Generalised model of brittle-ductile transition

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Abstract

In this paper a FM generalised model is proposed to solve quantitatively the problems of brittle ductile transition as a function of dimensions and strain rate, starting from traditional parameters like Charpy V and Drop Weight Tests Energy. This energy measurements are converted to FM parameters as Ctod, Ctoa, J and T making possible the passage from a qualitative approach to a quantitative one. In this model, using a very simple, inexpensive and traditional test , is solved the problem of transferability of results from small specimens to full scale structures.
The model was tested satisfactorily on materials of different toughness .

Nomenclature

a	crack length
a_i	initiation crack length, may be different from a_0
a_{ca}	crack arrest length
a_0	initial crack length
cmod	crack mouth opening displacement (clip gauge displacement)
ctod	crack tip opening displacement
$ctod_c$	critical ctod
$ctod_L$	plain stain limit ctod; end of elastic field
$ctod_{LK}$	kinematics limit ctod; cusp point in the motion's centroids
$ctod_i$	ductile initiation ctod
ctoa	crack tip opening angle
C	calibration function
Cr	cristallinity (% of cleavage fracture)
B	thickness
F	applied load
F_e	experimental applied load
F_{pc}	plastic collapse load
F_{fm}	linear elastic load derived from LEFM
J	J - integral
K_{1c}	critical stress intensity factor
K_1	stress intensity factor
L	ligament
M	applied bending moment
neck	elongation of ligament due to localised lateral contraction (necking)
r_a	rotational function that locates the apparent centre of rotation
r_i	rotational function that locates the instantaneous centre of rotation
R_c	Priest's law parameter
R_{anvil}	radius of curvature of anvil
R_{tup}	radius of curvature of tup
s	load application displacement
S	span length
S_a	% of shear area
S_c	Priest's law parameter
SE	total fracture specific energy (energy divided by initial ligament area)
S_t	external specimen length
(x_f,y_f)	fracture profiles co-ordinates
(x_c,y_c)	centroids co-ordinates

t_3 temperatureat which Priest's law is determined

t_{cat} minimum temperature at which the fracture surface is 100% shear (crack arrest temperature)

t_{nsat} maximum temperature at which the fracture surface is 100% cleavage (nihil shear area temperature)

t_{ndt} maximum temperature at which the fracture initiate in plane strain condition i.e. elastic field (nihil ductility temperature).

U_t total absorbed energy

U_i initiation absorbed energy

Y Young's modulus

α specimen half rotation angle

α_f angle at which the specimen skips from the anvil

α_i angle of ductile initiation it was assumed only for blunted notches equal to ctoa/2

Ω calibration geometrical function

β_0 calibratioin dimension and geometrical function

σ_f flow stress

σ_y yield stress

θ_2 constant equal to 4/3

Introduction

The problem of brittle ductile transition is analysed using as inputs energy and cristallinity transition curve of two sets of different ligaments specimens. Where used all boundary conditions that came from the experiences gained in this field and with the help of a generalised model was found the compatibility between quantitative data and analytical Fracture Mechanics.

It is more easy with this model to give physical meaning to FM and understand the parameters that govern the cleavage and ductile fracture and the energy scale factors. This approach is important for simplify FM test methodologies and for giving input in FE analysis when the problem of transferability from small specimen to full scale structure is important.

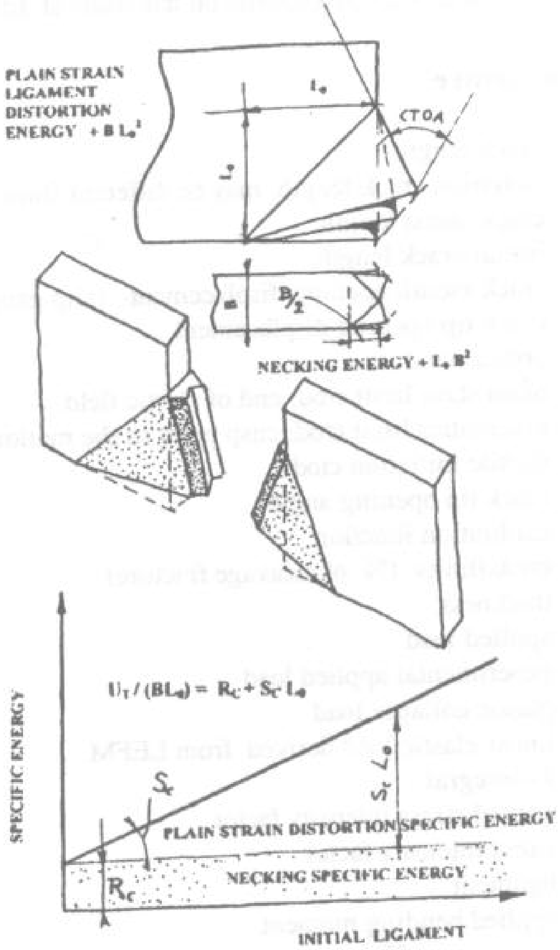

FIG. 1 PHYSICAL INTERPRETATION OF PRIEST'S LAW

1) Priest's Law

Priest's Law [2] [3] govern the energy scale factors for ductile fracture.

