Strength Degradation Analysis of Notched Concrete Beam Based on Numerical Simulation of Bending Test under Sequential Loads

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ABSTRACT. *Initial flaws and defects exist in all engineering materials and thus in all structural members; only the degree of imperfection varies. Under cyclic loading, the material weakening process of a structural member inevitably involves multiple cracking originated from some of these spatially distributed initial imperfections, and therefore diverse cracking behaviors can be expected. As a threshold value of crack propagation is approached, new cracking behaviors can abruptly emerge and replace the previous ones, causing strength degradation. In the present study, by applying sequential loads at different locations of the same FE model of a notched beam, it is shown that this unique strength degradation mechanism can repeatedly occur amid a variety of cracking behaviors, leading to a multistage, discontinuous reduction of the load-carrying capacity.*

INTRODUCTION

In testing and analysis of various fatigue problems, the time-varying nature of cyclic loads on structures induced by winds, waves, vehicles, etc. is often subjected to oversimplification with loads of varying amplitudes but fixed loading positions. Experimental studies have shown that during fatigue tests of reinforced concrete beams and plates, the use of a moving cyclic load in a simulation of traffic loads may cause a reduction in the maximum load obtained under fixed-point monotonic or cyclic load conditions [1, 2]. Though these experimental observations have shed some light on the effects of changing loading positions during cyclic loading on the load-carrying capacity of a structure, the exact cause of the reduction is not well understood.

There is no doubt that fatigue mechanisms are complicated, and it has become increasingly clear that the phenomenon is closely related to multiple-crack activities during cyclic loading. But exactly how the degradation of material strength of a structural member takes place during cyclic loading amid various cracking activities remains to be clarified. In a series of numerical studies in which the size of a specific notch was enlarged incrementally, the existence of a critical size at which the fracture process changed abruptly was found, and a significant reduction of the maximum load was obtained with the new failure mode [3]. To provide experimental evidence for the transition of failure mode and the subsequent reduction of the load-carrying capacity, an experimental study was recently conducted [4].

As shown in Fig. 1, the tests focused on the maximum loads of notched concrete beams under four-point bending and the corresponding failure modes, and a multistage strength degradation relation was obtained by arbitrarily increasing the sizes of the initial notches. Numerical analyses were also carried out to reproduce the fracture processes and obtain the maximum loads, which both compared well with the test results. The importance of this study is twofold. Firstly, it reconfirms the previous findings that, for a given load condition the potential failure mode and the maximum load are insensitive to the size of an initial notch, provided it is less than a critical value. With a larger notch than this threshold value, however, a drastic change in the failure mode takes place, and the load-carrying capacity can drop significantly. Secondly, it shows that this strength degradation process can repeat itself, i.e., the reduction of the load-carrying capacity can take place in multiple stages, and at each stage a very different cracking behavior governs the fracturing processes while the strength of the previously weakened material remains basically constant.

It is known that initial flaws and defects exist in all engineering materials and thus in all structural members; only the degree of imperfection varies. Under actual cyclic loading, not only the amplitude but also the loading position may change. Eventually, the material weakening process of a structural member caused by repeated loading inevitably involves multiple cracking originated from some of these spatially-distributed initial imperfections, and diversified cracking behaviors can be expected. In a numerical simulation by applying sequential loads at different loading points, cracks can propagate

