}$ and inelastic $\dot{\zeta}^{p}$ contributions. Both elastic terms are related to the stress and pore pressure rates $\dot{\boldsymbol{\sigma}}$ and $\dot{p}$ through the elastic incremental relation [1, 3, 4]:

$$\dot{\boldsymbol{\varepsilon}}^{e} = \frac{1}{E} \{(1 + \nu)\, \dot{\boldsymbol{\sigma}} + [(1 - 2\,\nu)\, \alpha\, \dot{p} - \nu\, tr\, \dot{\boldsymbol{\sigma}}]\, \mathbf{I}\}, \quad (1)$$

$$\dot{\zeta}^{e} = \frac{1}{E} [\Gamma\, \dot{p} + \alpha\, (1 - 2\,\nu)\, (3\,\alpha\, \dot{p} + tr\, \dot{\boldsymbol{\sigma}})], \quad (2)$$

where E is the elastic Young modulus, $\alpha = 3\,(\nu_u - \nu)\, /\, B\,(1 - 2\,\nu)\,(1 + \nu_u)$ is the Biot coefficient of effective stress ($0 < \alpha \leq 1$), $\Gamma = 3\,\alpha\,(1 + \nu)\,(1 - 2\,\nu_u)\, /\, B\,(1 + \nu_u)$, with ν and ν_u denoting the drained and undrained Poisson ratios and B the Skempton's pore pressure coefficient [1]. The Darcy's law and the mass continuity equation are

$$\mathbf{q} = -\,\rho_0\, \kappa\, \nabla p, \quad \rho_0\, \dot{\zeta} = -\, div\, \mathbf{q}, \quad (3)$$

where $\mathbf{q}$ is the mass flux, κ is the permeability coefficient and the reference density ρ_0 is assumed constant.

Drucker-Prager yield condition is assumed in terms of the effective stress $\boldsymbol{\sigma}^* = \boldsymbol{\sigma} + \alpha\, p\, \mathbf{I}$:

$$f(\boldsymbol{\sigma}^*, k) = \sqrt{J_2(\boldsymbol{\sigma}^*)} + \frac{\mu}{3}\, tr\, \boldsymbol{\sigma}^* - k = 0, \quad (4)$$

where $J_2(\boldsymbol{\sigma}^*) = |\, dev\, \boldsymbol{\sigma}^*\,|^2 / 2$ is the second invariant of the deviatoric effective stress, k is an internal variable governing the isotropic hardening behavior, and μ is the pressure-sensitivity factor, which defines the influence of the hydrostatic stress on the yield process.

Let $\mathbf{Q}$ and φ denote the derivative of the yield function (4) with respect to the effective stress tensor $\boldsymbol{\sigma}^*$ and to the pore pressure p, respectively:

$$\mathbf{Q} = \frac{\partial f}{\partial s} = \dot{e} + \dot{s}\,\mathbf{I}, \quad \varphi = \frac{\partial f}{\partial p} = \alpha\,\mu. \quad (5)$$

The evolution laws for the inelastic internal variable allow for non-associative flow-rule and linear isotropic hardening behavior, namely:

$$\dot{e}^p = \Lambda\,\mathbf{P}, \quad \dot{\zeta}^p = \Lambda\,\phi, \quad \dot{k} = \Lambda\,H, \quad (6)$$

where Λ is the (non negative) plastic multiplier, H is the hardening modulus, assumed to be strictly positive and constant for linear isotropic hardening behavior, and:

$$\mathbf{P} = \frac{dev\,s^*}{2\sqrt{J_2(s^*)}} + \frac{\beta}{3}\,\mathbf{I}, \quad \phi = \alpha\,\beta, \quad (7)$$

where β is the plastic dilatancy parameter, governing the volumetric plastic-flow. Note that for $\beta = \mu$, the evolution laws (6) become associative. Moreover, let $H = \eta\,G / (1 - \eta)$, where $\eta = G_t / G$ is the ratio between plastic and elastic shear moduli of the material. In the following, the dimensionless hardening modulus is denoted with $h = H / E$.