The law is given by the following relation:

$$SE = R_c + S_c (W - a_0) \qquad (1.1)$$

The specific energy SE (total energy divided by the initial ligament area) **is proportional to the ligament length through two constant R_c and S_c.** The total specific energy is divided into two components according the two constants, see fig.1.Both two constants depends on strain rate.They do not depend on thickness unless B (specimen thickness) tends to 0. **R_c depend on process zone (necking of specimen) S_c depend on plastification far from process zone i.e. ligament distortion deformation (far from the necking zone in plain strain condition). At least two diferent ligament spesimens are necessary for determining Sc and Rc**

2) Determination of CTOA

In this paper it was chosen for fracture parameters the geometrical ones because more easily to visualised and to understand from physical point of view. Later it is easy to pass to the energetic parameters like J (J integral) or T (tearing modulus)[4] [5].**The geometrical approach** using Ctod, Crack Opening Displacement and Ctoa , Crack Opening Angle, **makes possible the use of the Theory of Fracture Kinematics,** FK as a fundamental instrument [10] [11] [6] [8] (see Appendix A).According to this theory the specific ductile failure energy (see paragraph A.5) for a bending specimen is:

$$SE(t) \cong \theta_2 \sigma_f(t)(2\alpha_i/\tan(ctoa/2)+1/r_{a0})L_0\ tg[ctoa/2]/4 \qquad (2.1)$$

where α_i half of initation angle;$\theta_2 = 4/3$ is the constant for determining the plastic collapse load for bending;$r_{a0} = 0.4$ is the apparent rotational constant in the ctod calibration(see appendix) and $\sigma_f(t)$ is the flow stress at t temperature σ_f = 1.15 σ_y ;where σ_y is the yield stress. For dynamic tests a dynamic σ_{yd} must be assumed. Equating eq(3.1) and (1.1) one obtain ctoa(t):

$$ctoa = 2\ arctg\{[4\ R_c/L_0 + 4\ S_c]/[\ \sigma_f(t_3)\ \ \theta_2\ (\ 2\ \alpha_i/\tan(ctoa/2)+1/ra_0)\ ]\} \qquad (2.2)$$

where t_3 is the temperature at which is determined S_c and R_c for fatigue crack α_i is small compared ctoa/2;for blanted notch α_i may be equal to ctoa/2[12]. In this case eq (2.2) for low and medium toughness becomes:

$$ctoa \cong 2\ arctg\{[4\ R_c/L_0 + 4\ S_c]/[\sigma_f(t_3)\ \ \theta_2\ \ (2+1/r_{a0})]\} \qquad (2.3)$$

One can translate this energetic approach into a geometric one using as a link the flow stress.Eq (2.3) becomes:

$$ctoa = \ 2\ arctg[tg(ctoa_0/2) + neck/L_0] \qquad (2.4)$$

where $ctoa_0$ is the plane strain ctoa and is the parameter responsible for distortion energy of the ligament. It is given by:

$$ctoa_0(t) \cong \ 2\ arctg\{4\ S_c)/[\sigma_f(t_3)\ \ \theta_2\ \ (2+1/r_{a0})]\} \qquad (2.5)$$

and neck is the elongation of the ligament (displacement) due to localised lateral contraction (necking) responsible for the increment of energy respect to the one due to plane strain condition.Neck is given by:

$$neck(t) \cong 4\ Rc\ /[\sigma_f(t_3)\ \ \theta_2\ (2+1/ra0)] \qquad (2.6)$$

The ductile energy in plane strain SE_0 becomes according to eq (2.1)

$$SE_0 = \ \theta_2 \sigma_f(2\alpha_i/\tan(ctoa_0/2)+1/r_{a0})L_0\ tg(ctoa_0/2)\ /4 \qquad (2.7)$$

The value of ctoa is increased respect the plane strain one for the effect due to the localised lateral contraction of the ligament.

Ctoa in plain strain condition ($ctoa_0$) is a real material parameter (independent on dimensions temperature and strain rate).

Both ctoa and neck are not dependent on temperature, yield and strain rate.. Ctoa depend on initial ligament.
There are some experimental evidence produced at CSM [7] that shows the independence of ctoa from strain rate.The consequence of this is that **ductile specific energy depends on flow stress** (i.e. temperature strain rate), **ligament and ctoa**.

3) Generalised model of brittle ductile fracture with temperature

3.1) Flow stress variation with temperature

For the variation of flow stress with the temperature the following relation was assumed:

$$\sigma_f(t)=\sigma_{f0}\exp[-(273+t)/t_{\sigma f}] \tag{3.1.1}$$

Where σ_{f0} $t_{\sigma f}$ are two constants obtained by the values of σ_f determined at two different temperature one of which is t3

3.2) Brittle Fracture Initiation in Plane Strain Condition

Now the following hypothesis based on experience were assumed :
The steel has an elastic-plastic behaviour.One has practical plain strain condition when there are elastic condition.A ctod limit was established that defined this field, equating the load F_{fm} obtained from LEFM with the F_{pc} determined by PC (plastic collapse),taking into account the hypothesis of elasto-plastic material.The two theoretical loads are:

$$F_{fm}=K_1 B\sqrt{(W)}/f(a_0/W) \tag{3.2.1}$$

where K_1 is the Stress Intensity Factor in mode one and $f(a_0/W)$ is a FM calibration function.Choosing ctod instead of K_1 as fracture parameter, using the following relation

$$K_1=[Y\sigma_f(t)\,\text{ctod}]^{1/2} \tag{3.2.2}$$

where ctod is given (see also appendix A):

$$\text{ctod}=2\,r_a L_0\sin(\alpha)$$

where Y is the Young Modulus and r_a is a rotational function.
Starting from equality of internal and external work

$$F_{pc}\,ds=2\,M_{pc}\,d\alpha \tag{3.2.3}$$

where M_{pc} is the plastic collapse resisting bending moment, due to internal forces, **this time with the real ligament** given by:

$$M_{pc}=\theta_2\sigma_f(t)B[L_0/\cos(\alpha)]^2/4 \tag{3.2.4}$$

where F_{pc} is the external force applied and s is the displacement given by [8]:

$$s=S\,tg(\alpha)/2-(W+R_{tup}+R_{anvil})[1-\cos(\alpha)]/\cos(\alpha) \tag{3.2.5}$$

the derivative of s respect to α is:

$$ds/d\alpha=[S/2-(W+R_{tup}+R_{anvil})\sin(\alpha)]/\cos^2(\alpha) \tag{3.2.6}$$

