Rotating Bending with Constant Torsion and Rotated Bending with Constant or Variable Torsion

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ABSTRACT. *A unique fatigue testing machine was created for rotating bending with constant torsion that is multiaxial non-proportional loading. The machine is based on a statically indeterminate shaft system and thus no working power is consumed but only power on dissipation. Another more idea for technical equipment for bending with torsion, the idea of inversion, was implemented: the specimen is immovable but the (plane of) bending is rotated. Again no working power is consumed. Thanks to the specimen's immovability, essential advantages of this technical solution are achieved. Besides, cyclic torsion can also be produced by means of a simple built-in cam mechanism. Fatigue testing results are presented indicating that adding constant torsion to rotating bending produces (to some extent) an effect of increase in the fatigue life instead of decrease. This effect could not be revealed by means of some known fatigue life evaluation methods but by means of the IDD method (Integration of Damage Differential). The IDD application done and verification of the IDD parameters* f_c, f_τ, N_c *and* N_τ *are presented.*

INTRODUCTION

In rotating machine shafts, normal stress $\sigma_x(t) = \sigma_a \sin\omega t$ occurs due to bending together with shear stress τ_{xy} = constant due to torsion. Although this multiaxial and non-proportional loading is so often met in the mechanical engineering, it is not paid with sufficient attention on the part of researchers. Anyway, there are research traditions and experience under such loading at least in Portugal [1] and Bulgaria [2 – 5].

The goals of this paper are to represent and discuss at ICMFF9 the development of the following subjects.

- Different technical solutions for implementation of multiaxial fatigue tests under combined rotating bending and torsion, emphasizing that such implementation has always been actual and important for the fatigue life of real rotating machine shafts.
- A new technical solution and equipment where the bending is rotated instead of

rotating (implementation of an idea of inversion).

- New application and verification of the IDD method (Integration of Damage Differentials) [6 – 8] for fatigue life evaluation under the (non-proportional) stress-time functions $\sigma_x(t) = \sigma_a \sin\omega t$, $\sigma_y(t) = 0$, $\tau_{xy}(t) = \text{constant} = \tau$.

TESTING MACHINE BASED ON STATICALLY INDETERMINATE SYSTEM OF TWO ROTATING SHAFTS

A usual way of subjecting a specimen to rotating bending with constant torsion is to include it between an electromotor and a hydraulic brake. With that, the working power is wasted in the brake causing heating. To avoid this, a solution is to use a generator [1] instead of a brake in order to restore the energy from the electromotor. Another solution was applied in the Technical University of Gabrovo: statically indeterminate system of two rotating shafts. They, by means of gears, mutually twist each other [2 – 4] (Fig. 1). Then, the motor is only used to rotate the two shafts and cover power dissipation.

The specimen, incorporated in the front (connection) shaft (Fig.1), is hold by threaded inserts (15) and cone inserts (17). This fixture (see also the next Fig. 2(b)) sets the specimen and the connection shaft as a whole for transmission of the bending. The torsion is transmitted by square nests of the connection shaft in which square ends of the specimen are inserted. The specimen is bended as a simple beam on two articulated supports (Cardan joints 13). Two forces load the specimen symmetrically by two self-aligning bearings in housing (20). The two forces are produced by means of weight set