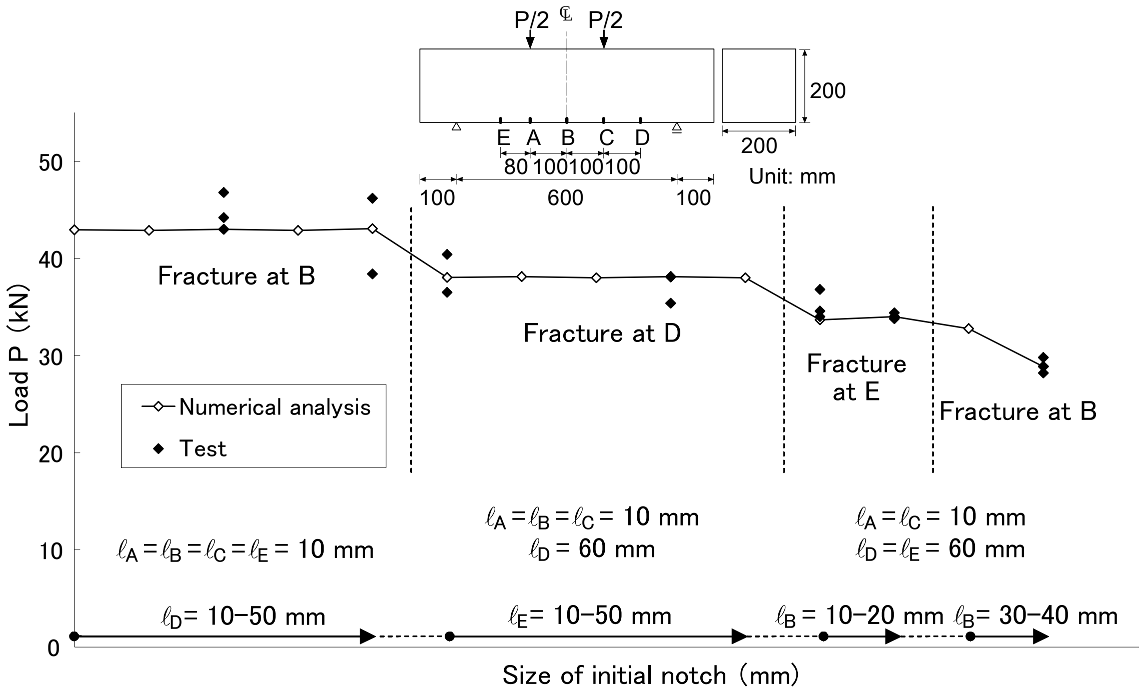


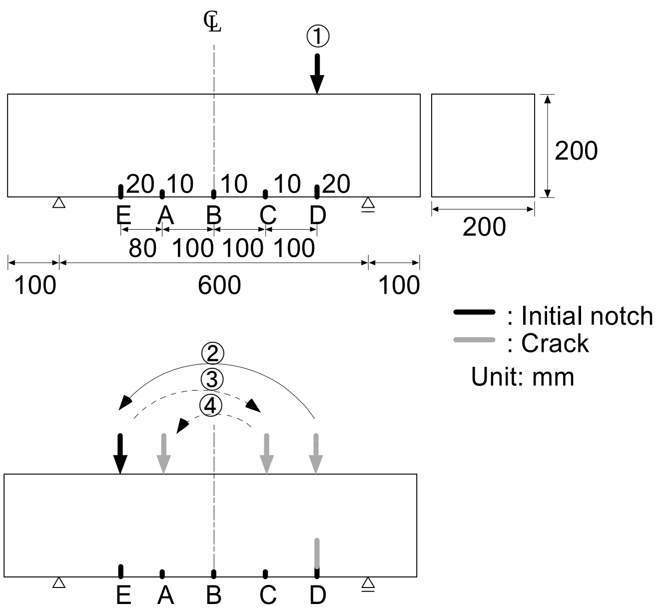
Figure 1. Numerical and experimental results on multistage strength degradation due to change of failure mode in bending tests of notched concrete beams [4]

from various locations of the notched beam to simulate a variety of cracking behaviors under cyclic loading. Thus, the present study aims to show that the multistage strength degradation theory, which in principle cannot be verified directly on the same test specimen by experiments (because the load-carrying capacity can only be obtained by actually fracturing that test specimen), can indeed be corroborated based on the results of these numerically-simulated crack propagations. Note that by applying a single load sequentially at different locations of the same specimen, the stress tensors at the tip of a crack rotate.

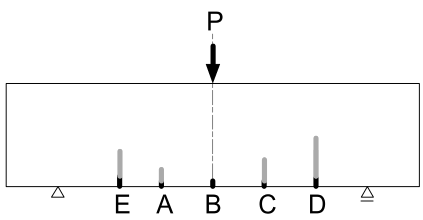
SCHEME OF NUMERICAL STUDY

As a further investigation on the multistage strength degradation phenomenon, the same notched beam problem as in the previous study [4] discussed above is selected as illustrated in Fig. 2. As seen, the beam contains five initial notches equally spaced at an interval of 100 mm, except the spacing between notches A and E which is deliberately set at a smaller value of 80 mm to embed an unsymmetrical effect in the problem. The notch size for notches A, B and C is assumed to be 10 mm, and for notches D and E it is set at 20 mm. For further geometric details of the problem, refer to Fig. 2.

To propagate cracks sequential loads are applied above the initial notches in four loading steps, following a random order of D, E, C and A, as shown in Fig. 2(a). In each



(a) Geometries of a notched beam and the sequence of loading



(b) Computation of load-carrying capacity of a notched beam with pre-existing cracks

Figure 2. Scheme of numerical studies on a notched beam

loading step, a single load is applied to extend the crack from the notch below to a certain length before moving to the next loading point. Obviously, during each of the four loading processes multiple cracks may propagate from other notches, too. At each step of crack growth the load-carrying capacity of the beam under three-point bending is evaluated through crack analysis, as schematically illustrated in Fig. 2(b) where the cracks at the initial notches are treated as pre-existing cracks for strength evaluation.