From equation (3.2.3) one obtains:

$$F_{pc}=\theta_2\,\sigma_f(t)\,B\,L_0^2/\{2[S/2-(W+R_{tup}+R_{anvil})\sin(\alpha)]\} \tag{3.2.7}$$

for small value of α (i. e. medium and low toughness steels) eq (3.2.5) and (3.2.7) becomes:

$$s\cong S\,tg(\alpha)/2 \tag{3.2.8}$$

$$F_{pc}\cong\theta_2\,\sigma_f(t)B\,L_0^2/S \tag{3.2.9}$$

where S is the span length, R_{tup} and R_{anvil} the curvature radius of tap and anvil
The limit of plain strain conditions is reached when the critical LEFM load F_{fm} is equal to the PC one F_{pc}.So equating eq (3.2.1) and (3.2.9) using (3.2.2) one obtains the value of ctod at which this condition is reached:

$$ctod_L = \sigma_f(t)\,[f(a_0/W)\,\theta_2\,L_o^2]^2/(S^2 W\,Y) \quad (3.2.10)$$

The $ctod_L$ is geometry ,dimension and yield stress dependent, and thus also temperature and strain rate dependent.At this value of ctod the plane strain condition and the validity of LEFM end and the Plastic Collapse Failure Mode begins.Now a **critical ctod is defined, $ctod_c$, as the one at which cleavage fracture takes place.The $ctod_c$ is only function of temperature and is not dependent on dimensions even if the plane strain conditions are not respected. Also it seems that $ctod_c$ does not depend on specimen orientation (longitudinal or transverse according to experience)Then the condition of initiation of cleavage fracture in plane strain is:**

$$ctod_c = ctod \qquad ctod_c < ctod_L \qquad ctoa = 0 \qquad F = f(a_0) \quad (3.2.11)$$

When this condition occurs the fracture initiates in a brittle mode and ctoa is equal to 0. If $ctod_c$ is lower than $ctod_L$ the fracture is in plane strain and is valid LEFM.The load during this phase is given by eq (3.2.1), and thus is function of a_0 and $ctod_c$

3.3) Brittle Fracture Initiation after Plain Strain Condition

In this case **the condition of initiation of cleavage fracture in plane stress is:**

$$ctod_c = ctod \qquad ctod_L < ctod_c \qquad ctoa = 0 \qquad F \cong f(L_o) \quad (3.3.1)$$

The load in this case is given by eq (3.2.7) or (3.2.9), so is function of L and σ_f.

3.4) Ductile Fracture

Now **a ductile initiation ctod is defined, $ctod_i$** (see paragraph A.1),**as the one at which ductile fracture initiates** (This is an engineering model. There is a good agreement only if one considers the global propagation and ignores the crack growth transient initial part) **.When this condition occurs the fracture starts ductile** and the ctoa is different from 0 and the value is given by eq (2.4).To define a $ctod_i$ the initiation angle must be known.The hypothesis is assumed that for ductile fracture the initiation angle is lower or equal to ctoa depending how sharp is the notch [12].**For blunted notches ctoa was assumed equal to the initiation angle, the half of which will be called α_i .The model that follows is based on this assumption.Ductile fracture initiates when:**

$$ctod = ctod_i(L_o) \qquad ctod_c > ctod_i(L_o) \qquad ctoa > 0 \quad (3.4.1)$$

Where:

$$ctod_i(L_o) = 2\,r_a\,L_o \sin(\alpha_i) \cong 2\,r_{a0}\,L_o \sin(ctoa/2) \quad (3.4.2)$$

For determining $ctod_i$ see paragraph A.1); any case $ctod_i$ is ligament and ctoa dependent.

3.5) Load determination during initiation and propagation

The relations (3.2.5)(3.2.7),(3.2.8),(3.2.9) are valid for all points of propagation with a instead of a_0. **For initiation is more accurate to use:**

$$a = a_0 + ctod \sin(\alpha) \qquad a_i = a_0 + ctod \sin(\alpha_i) \qquad a_i \cong a_0 \quad (3.5.1)$$

where a_i is the **a** at initiation (in this case the crack extension is due only to geometrical variation of ligament not to crack growth).**During propagation a(α)** has to be obtained by integration of eq (a.4.1) Appendix A

$a(\alpha) = W - L_0 \cos(\alpha-\alpha_i)^{ra0} \exp[-r_{a0}(\alpha-\alpha_i)/tg(ctoa/2)]$

The simplified formula obtained from eq (a.4.2) is:

$a(\alpha) = W - L_0 \exp[-r_{a0}(\alpha-\alpha_i)/tg(ctoa/2)]$

$F_{pc}(\alpha) = \theta_2 \ \sigma_f(t) \ B \ L(\alpha)^2 / \{2[S/2-(W+R_{tap}+R_{anvil})\sin(\alpha)]\}$ (3.5.2)

Simplified expressions for load and displacement valid for low and medium toughness steels are:

$F_{pc}(\alpha) \cong \theta_2 \sigma_f(t) \ B \ L(\alpha)^2 / S$ (3.5.3)

$s \cong S/2 \ tg(\alpha)$ (3.5.4)

where L is the component of the ligament on the x axis, see fig A.3.