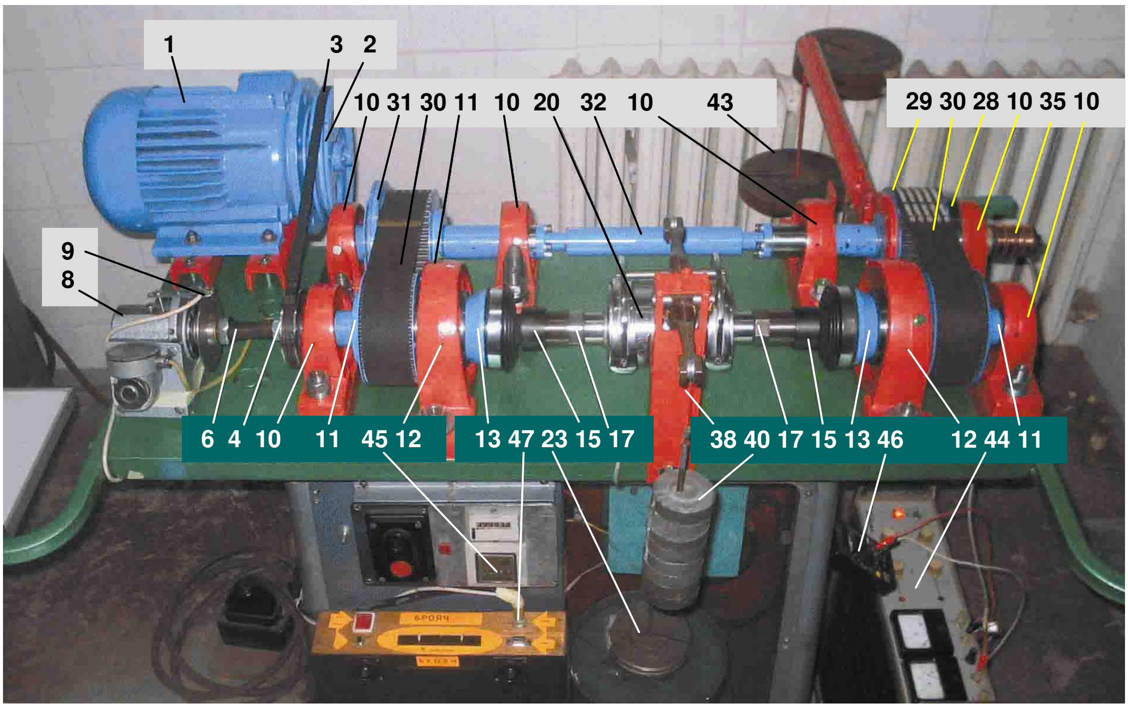

Figure 1. The testing machine based on the statically indeterminate shaft system

(23). To avoid initial bending of the specimen due to the weights of the housing (20) etc., there is a balancing system: the weight (40) and a sheave (38) with a steel cord.

The second shaft of the system consists of: a primary shaft (32) connected with a right secondary shaft, a torque transducer (not visible in Fig. 1), a left secondary shaft and a spring-clutch (not visible). One (29) of four toothed pulleys is in a close-sliding fit whereas the rest three toothed pulleys (11) are permanently connected to their shafts by splines. Instead of the pulley (29), a flange (28) is permanently connected to the right secondary shaft by splines to transmit the torsion.

For exerting a necessary initial torsional moment, the system of the weight (43) is connected with the clamping pulley (29). The initial moment is "weight times arm" and is transmitted by toothed belts (30) to the specimen before the pulley (29) and the flange (28) are clamped. The idea of the statically indeterminate system is that, after exerting the initial torsional moment, the pulley (29) and the flange (28) are clamped and thus the initial moment remains in the system as a permanent constant internal torsional moment. The two shafts mutually twist each other by this moment. Then, the electric motor (1) rotates the specimen and the connected shafts (with 3000 r.p.h.).

The scheme of a simple beam loaded symmetrically by two forces, Fig. 2(b), was preferred as it provides a constant bending moment $M = Pl$ along the specimen. In [1], the scheme of a cantilever beam, Fig. 2(a), was used.

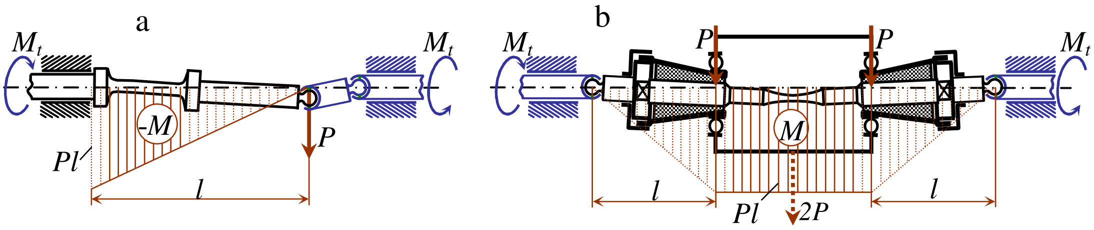

Figure 2. Bending a specimen as a cantilever beam (a) [1], or as a simple beam (b) symmetrically loaded by two forces

The machine (Fig. 1) successfully fulfilled the tasks for which it was designed (experimental data follow below). It is also of interest to any designer of low-energy consuming testing machines, not only for fatigue tests but also for operational tests of gears, belts, etc. However, especially for fatigue tests, as the usual case is with creating a unique machine, it left what to be desired. Specific disadvantages of this machine are:

- the spring-clutch cannot keep constant torsional moment in case the specimen over-twists plastically too much: great values of the ratio τ/σ_a cannot be enabled;
- the kinematics is complicated, contains a lot of rotating parts (as every other machine for rotating bending) and requires special safety precautions.

General disadvantages of a machine for rotating bending are:

- the rotation of the specimen complicates the strain-gauge measurement on it (requires current collectors etc.) and affects the reliability of the signals;
- for the same reason, the machine requires building-in a special torque transducer for inspecting the torsional moment, whose signal could not be reliable.