The consitency condition $\dot{f}(\sigma^*, k) = 0$ together with (5) and (6_3) allow us to obtain the value of the plastic multiplier Λ :

$$\Lambda = H^{-1} \langle\, \mathbf{Q} \bullet \dot{s}^* \,\rangle = H^{-1} \langle\, \mathbf{Q} \bullet \dot{s} + \alpha\,\mu\,\dot{p} \,\rangle\,. \quad (8)$$

It should be remarked that plastic flow occurs when the stress point lies on the yield surface (4) and $\mathbf{Q} \bullet \dot{s}^* \geq 0$, whereas elastic unloading occurs when $\mathbf{Q} \bullet \dot{s}^* < 0$. Finally, the elastic-plastic incremental constitutive equations relating the stress rate $\dot{s}$ to the total strain rate $\dot{e}$, can be written in the form:

$$\dot{e} = \frac{1}{E}\,\{(1 + \nu)\,\dot{s} + [\alpha\,(1 - 2\,\nu)\,\dot{p} - \nu\,tr\,\dot{s}]\,\mathbf{I} + h^{-1}\,(\mathbf{Q} \bullet \dot{s} + \alpha\,\mu\,\dot{p})\,\mathbf{P}\}, \quad (9)$$

$$\dot{\zeta} = \frac{1}{E}\,\{\alpha\,(1 - 2\,\nu)\,tr\,\dot{s} + [3\,\alpha^2\,(1 - 2\,\nu) + \Gamma]\,\dot{p} + h^{-1}\,\alpha\,\beta\,(\mathbf{Q} \bullet \dot{s} + \alpha\,\mu\,\dot{p})\}. \quad (10)$$

Equations (9)-(10) hold when the stress state satisfies the yield condition (4). Otherwise, the incremental constitutive relationship reduces to the poroelastic behavior, obtained for $\Lambda = 0$.

3. Crack propagation problem

The problem of a plane crack propagating at constant velocity *c* along a rectilinear path in an infinite medium is considered. The mechanical behavior of the material is described by the incremental elastic-plastic constitutive model presented in Section 2. This framework allows us to consider the possibility of elastic unloading sectors, which may appear in the proximity of the crack-tip, during crack propagation. A cylindrical co-ordinate system (O, $\mathbf{e}_r$, $\mathbf{e}_\vartheta$, $\mathbf{e}_3$) moving with the crack-tip towards the $\vartheta = 0$ direction is considered, with the x_3-axis along the straight crack front. The steady-state condition yields the following time derivative rule, for any scalar, vectors or second order tensor **A**:

$$\dot{\mathbf{A}} = c\,(r^{-1}\,\mathbf{A}_{,\vartheta}\,sin\,\vartheta - \mathbf{A}_{,r}\,cos\,\vartheta), \quad (11)$$

where r, and ϑ are the polar coordinates in the plane orthogonal to the x_3-axis.

The quasi-static equilibrium equations and the kinematic compatibility conditions between strain rates and velocities are:

$$div\, \sigma = \mathbf{0}, \quad \dot{\varepsilon} = \tfrac{1}{2}(\nabla\, \mathbf{v} + \nabla\, \mathbf{v}^{T}). \quad (12)$$

In addition to equation (12_2) the plane strain condition $\dot{\varepsilon}_{33} = v_3 = 0$ must be considered.