$L(\alpha) = W - a(\alpha)$ (3.5.5)

The real ligament is $L/\cos(\alpha)$

3.6) Arrest of Brittle Fracture

The condition of brittle propagation is

$ctod_c < ctod_i(L)$ (3.6.1)

The condition of ductile propagation is:

$ctod_c > ctod_i(L)$ (3.6.2)

In propagation $ctod_i$ is given by eq(3.4.2) changing L_0 in L :

$ctod_i(L) = 2 \ r_a \ L \sin(\alpha_i) \cong 2 \ r_{a0} \ L \sin(ctoa/2)$ (3.6.3)

For determining $ctod_i$ see Appendix 1.So because **a** grows during propagation $ctod_i$ decreases.**At a certain value of a there is a passage from the condition of brittle propagation (3.6.1) to the one of ductile propagation (3.6.2).**

The load also in this case is given by eq (3.5.2) or (3.5.3) if the fracture starts after plane strain condition .The load is given by eq (3.2.1) if fracture initiates in plane strain condition.**The crack arrest condition is:**

$ctod_c = ctod_{ca} = ctod_i(L)$ and $a = a_{ca}$ (3.6.4)

3.7) Experimental determination of $ctod_c$ as a temperature function

It was assuming the hypothesis that the mathematical function describing physically the increase of $ctod_c$ with the temperature is an exponential function of the following type:

$ctod_c(t) = g \exp(h \ t)$ (3.7.1)

Where the constants **g** and **h** must be determined statistically from the experimental cristallinity transition data, imposing the condition that the eq.(3.7.1) is the best fit of the points determined by the crack arrest condition eq.(3.6.4).During the ductile-brittle transition the remaining ligament L after cleavage propagation is:

$L = L_0 [100 - Cr_e(t)] / 100$ (3.7.2)

$ctod_c(t) = ctod_i(L) = 2r_a\{L_0[100-Cr_e(t)]/100\}\sin(\alpha_i) \cong 2r_{a0}L \sin(ctoa/2)$ (3.7.3)

Where $Cr_e(t)$, experimental cristallinity, is the percentage of cleavage fracture at **t** temperature.**All the value of $Cr_e(t)$ equal to 0 or to 100 must be taken away.**

4) Shear area determination

Considering the part of ligament in which was developed cleavage fracture delimited by a_i and a_{ca} one easily obtain that cleavage fracture percentage **Cr**, cristallinity, is:

$$Cr = 100\,[\,L_0 - (a_{ca} - a_i)\,]\,/L_0 \qquad Sa = 100 - Cr \qquad (4.1)$$

Where S_a is the shear area percentage

Remembering that PC loads are proportional to the square of the ligament ; it is possible to obtain the same formula substituting the square root of corresponding loads for instance from experimental diagrams.

5) Generalised FM relationships

5.1) Fundamental Fracture Parameters used in this paper

The following FM parameters where used. **It is important to note that fracture is deformation controlled and not stress controlled.**

Ctod responsible of the deformation at crack tip .

Ctod$_L$ maximum ctod at which the specimen is in a plane strain condition

Ctod$_c$ ctod at which cleavage fracture take place both in plane strain and plane stress. Also it seems that ctod$_c$ is not depending on specimen orientation (longitudinal or transverse)

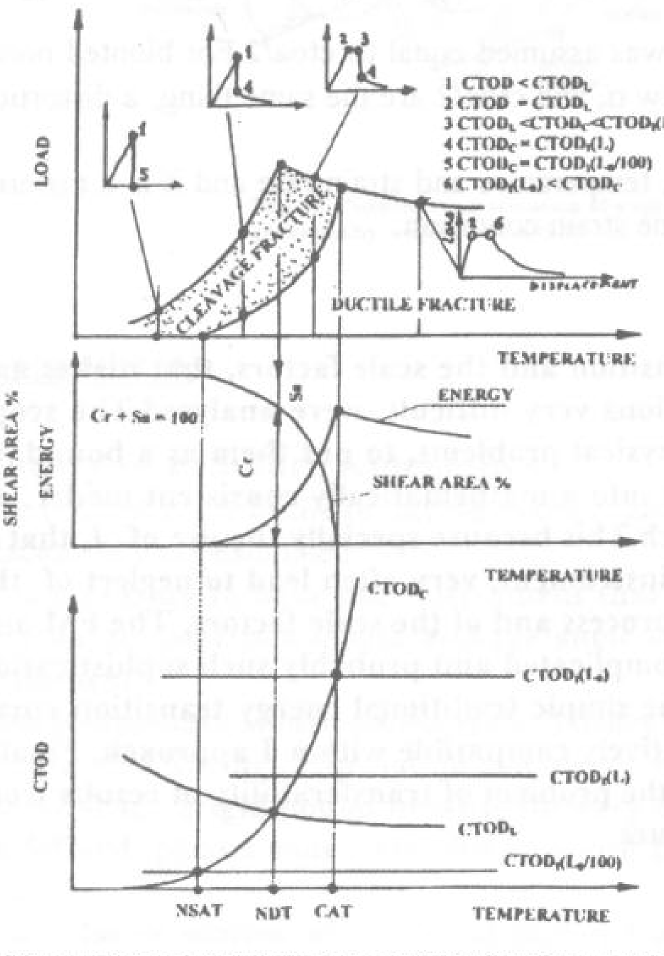

FIG. 2 PHYSICAL QUANTITATIVE INTERPRETATION OF BRITTLE -DUCTILE TRANSITION PHENOMENA

Ctod$_i$ ctod at which ductile fracture initiates.

Ctoa$_0$ responsible of ductile crack propagation through ligament in plane strain condition

Ctoa responsible of ductile crack propagation through ligament not in plane strain condition

Of course if ctoa is equal to 0 one has cleavage fracture propagation.

Fig. 2 reassume all the conditions and gives a physical quantitative interpretation of the brittle - ductile transition phenomena.