TESTING DEVICE FOR ROTATED BENDING WITH CONSTANT OR VARYING TORSION

The idea of inversion of mobility and immobility entailed better technical solutions in many cases in the history of the engineering. In the case considered, if the specimen is made immovable and the plane of the bending (of the two forces, Fig. 2(b)) is made rotating, this may immediately remove the above-counted disadvantages. Based on this idea, a new testing device is in a process of designing, development and manufacturing.

The initial design of the device is illustrated in Fig. 3. The specimen (1) represents again a simple beam on two articulated supports loaded symmetrically by two forces. The latter are exerted by the bearings (2). The specimen has a square end put into the square nest (3) which has a cone center (4) in its bottom. The other end of the specimen is supported by a cone center (5). That specimen's end, also square, is gripped by a lever (6) with weights (7).

The specimen is placed in a hollowed and shortened shaft (8) of a usual electric multiple-speed motor. The cone center (5) is screwed (axially adjustable) into a threaded hole of the cap (11). A fan turbine (13) is mounted at the back of the motor's shaft (8). Throughout the whole electric motor, two vertical radial thru holes (17) pass (aside from the motor's electrical part). They also pass through the hollow shaft (8) where they are threaded. Screws (18) are driven there to displace the bearings (2) and bend the specimen. These (forcing) screws (18) are also envisaged to be exchanged with

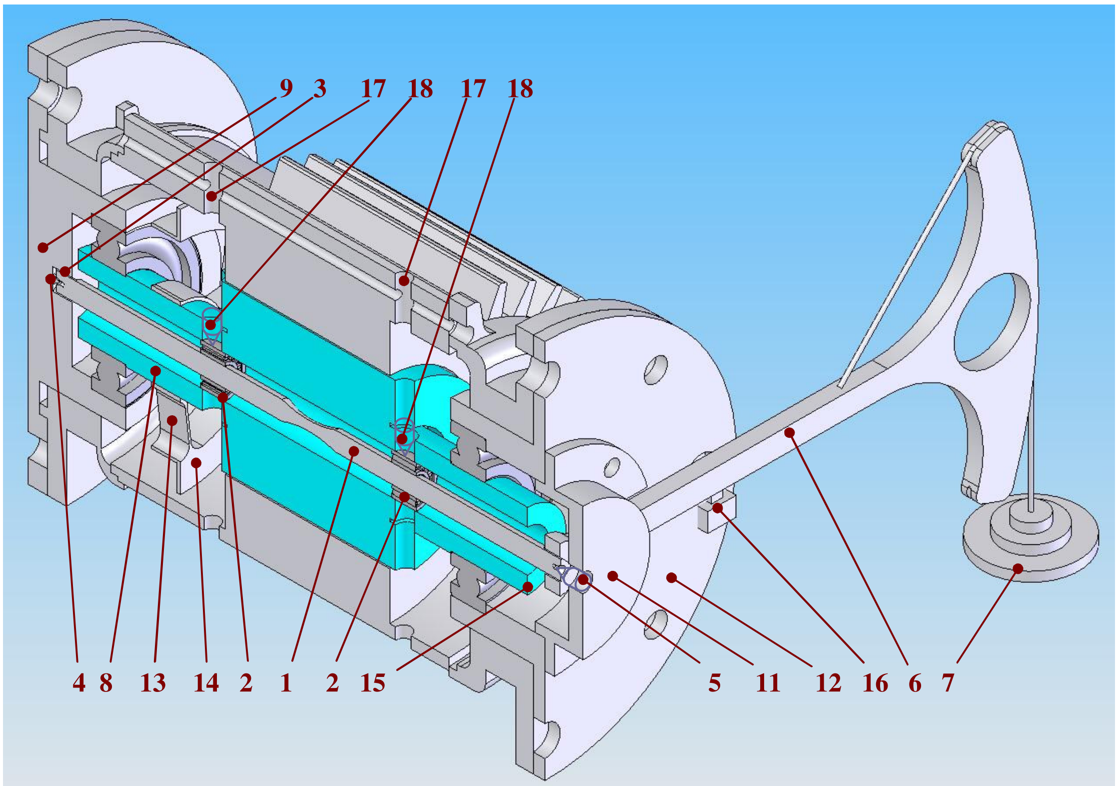

Figure 3. The initial design of the device

different ones containing springs if bending stress relaxation is expected.