Constitutive equations (9)-(10) together with quasi-static equilibrium and kinematic compatibility conditions (12), form a system of first order PDEs which governs the problem of the crack propagation. The solution is sought in a separated variables form, by considering single term asymptotic expansions of near crack-tip fields. To this purpose, note that if the order of the stress singularity is $\sigma = O(r^{s})$ as r tends to zero, then the constitutive relation (10) implies that at least one of the conditions $p = O(r^{s})$ or $\zeta = O(r^{s})$ must be met. Let us assume the validity of the former: it follows that $\nabla\, p = O(r^{s-1})$. In this case, the Darcy's law (3_1) also implies that $\mathbf{q} = O(r^{s-1})$, thus $div\, \mathbf{q} = O(r^{s-2})$, and finally the mass continuity equation (3_2) yields $\dot{\zeta} = O(r^{s-2})$, but this result is not compatible with the constitutive equation (10), being $\dot{p} = O(r^{s-1})$ and $\dot{\sigma} = O(r^{s-1})$. Therefore, only the condition $\zeta = O(r^{s})$ must be considered. In this case, the time derivative rule (11) gives $\dot{\zeta} = O(r^{s-1})$ and thus the mass continuity equation (3_2) implies $\mathbf{q} = O(r^{s})$ so that the Darcy's law (3_1) is satisfied if and only if $p = O(r^{s+1})$. As a conclusion, the pore pressure does not display a singularity at the crack-tip for $-0.5 \le s \le 0$ but tends to vanish as r approaches zero. This condition agrees with the results obtained for quasi-static crack propagation in poroelastic materials [1], where the pore pressure behaves as $r^{1/2}$ and the stress field in the solid skeleton has the well-known square root singularity. The vanishing of the pore pressure as r tends to zero means that for quasi-static conditions the pore fluid diffuses away from the crack-tip, where the material is effectively drained.

From the above argumentation, the following assumption for the velocity, stress, pore pressure and mass flux asymptotic fields can be accepted:

$$\mathbf{v}(r,\vartheta) = \frac{c}{s}\left(\frac{r}{R}\right)^{s}\mathbf{w}(\vartheta), \quad \sigma(r,\vartheta) = E\left(\frac{r}{R}\right)^{s}\mathbf{T}(\vartheta), \quad p(r,\vartheta) = R\left(\frac{r}{R}\right)^{s}P(\vartheta),$$

(13)

$$\mathbf{q}(r,\vartheta) = \rho_0\, c\left(\frac{r}{R}\right)^{s}\mathbf{z}(\vartheta), \quad \dot{\zeta} = \frac{c}{r}\left(\frac{r}{R}\right)^{s}\Gamma(\vartheta), \quad k(r,\vartheta) = E\left(\frac{r}{R}\right)^{s}\chi(\vartheta),$$

where s is the exponent of the stress singularity and R denotes a characteristic dimension of the plastic zone, which remains undetermined, since the asymptotic problem is homogeneous.

By applying the derivative rule (11) to the fields representations (13), the velocity of deformation and the rates of stress and pore pressure assume the following expressions:

$$\dot{\varepsilon}(r,\vartheta) = \frac{c}{r}\left(\frac{r}{R}\right)^{s}\mathbf{D}(\vartheta), \quad \dot{\sigma}(r,\vartheta) = E\,\frac{c}{r}\left(\frac{r}{R}\right)^{s}\Sigma(\vartheta), \quad \dot{p}(r,\vartheta) = \frac{c^{2}}{\kappa}\left(\frac{r}{R}\right)^{s}\Pi(\vartheta), \quad (14)$$

where the time derivative rule (11) can be used to define the angular functions Σ and Π :

$\Sigma = \mathbf{T}' \sin\vartheta - s\, \mathbf{T} \cos\vartheta$, $\Pi = P' \sin\vartheta - (1 + s)\, P \cos\vartheta$, (15)

and the suffix $(\ )'$ denotes differentiation with respect to the argument ϑ.

It is worth noting that asymptotically the effective stress σ^* is equal to the stress σ, since the pore pressure gives higher order contribution. The introduction of the asymptotic fields (13) in the yield condition (4) results in $f(\sigma^*, k) = E\,(r / R)^s f(\mathbf{T}, \chi) + o(r^s)$, and thus, in a neighborhood of the crack-tip the yield condition (4) may be equivalently written as:

$$f(\mathbf{T}, \chi) = \sqrt{J_2(\mathbf{T})} + \frac{\mu}{3}\, tr\, \mathbf{T} - \chi = 0. \quad (16)$$

Moreover, the yield function gradient and flow-mode tensors defined in (5) and (7) reduce to:

$$\mathbf{Q} = \frac{dev\,\mathbf{T}}{2\sqrt{J_2(\mathbf{T})}} + \frac{\mu}{3}\mathbf{I},\quad \mathbf{P} = \frac{dev\,\mathbf{T}}{2\sqrt{J_2(\mathbf{T})}} + \frac{\beta}{3}\mathbf{I}, \quad (17)$$

which are independent of r.