5.2) Equivalence of J and T with Ctod and Ctoa

It is possible to pass from ctod and ctoa to J and T

[8] and Appendix A.

$$J = \theta_2 \ \sigma_f \ L_0 \ \alpha = \theta_2 \ \sigma_f \ \alpha \ ctod \ / \ (2 \ r_a \sin(\alpha)) \cong \theta_2 \ \sigma_f ctod \ / \ (2 \ r_{a0}) \quad (5.2.1)$$

$$J_c \cong \theta_2 \ \sigma_f ctod_c \ / \ (2 \ r_a) \quad \text{and} \quad T = 0 \quad (5.2.2)$$

If $ctod_c$ is less than $ctod_L$ then J_c is a J_{1c} (plane strain condition).

$$J_i = \theta_2 \ \sigma_f \ L_0 \ \alpha_i = \theta_2 \ \sigma_f \ \alpha_i \ ctod_i \ / \ (2 \ r_a \ \sin(\alpha_i)) \cong \theta_2 \ \sigma_f ctod_i \ / \ (2 \ r_{a0}) (5.2.3)$$

$$J_r \cong J_i + \theta_2 \ \sigma_f (a-a_0) \ tg(ctoa/2) \ / \ r_{a0} \quad (5.2.4)$$

$$T \cong \theta_2 \ Y \ tg(ctoa/2) \ / (\ r_{a0} \ \sigma_f) \quad (5.2.5)$$

J_i J_r and T as well as $ctod_i$ and ctoa are temperature, strain rate, ligament and thickness dependent. For this reason they are not material parameters.

5.3) Scale Factors

σ_f is temperature and strain rate dependent.

Ctod for the same angle of rotation depend on a_0/W, L_0, Y, σ_f (temperature ,strain rate)

$Ctod_L$ has the same dependence of ctod

$Ctod_c$ is independent of geometry and dimensions and specimen's orientation, it depends only on temperature and it is a material parameter as K_{1c} , but it is valid both in plane strain and plane stress.

$Ctod_i$ has the same dependence of ctod. It is not dependent on temperature and strain rate, but is dependent on ctoa ligament and on notch severity.

Ctoa depends on dimensions and specimen's orientation ,not on temperature and strain rate.

α_i is proportional to ctoa and was assumed equal to ctoa/2.For blunted notch Probably from a physical point of view α_i and ctoa/2 are the same thing, a distortion angle.

$Ctoa_0$ is independent of dimensions, temperature and strain rate and it is a material parameter as K_{1c}. It is the ctoa in plane strain condition.

Conclusions

The problem of brittle ductile transition and the scale factors, that makes any interpretation of ductile deformations very difficult, were analysed.The scope of this paper is to return to the physical problems, to put them as a boundary condition and to transform them into a mathematically consistent model to be used in a finite element approach.This because specially the use of J, that is an extremely valid mathematical instrument, very often lead to neglect of the physical meaning of the fracture process and of the scale factors. The FM and J testing methodologies are very complicated and probably such sophistication is not necessary.For this reason the simple traditional energy transition curve were analysed and made quantitatively compatible with a J approach. Finally the model proposed tries to solve the problem of transferability of results from small specimens to full scale structure.

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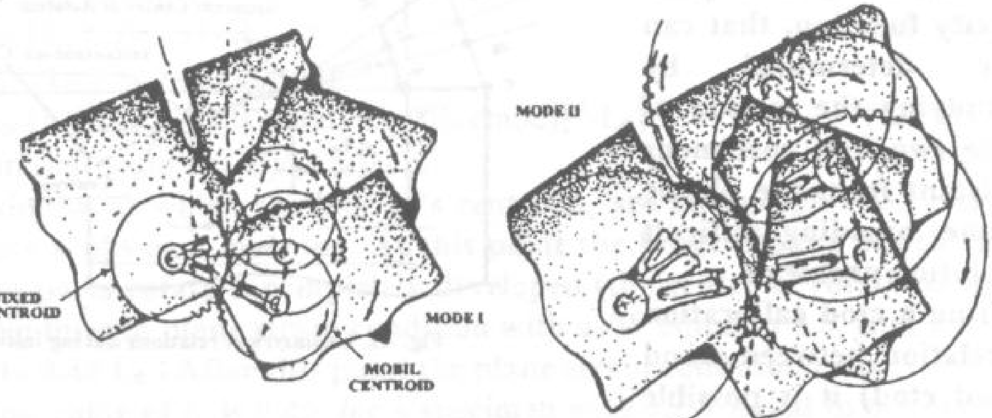

Fig. A1 Pictorial representation of a specimen during crack growth governed by motion's centroids

Appendix A

Theory of Fracture Kinematics

The condition for brittle or ductile fracture and the relation between loads and displacement were examined.

It is necessary to have one instrument that links FM parameters like, ctod or ctoa, to the crack length and the angle of rotation of the specimen i.e. displacement .

This mathematical instrument is the Theory of Kinematics of Rigid Bodies.

A specimen during deformation can be represented by the two sections in the elastic field (rigid bodies because the very high Young Modulus) linked by an undefined plastic zone . For this reason it is possible to apply the theory of FK.

The relative motion of two rigid bodies (one fixed on the other mobile) is completely defined when one has the motion's centroids.

The motion's centroids are the locus of the instantaneous centres of rotation.

The centre of instantaneous rotation is defined when one knows at the same time during the motion two different relative displacements, in a specimen for instance cmod and ctod.

Two type of centroids can be defined one fixed and one mobile. Both the centroids are fixed to their own rigid bodies (the two specimen's sections with the exception of the plastic part).

The motion of two rigid bodies is obtained making purely rotate the mobile centroid on the fixed one fig A.1.