The specimen (1), together with the bearings (2) put on it, is inserted into the device through the opening of the front shield (12) as to be taken from the nest (3) and the cone center (4). The lever (6) is connected. The cap (11) and the center (5) are fixed. The screws (18) are driven and the bending deflection is inspected through the holes (17) from below. The weights (7) are put and thus constant (steady) torsion is produced. The motor is switched on and thus the fatigue loading starts. The loading cycles are the revolutions of the motor's shaft. The turbine (13) cools the specimen and the whole device. The test is terminated when the lever (6) contacts and activates the limit switch (16) due to fracture of the specimen or due to covered allowance of over-twisting.

In contrast to the above-counted disadvantages, now the following advantages can be counted, as follows.

- The specimen is immovable what makes much easier to do the torsion, and also the strain-gauge measurement of the bending and torsion.
- The device has the simplest possible kinematics: there are not at all any kinematical drives. There are no rotating parts outside at all and no need of respective precautions. Everyone can operate the device without any special instruction.
- The power consumption is a minimum one, for covering energy dissipation only (the motor practically works in idle running).
- The device is as small, compact as an electric motor, light and cheap.
- The magnitude of the torsional moment is guarantied. Besides, this magnitude can be changed during the test without stopping it by just adding or removing weights.
- Different cycle frequency can be enabled by just switching over the speeds of the motor.

In addition, other morel loading components can be enabled: compression by screwing up the cone center (5) and cyclic variation of the torsional moment by using a built-in cam mechanism at (15). There are possibilities for various loading combinations. The device can be more universal than expected. As well, thanks to the specimen's immovability, possibilities for tests with hollow specimens under internal pressure or vacuum, also applying aggressive chemical agents, etc., are not excluded.

Especially for cyclic variation of the torsional moment, the mentioned cam mechanism includes additional structural components (slides, springs etc.) not shown in Fig. 3. The mechanism provides reversed rotary actuation of a two-shoulder lever instead of the lever (6) in Fig. 3. Thus, the cycle of τ_{xy} can be set fully-reversed (symmetrical), partly-reversed (asymmetrical) or pulsating, with lower frequency than the cycle of $\sigma_x(t)$. The kinematics and dynamics of the cam mechanism are described in outline in [5] and are still in a process of designing, manufacturing and testing.

EXPERIMENTAL RESULTS AND IDD APPLICATION

Figure 4 illustrates the experimental lives under rotating bending and constant torsion. It is remarkable that adding $\tau = 0{,}2\sigma_a$ to the pure rotating bending disposes □ to the right from ○, instead of to the left, and adding $\tau = 0{,}3\sigma_a$ disposes △ even more to the right.

This is the interesting effect of increasing the life when adding constant torsion to the rotating bending until the ratio τ/σ_a is not high. This effect proved to be available for $\tau/\sigma_a = 0{,}2$ and $\tau/\sigma_a = 0{,}3$. It remained not clear to which higher τ/σ_a value the effect will still be available: the testing machine (Fig. 1) could not provide correct experimental data when trying to produce τ/σ_a greater than 0,3. By the way, for this reason, also out of research interest, the other technical solution (Fig. 3) was suggested.

Other fatigue life evaluation criteria may hardly assess the effect in question. But the IDD method (Integration of Damage Differentials) confirms the effect as represented below. For understanding the IDD procedures carried out next, it is necessary to read [6] (general representation of IDD) and [7, 8]. Besides, full presentation of IDD and any detail can be found of the IDD site: http://www.freewebs.com/fatigue-life-integral/. All the data files and IDD software files used and mentioned below will be placed on the IDD site after ICMFF9 is held.

The reproduced *S-N* line under pure bending (Fig. 4), which agree very well with the experimental lives, has an R_r-prototype [1] with $m = 5$, $N_r = 2{,}2.10^6$ and $s_r = 496{,}54$ MPa valid for $\tau/\sigma_a = 0$. In case $\tau/\sigma_a > 0$, the R_r-prototype changes according to the value of τ. This is the mean stress effect that relates to the well-known Haigh diagram σ_a-σ_m described with Goodman line, Gerber parabola, etc. According to Fig. 4, there are 8 lives to be assessed with the following values of σ_a and τ: 740 and 148 (for $\tau/\sigma_a = 0{,}2$), 740 and 222 (for $\tau/\sigma_a = 0{,}3$), 650 and 130 (for $\tau/\sigma_a = 0{,}2$) and so on. Correspondingly, 8 L-files [6] L740-02, L740-03, L650-02 and so on were composed containing different R_r-prototypes. As well, 8 C-files [6] were done: C740-02, C740-03, C650-02 and so on.