A substitution of the asymptotic fields (13) and (14) into equilibrium, compatibility conditions (12) and constitutive relation (9) yields at the lowest order:

$$\mathbf{T}'\, \mathbf{e}_\vartheta + s\, \mathbf{T}\, \mathbf{e}_r = \mathbf{0},\quad \left(\frac{1}{s}\, \mathbf{w}' \otimes \mathbf{e}_\vartheta + \mathbf{w} \otimes \mathbf{e}_r\right)_{Sym} = (1 + \nu)\, \Sigma - \nu\, tr\, \Sigma + h^{-1} (\mathbf{Q} \bullet \Sigma)\, \mathbf{P}. \quad (18)$$

Note that the field equations (18) together with the yield condition (16) and the definitions (17) coincide with the corresponding equations governing the quasi-static crack propagation in the drained elastic-plastic material of the solid phase. This problem has been previously solved for associative [2] and non-associative [6] flow rule. The introduction of the fields (13) and (14) into the Darcy's law and the mass continuity equation (3) and in the constitutive relation (10) yields at the lowest order:

$$z_r = -(1 + s)\, P,\quad z_\vartheta = -P',$$

(19)

$$z_{\vartheta,\vartheta} + (1 + s)\, z_r + \alpha\, (1 - 2\nu)\, tr\, \Sigma + h^{-1} \alpha\, \beta\, (\mathbf{Q} \bullet \Sigma) = 0,$$

and thus by substituting ($19_{1,2}$) into (19_3), one obtains:

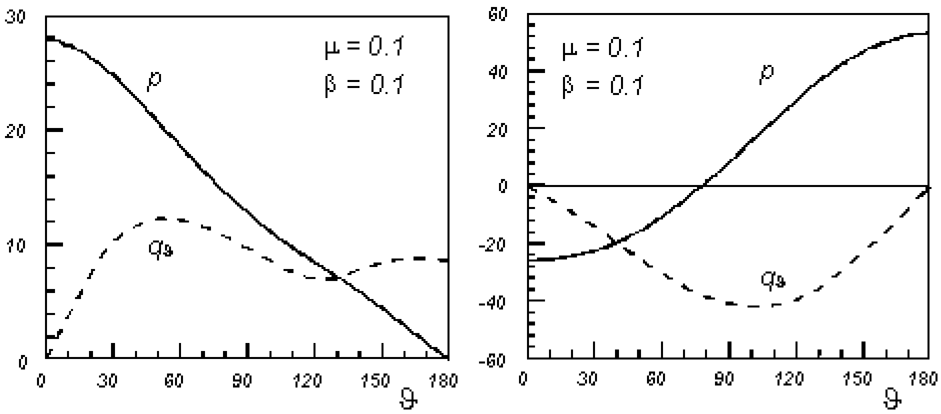
$$P'' + (1 + s)^2\, P = \alpha\, (1 - 2\nu)\, tr\, \Sigma + h^{-1} \alpha\, \beta\, (\mathbf{Q} \bullet \Sigma). \quad (20)$$

Once the stress field **T** is known from the solution of the drained problem (18), the expression of the angular function Σ and **Q** in (15) and (17) may be introduced into the second order ODEs (20), which can be solved for P.