The centroids assume in this theory the physical meaning of a plasticity function, that can be determined by studying the motion of the two half specimens distant from the plastic zone, avoiding difficult solution problems.

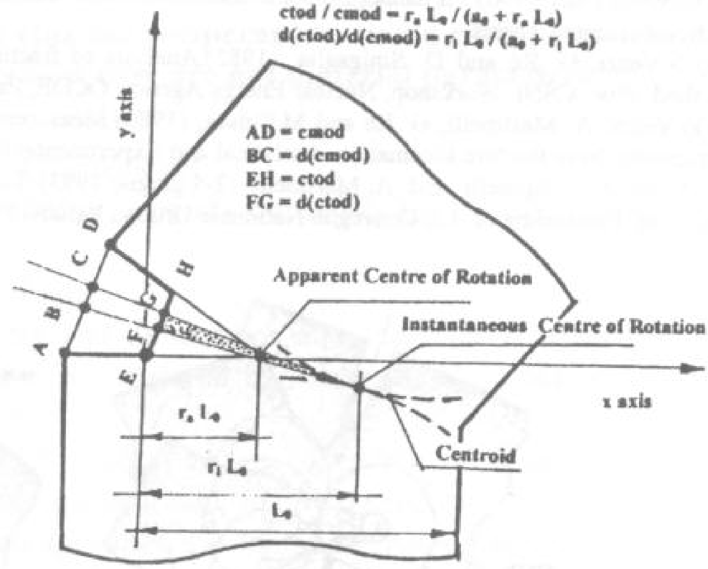

Fig. A2 Geometrical relations during initiation

From a ctod calibration (relation between cmod and ctod) it is possible to derive mathematically the motion's centroids during initiation.The distance of the instantaneous centre of rotation r_i L_o, during initiation may be obtained, instant by instant from eq. (a.1.1) and (a.1.4) The ctod's calibration and particularly the rotational function r_a becomes a plasticity function that can be determined experimentally avoiding the crack tip singularity.During propagation the following hypothesis must be assumed:

The fracture surfaces form with the symmetry axis an angle equal to half of ctoa.Ctoa is constant during all propagation. This hypothesis is engineering consistent with experimental evidence [12][8].The instantaneous centre of rotation is on the symmetry axis at a distance equal r_i L; r_i can be assumed equal to r_{a0}, so also in this case it easy to obtain the centroids.Finally it is possible to obtain from the broken specimen's profiles ctoa, ctod and load displacement curve applying all the hypothesis done see fig A.4.From one of the four diagrams of fig A.4 is possible to obtain the other three.

A.1) Ctod calibration

With reference [8] [10][11] to fig.A.2

$$\text{d(ctod) / d(cmod)} = \Omega_i \quad \text{(a.1.1)}$$

$$\text{ctod / cmod} = \Omega_a \quad \text{(a.1.2)}$$

where the index i and a indicate instantaneous and apparent and cmod the crack mouth opening displacement, (the opening of the clip gauge).

$$\Omega_a = [1 + a_0 / (r_a L_0)]^{-1} \quad (a.1.3)$$
$$\Omega_i = [1 + a_0 / (r_i L_0)]^{-1} \quad (a.1.4)$$

the limit of Ω_a for cmod going to infinite is Ω_{a0}.

$$\Omega_{a0} = [1 + a_0 / (r_{a0} L_0)]^{-1} \quad (a.1.5)$$

r_{a0} rotational constant has a value of 0.4 for specimens with a ratio a_0/W in the range of 0.4 , 0.7

The experimental calibration is given by [8] [10] [11]:

$$\mathbf{ctod = \Omega_a\, cmod} \quad (a.1.6)$$
$$C = 1 - \exp(-\beta_0\, cmod) \quad (a.1.7)$$
$$\beta_0 = 4\, \Omega_{a0} Y / (\pi\, \sigma_y L_0) \quad (a.1.8)$$

from previous equations and from fig.(A.2) one obtains:

$$ctod = 2\, r_a L_0 \sin(\alpha) \quad (a.1.9)$$
$$r_a = a_0\, \Omega_{a0}\, C / [L_0 (1 - \Omega_{a0} C)] \quad (a.1.10)$$
$$r_i = a_0 / \{L_0 [d(cmod)/d(ctod) - 1]\}$$
$$= a_0 / \{L_0 [\Omega_{a0} (1 - (1 - \beta_0\, cmod) \exp(-\beta_0\, cmod)]^{-1} - L_0\} \quad (a.1.11)$$
$$\alpha = \arcsin[(cmod - ctod)/(2\, a_0)] \quad (a.1.12)$$

r_i has a maximum at which the motion's centroid, see paragraph A.3, has a cusp that has a physical meaning. At this point the hinge around which the two specimen parts rotate is completely developed (the centre of rotation, according to slip lines in plane strain condition with a_0/W equal to 0.5, is at a distance equal to 0.45 L_0).After this point the plane strain condition ends .
The maximum value of r_i is 0.45 for a specimen with a_0/W equal to 0.5. After r_i and r_a tends to the same limit that is r_{a0}; practically for ductile material during propagation r_i r_a r_{a0} coincide.The cusp condition is reached when $dr_i / d(cmod) = 0$ and $\beta_0\, cmod = 2$. At this point is defined a $ctod_{Lk}$ limit kinematics at which is given a physical interpretation of end of plane strain condition. In the past the physical interpretation of beginning of ductile initiation was also given [11]. During deformation between $ctod_{lk}$ and $ctod_i$ ctoa decrease from 180 degree
to the constant value of propagation with a very small crack growth. From engineering point of view it was assumed initiation at $ctod_i$ with constant propagation ctoa and was ignored the crack growth transient phenomena.