To carry out crack analysis the extended fictitious crack model (EFCM), which analyzes multiple cracks discretely based on relevant cracking modes, is employed; for details of the EFCM, refer to [4]. To facilitate the convergence of numerical solutions under sequential loading, a crack path from an initial notch is fixed and is assumed to be vertical. The material properties employed for the present study are summarized in Table 1, which include the elastic modulus E, Poisson's ratio ν, the compressive strength f_c, the tensile strength f_t, and the fracture energy G_F.

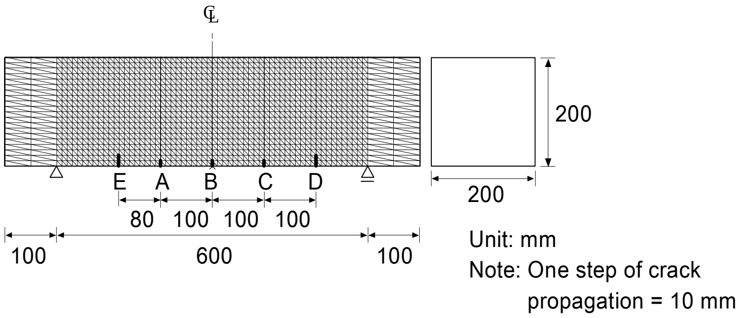
Table 1. Material properties of a notched beam

E (kN/mm^2)	ν	f_c (N/mm^2)	f_t (N/mm^2)	G_F (N/mm)
26.6	0.2	37.5	2.86	0.1

RESULTS AND DISCUSSION

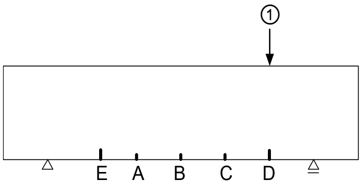
Figure 3 presents four selected scenes of crack propagation as examples to summarize the results of crack analysis under sequential loading and three-point bending, respectively. Note that these cases are connected to two threshold regions where the transition of failure modes and decrease of the load-carrying capacity under three-point bending take place, as the size of a propagating crack approaches a critical value. The obtained relation between the load-carrying capacity and the crack length is shown in Fig. 4, which clearly bears a multistage strength degradation characteristic.

In case 1 of Fig. 3(a), with a single load applied above notch D a crack propagates from that notch for six steps, accompanied by a one-step growth of a crack from notch C. No cracks emerge from other notches. Based on the FE mesh shown in the same figure, it is known that one step of crack growth equals the spacing between two neighboring nodes, i.e., 10 mm. The initial conditions set for calculating the load-carrying capacity under three-point bending are shown in the notched beam with two pre-existing cracks, i.e., a crack of 10 mm at notch C (one step of propagation) and a crack of 60 mm at notch D (six steps of propagation). To account for the cumulative damage effect by cyclic loads on the crack surface, the cohesive forces at the pre-existing cracks are deliberately ignored in calculating the beam strength. The results of crack analysis show that the potential failure mode is beam fracture at notch B, and there is no strength degradation of the beam up to this stage of crack propagation, as shown in Fig. 4.



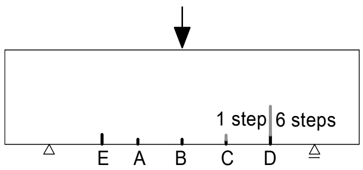
Case 1

Crack propagations due to sequential loading



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	0	0	1
	0	0	0	1	2
	0	0	0	1	3
	0	0	0	1	4
	0	0	0	1	5
	0	0	0	1	6

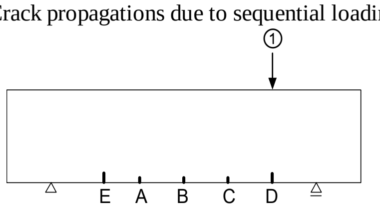
Computation of load-carrying capacity under three-point bending



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	1	1	1
	0	1	2	1	2
	1	1	4	1	3
	1	1	5	1	3
	1	1	6	1	2
	0	0	7	0	2
	0	0	8	0	2
	0	0	9	0	1
	0	0	10	0	1
	0	0	11	0	1
	0	0	12	0	1
	0	0	13	0	1
	0	0	16	0	0
	0	0	17	0	0
	0	0	18	0	0