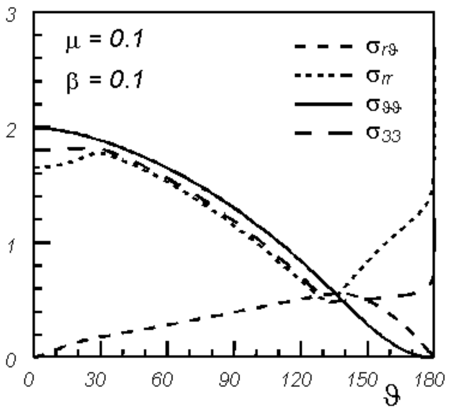
The symmetry of the Mode I crack propagation problem implies the vanishing of the mass flux component z_ϑ at $\vartheta = 0$, and thus by (19) the condition $P'(0) = 0$. Moreover, two different boundary conditions must be introduced for permeable and impermeable crack faces, respectively. In particular, for permeable crack flanks the pore pressure must vanish at $\vartheta = \pi$, namely $P(\pi) = 0$, whereas for impermeable crack surfaces the mass flux must vanish, leading to the condition $P'(\pi) = 0$.

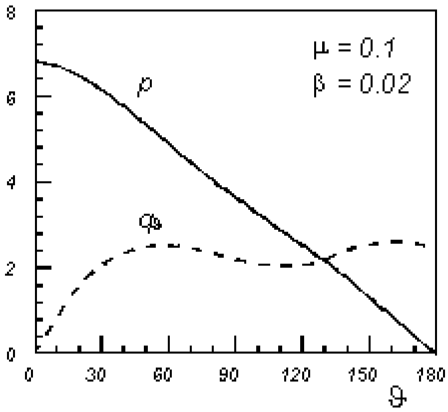
4. Results and conclusions

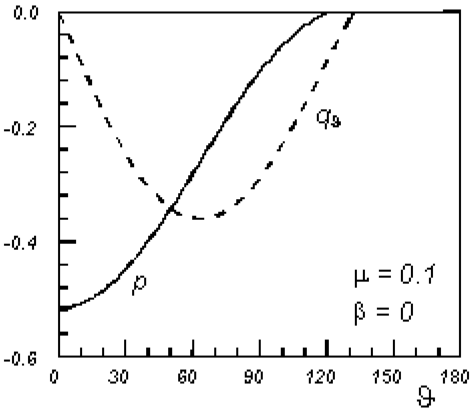
The asymptotic pore pressure field near the tip of a crack steadily running in an elastic-plastic fluid-saturated porous solids, under Mode I plane strain conditions has been obtained, based on the results of the stress and velocity crack-tip fields in the drained material of the solid phase already known [2, 6]. In Figs. 1a and 1b the angular variation of the pore pressure *p* and of the mass flux component q_ϑ are reported, relative to the associative flow-rule and the following values of the material parameters: $\alpha = 0.5$, $\nu = 1/3$ and $\eta = 0.001$. Note that from (19_1) the radial component of the mass flux is proportional to the pore pressure, with reversed sign, and thus has not been plotted. In particular, figs.1a and 1b refer to the cases of permeable and impermeable crack faces, respectively. For permeable crack faces the pore pressure is positive in a neighborhood of the crack-tip and attains its maximum value ahead of the crack-tip (Fig. 1a). For impermeable crack faces a decrease is found in the pore pressure directly ahead of the crack-tip for $\vartheta < 80°$ (Fig. 1b), which has the effect of weakening the material ahead of the crack-tip and dissipating energy. The angular variations of the corresponding singular stress components have been reported in Fig.2. Since all the field equations and the assigned boundary conditions are of homogeneous type, the solution may be determined within an arbitrary amplitude constant.



(a) (b)







(a) (b)

In Figs. 3 the effect is shown of small (a) and null (b) plastic dilatancy on the pore pressure and mass flux fields in a neighborhood of the crack tip. In the former case the behavior is similar to the associative case of Fig.1a, except for the magnitude of the fields. In the latter case the pore pressure in the diffusing fluid is coupled with the elastic volumetric dilatation only, and thus it displays a variation similar to the poroelastic case [1]. Note also that the results for permeable crack faces coincide with those for impermeable crack faces, since both pore pressure and mass flux tend to vanish at the crack flanks, namely for $\vartheta > 130°$.

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Atti del convegno

[Precedente] [Successiva]