$$\mathbf{ctod_{Lk} = 2\, \Omega_{a0} (1 - \exp^{-2}) / \beta_0} \quad (a.1.13)$$

remembering eq(3.2.10)

$$\mathbf{ctod_L = \sigma_f(t)\, [f(a_0/W)\, \theta_2\, L_0^2]^2 / (S^2 W\, Y)}$$

(3.2.10)

equating $ctod_{Lk}$ (kinematics)to **$ctod_L$** (theorical PC load =LEFM load) **one can obtain a theoretical β_{0T}**, instead of the experimental one β_0 eq (a.1.8), **and this makes compatible the kinematics calibration with LEFM laws.**

$$\mathbf{\beta_{0T} = 2\, \Omega_{a0} (1 - \exp^{-2}) / ctod_L} \quad (3.2.11)$$

The $ctod_i$ at the initiation is eq. (a.1.9):

$$\mathbf{ctod_i = 2\, r_a L_0 \sin(\alpha_i)} \quad (a.1.14)$$

There is some evidence [12] that ctoa and 2 α_i can be the same angle unless either a sharp notches or brittle fracture makes start fracture before **α_i =ctoa/2**

A.2)Fracture Profiles

Assuming x,y axis as in fig A.2 the fracture profiles co-ordinates during initiation remembering eq(3.5.1) are:

$$x_f = a - a_0 = ctod \sin(\alpha) / 2 \qquad x_f = ctod \cos(\alpha) / 2 \qquad (a.2.1)$$

during propagation:

$$x_f = x_{fi} + a - a_i \qquad y_f = y_{fi} + \int (tg(ctoa/2-\alpha)\, da \qquad (a.2.2)$$

A.3) Motion's Centroids

Assuming x,y axis as in fig A.2 the centroids co-ordinates during initiation are:

$$x_c = x_f + r_i L \cos(\alpha) \qquad y_c = y_f - r_i L \sin(\alpha) \qquad (a.3.1)$$

during propagation

$$x_c = x_f + r_{a0} L \cos(\alpha) \qquad y_c = y_f - r_{a0} L \sin(\alpha) \qquad (a.3.2)$$

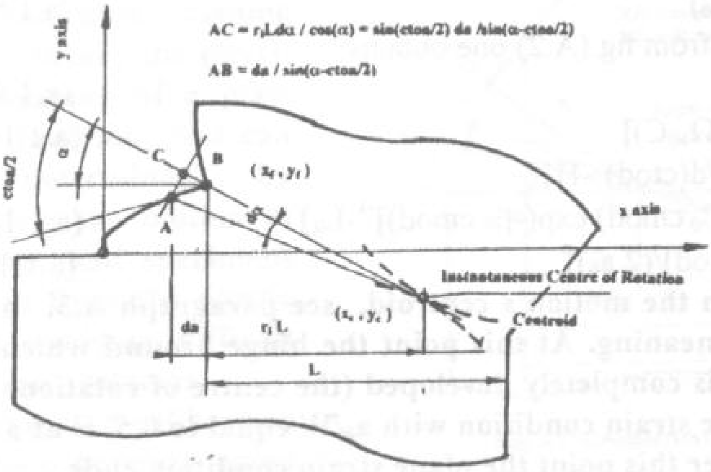

Fig. A3 Geometrical relations during propagation

The centroids, for ductile materials, have two cusps that have the two following physical meaning: a) End of linear elastic field (plane strain condition) b)Geometrical decreasing of the ligament during initiation or crack extension.

A.4) Crack propagation

With reference to fig. A.3, remembering the hypothesis of propagation it possible to derive from simple geometrical consideration the **propagation fundamental formula:**

$$d\alpha/da = [\sin(ctoa/2)\cos(\alpha)]/[\cos(ctoa/2-\alpha) r_i L] \qquad (a.4.1)$$

A simplified formula is:

$$d\alpha/da \cong tg(ctoa/2)/(r_i L) \qquad (a.4.2)$$

where ri is the instantaneous rotational function taken at the moment of initiation For ductile materials r_i can be taken constant equal to r_{a0}. The ctoa also can be assumed constant for all propagation, there is much evidence of this [12].

The integration of differential eq.(a.4.1) and (a.4.2) from $\alpha=\alpha_i$ to $\alpha=\alpha_f$ must be performed.Where α_f is half of the rotation angle at which the specimen, not completely broken, skips from the anvil. The geometrical relation is:

$$a_f = [S / 2 - R_{app} - (S_t/2 + y_f(\alpha_f)) \cos(\alpha_f)] / \sin(\alpha_f) \qquad (a.4.3)$$

Where S_t, total length of the specimen and $y_f(\alpha_f)$ is one of the co-ordinate of fracture profiles, see paragraph A.2.Where a_f has to be obtained by integration of eq (a.4.1)

$$a = W - L_0 \cos(\alpha_f - \alpha_i)^{ra0} \exp[-r_{a0} (\alpha_f - \alpha_i)/tg\,(ctoa/2)] \qquad (a.4.4)$$

The simplified formula obtained from eq (a.4.2) is:

$$a = W - L_0 \exp[-r_{a0} (\alpha_f - \alpha_i)/tg\,(ctoa/2)] \qquad (a.4.5)$$

α_i could be taken equal to **ctoa / 2**

A.5) Specific energy determination

A.5.1) Initiation specific energy

Starting from

$$d(SE) = 2\,M_0 / (BL_0)\, d\,\alpha \qquad (a.5.1.1)$$

where

$$M = \theta_2\, \sigma_y\, B\, L_0^2/4 \qquad (a.5.1.2)$$

One obtains:

$$\mathbf{SE_i = \theta_2\sigma_y\, L_0\, \alpha_i /2} \qquad (a.5.1.3)$$

from eq.(a.1.14) multiplying and dividing for $2\, r_a \sin(\alpha_i)$ one obtain the classical FM relation :

$$\mathbf{SE_i \cong \theta_2\sigma_y\, ctod_i\, \alpha_i /(4\, r_a \sin(\alpha_i)) \cong \theta_2\sigma_y\, ctod_i /(4\, r_{a0})} \qquad (a.5.1.4)$$