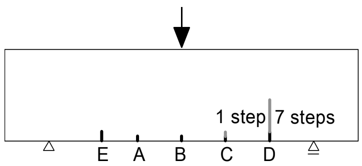
Case 2

Crack propagations due to sequential loading



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	0	0	1
	0	0	0	1	2
	0	0	0	1	3
	0	0	0	1	4
	0	0	0	1	5
	0	0	0	1	6
	0	0	0	0	7

Computation of load-carrying capacity under three-point bending

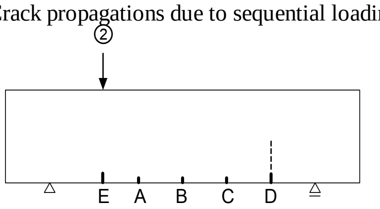


Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	1	0	1
	0	1	3	1	3
	0	1	3	1	4
	0	1	2	1	5
	0	0	2	0	6
	0	0	1	0	7
	0	0	1	0	8
	0	0	0	0	9
	0	0	0	0	10

Figure 3(a). Selected cases of crack propagations under sequential loading and computation of load-carrying capacity

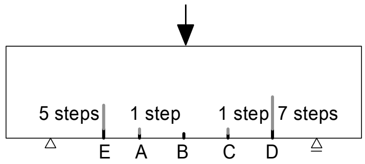
Case 3

Crack propagations due to sequential loading



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	0	0	1
	0	0	0	1	2
	0	0	0	1	3
	0	0	0	1	4
	0	0	0	1	5
	0	0	0	1	6
	0	0	0	0	7
	1	0	0	0	0
	2	1	0	0	0
	3	1	0	0	0
	4	1	0	0	0
	5	1	0	0	0

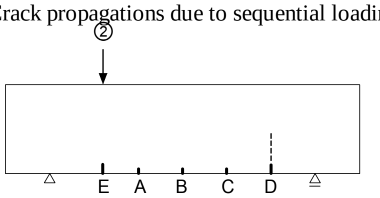
Computation of load-carrying capacity under three-point bending



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	1	0	1	0	1
	3	1	3	1	3
	3	1	3	1	4
	3	1	2	1	5
	2	0	1	0	6
	2	0	1	0	7
	1	0	0	0	8
	1	0	0	0	9
	0	0	0	0	10

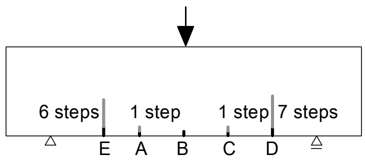
Case 4

Crack propagations due to sequential loading



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	0	0	0	0	1
	0	0	0	1	2
	0	0	0	1	3
	0	0	0	1	4
	0	0	0	1	5
	0	0	0	1	6
	0	0	0	0	7
	1	0	0	0	0
	2	1	0	0	0
	3	1	0	0	0
	4	1	0	0	0
	5	1	0	0	0
	6	0	0	0	0

Computation of load-carrying capacity under three-point bending



Crack	E	A	B	C	D
Nodal step of crack propagation	0	0	0	0	0
	1	0	1	0	1
	2	0	1	1	2
	3	0	2	1	3
	4	0	2	1	3
	5	0	2	1	2
	6	0	1	1	2
	7	0	1	0	2
	8	0	0	0	1
	9	0	0	0	1
	10	0	0	0	0
	11	0	0	0	0

Figure 3(b). Selected cases of crack propagations under sequential loading and computation of load-carrying capacity

In case 2 of Fig. 3(a), if crack D propagates one step further as compared to case 1 under the increasing load at notch D, the failure mode of the notched beam will abruptly change and the beam will break up at notch D. As shown in Fig. 4, this failure mode transition triggers a simultaneous decrease of the load-carrying capacity by approximately 5% from the previous case.