A.5.2) Propagation Energy

Starting from:

$$d(SE_p) = 2\,M\,(d\alpha /da)\; /(BL_0)\; da \qquad (a.5.2.1)$$

where

$$M = \theta_2\, \sigma_y\, B\, L^2/ 4 \qquad (a.5.2.2)$$

for eq. (a.4.2) we obtain:

$$d(SE_p) = 2\,M\; tg(ctoa / 2) / (r_i\; B\, L\, L_0)\, da \qquad (a.5.2.3)$$

Integrating with ctoa and r_i constant from $a=a_0$ to W remembering that $da = -dL$, one obtains:

$$\mathbf{SE_p = \theta_2\, \sigma_y\;\; L_0\; tg(ctoa/2) / (4\, r_i)} \qquad (a.5.2.4)$$

the total specific energy $SE = SE_i+SE_p$,assuming that for small angles the tangent is equal to the angle, and that r_i for ductile propagation can be taken equal to r_{a0} is:

$$\mathbf{SE = \theta_2\sigma_y\, L_0\, (\, 2\, \alpha_i/tan(ctoa/2) + 1/r_{a0})\; tg(ctoa/2)\; /4} \qquad (a.5.2.5)$$

remembering Priest's Law:

$$\mathbf{SE = R_c + S_c\, L_0}$$

one obtains:

$$\mathbf{ctoa = 2\; arctg\; \{(R_c+S_c\, L_0)/[\theta_2\, (2\alpha_i/tan(ctoa/2)+1/ra_0)\; \sigma_y L_0/4]\}} \qquad (a.5.2.6)$$

for fatigue crack α_i is small compared ctoa/2for blanted notch α_i may be equal to ctoa/2. In this case eq (a.5.2.6) for low and medium toughness becomes:

$$\mathbf{ctoa \cong 2\; arctg\{(R_c+S_c\; L_0)/[\theta_2\; (\; 2\; +1/ra_0)\; \sigma_y L_0/4]\}} \qquad (a.5.2.7)$$

In paragraph 2 the formulas are different because they take into account the modified Priest's Law and also it was used instead of σ_y, σ_f

A.5.3) Methodology for determining α_i and Ctoa from experimental load-displacement diagrams

With reference to [8] it is possible obtain directly from a load displacement curve ctoa and α_i. The relation is the following.

$$\mathbf{ctoa = 2\; arctg\; [r_{a0}\; (U_t-U(s)) \;/\; (F_e(s)\; S/4)]} \qquad (a.5.3.1)$$

Where U_t is the total energy U(s) is the energy adsorbed at s displacement and $F_e(s)$ is the experimental load taken from a load -displacement curve at s displacement. $F_e(s)$ must be taken during propagation and it's value must be approximately half of the maximum load.

For eq.(a.5.2.4)

$$U_t = U_i + U_p = \;U_i + SE_p\, L_0\, B = Ui + \theta_2\sigma_y\; B\; L_0^2\; tg(ctoa/2) /(\,4\; r_{a0}) \qquad (a.5.3.2)$$

one obtains:

$$U_i = U_t - \theta_2 \sigma_y B L_0^2 \, tg(ctoa/2) / (4 \, ra_0) \quad (a.5.3.4)$$

For eq (5.1.3)

$$\alpha_i = U_i / (\theta_2 \sigma_y L_0^2 B / 2) \quad (a.5.3.5)$$

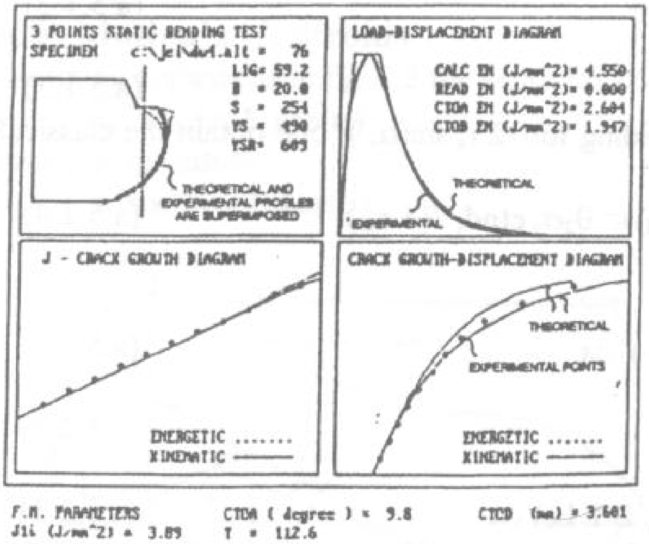

Fig. A4 Analysis of load-displacement curve using FK. It is possible to see the agreement between the experimental and the theoretical data.

Analysing with this methodology different ligament specimens of different steel it was possible to verify the assumptions of this paragraph and that for some conditions α_i is near half of ctoa.

Was also possible at the same time derive load-displacement, crack extension-displacement, fracture profiles and J_r-crack extension curve.

The agreement of experimental with teorethical results obtained from FK, having as input ctoa and α_i, is good [8] [12] see fig.A.4.

It is also possible to demonstrate the experimental evidence that load-displacements curve are omothetic [6] and that the energetic relation during propagation is:

$$L(s) = L_0 \, [(U_t - U(s))/(U_t - U_i)]^{1/2} \quad (a.5.3.6)$$