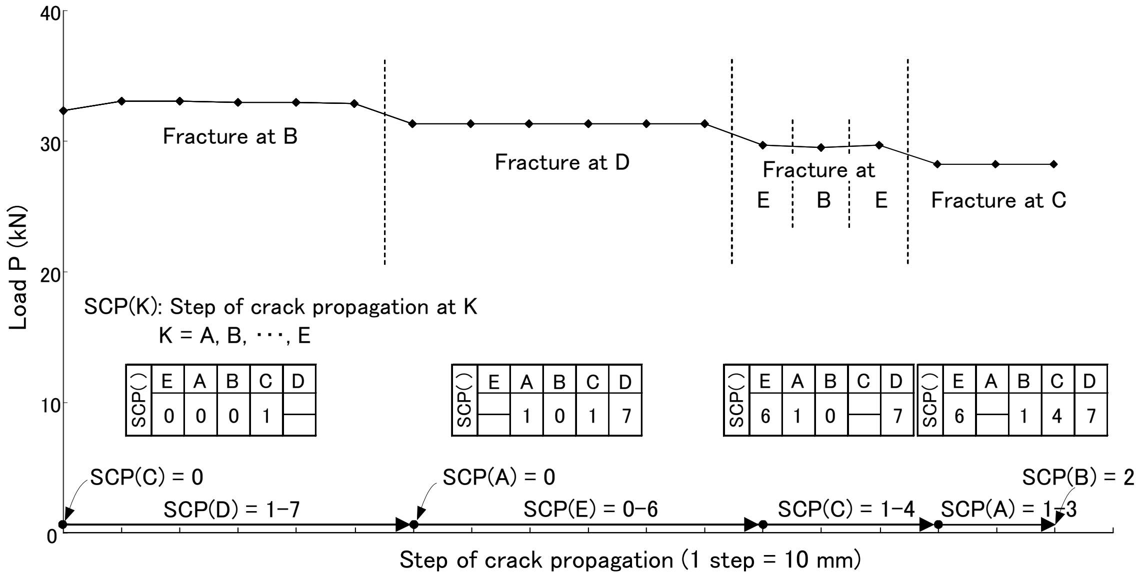


Figure 4. Numerical results on multistage strength degradation due to crack propagation and change of failure mode in a notched beam

Case 3 and case 4 of Fig. 3(b) display cracking behaviors of the beam in the second stage of the sequential loading, i.e., while retaining the previous crack history, the sequential load is now moved to notch E to propagate cracks from that side of the beam. As a result, two new cracks emerge at notches E and A, while the older cracks at notches D and C are forced to close due to the rotation of stress tensors at the tips of these cracks. As with cases 1 and 2, cases 3 and 4 are also two consecutive cases in terms of crack growth: Crack E propagates five steps in case 3 and six steps in case 4. In both cases crack A undergoes only one step of growth.

Since material damage due to fracture is irreversible, it is clear that in calculating the load-carrying capacity of the beam the older cracks from the previous loading history should also be treated as pre-existing cracks as with the new cracks. Therefore, the notched beam under three-point bending contains four pre-existing cracks in cases 3 and 4, as shown in Fig. 3(b). The sizes of the pre-existing cracks in the order of E, A, C, D are 50 mm, 10 mm, 10 mm and 70 mm for case 3, and 60 mm, 10 mm, 10 mm and 70 mm for case 4. As is observed from the results of crack analysis under three-point bending, the beam experiences a transition in failure mode as the sequential load extends crack E from five steps in case 3 to six steps in case 4, i.e., from a notch D fracture to a notch E failure. As shown in Fig. 4, the load-carrying capacity remains constant up to the fifth step of growth by crack E, and then drops by approximately 5% at the crack's sixth step of growth.

As the sequential loading continues at notch C and then at notch A to propagate more cracks at different locations, the load-carrying capacity of the beam is shown to decrease in a similar fashion. For details of the numerical analyses, refer to Fig. 4. Obviously, the obtained multistage strength degradations can continue as more loads are added to the sequential loading. In reality, however, this process of strength degradation under repeated loading can not continue forever, because an abrupt structural failure will terminate the process when the much reduced load-carrying capacity of the beam can no longer sustain the design loads.

As is known, concrete is a heterogeneous material consisting of aggregates and cement pastes bonded together at the interface, and the material is inherently weak in tension due to the limited bonding strength and various pre-existing microcracks and flaws formed during hardening of the matrix. As such, fracture of concrete involves complicated micro-failure mechanisms that include microcracking, crack deflection, crack branching, crack coalescence and debonding of the aggregate from the matrix. Obviously, each of these micro-failure mechanisms can extensively develop under cyclic loads, and their transitions (from one form to another in hierarchical order of crack formation) at certain threshold points of crack propagation should signify the emergence of new cracking behaviors and new failure modes. Based on this analysis and the results of the present study, it seems reasonable to conclude that the material weakening process of concrete caused by cyclic loading possesses a multistage strength degradation characteristic.

CONCLUSIONS

1. The material weakening process of concrete under cyclic loading should possess a multistage strength degradation characteristic. The strength degradation takes place amid multi-crack activities that involve changing of the cracking behavior and the potential failure mode.
2. Under cyclic loading a threshold value of crack propagation exists, and the load-carrying capacity of a structural member can remain constant until this region is approached. Beyond it, a new failure mode emerges and a reduction in the load-carrying capacity occurs.
3. This strength degradation can take place in multiple stages, and at each stage a very different cracking behavior dominates the fracturing process while the maximum load remains basically unchanged. This process of strength degradation can continue until the remaining material strength of the structure can no longer sustain the level of stress produced by the design load, leading to abrupt structural failure.

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