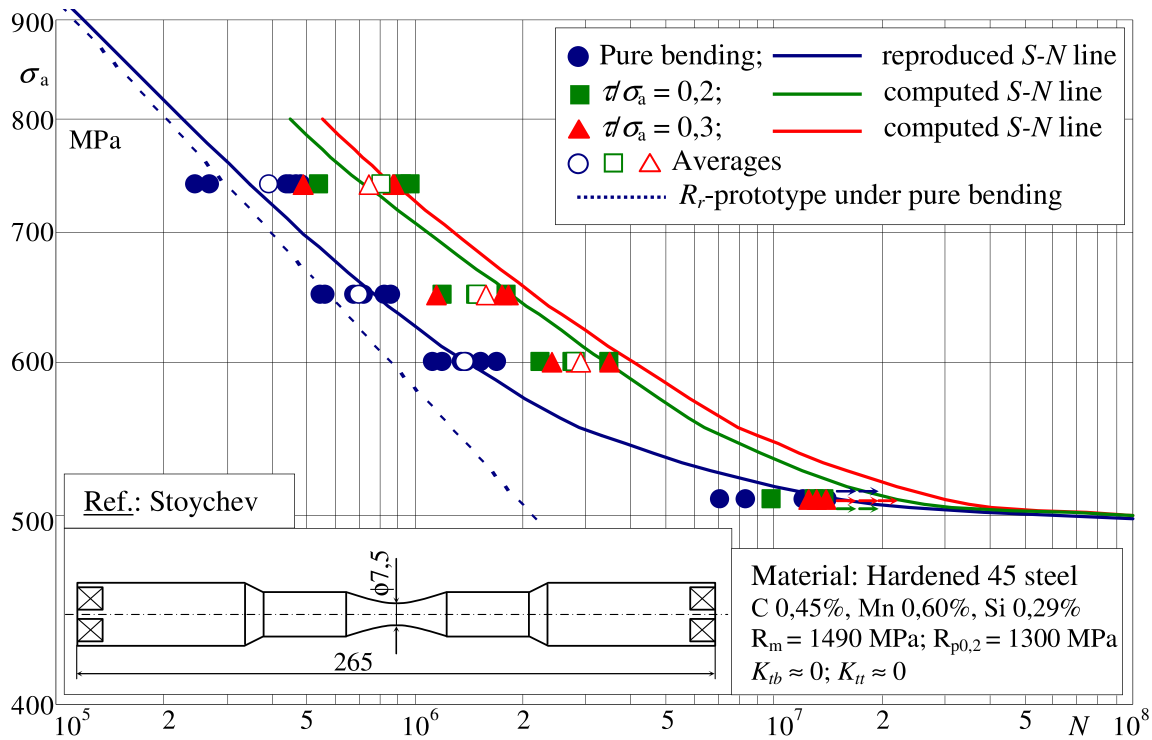

Figure 4. Illustration to the experimental lives and IDD verification

For σ_m in the Haigh diagram, an equivalent static stress $\sigma_{equ,m}$ = 1,3τ was used: 1,3.148, 1,3.222, 1,3.130 and so on. Actually, the ratio 1,3 of R_m = 1490 MPa and the static ultimate strength under torsion, 1130 MPa [2], was taken for $\sigma_{equ,m}/\tau$. For the Haigh diagram, an equation proposed [2], with the participation of $R_{p0,2}$, was used (if R_m participates instead of $R_{p0,2}$, the computed results below will not be significantly influenced). That equation makes the Haigh diagram pass along the middle between the Gerber parabola and the Goodman line. For a certain σ_a at a certain $\sigma_{equ,m} > 0$, another higher σ_a at $\sigma_m = 0$ was calculated and supposed to be valid for the same N.

For example, σ_a = 740 at 1,3.222 = 288,6 is increased by the Haigh diagram to 856,1. According to the above basic R_r-prototype, $856{,}1^5 N = 496{,}54^5 2{,}2.10^6$, and hence $N = 1{,}444.10^5$. Then, the new R_r-prototype passes through the point (s_{max}, N) ≡ (740+288,6; N) ≡ (1028,6; $1{,}444.10^5$). Upon comparing with the life from the basic R_r-prototype in Fig. 4, which is 3.10^5 at the level 740 MPa, the new R_r-prototype is shifted at that level to the left: the life becomes 2,08 times as short. A second point (s_{max}, N) relates to the level 510 MPa, i.e. (s_{max}, N) ≡ (510+288,6; N); the same calculation procedure is repeated substituting 510 for 740. The second point proves (798,6; $9{,}258.10^5$). The new R_r-prototype, passing through the two points, accepts the parameters m and s_r shown in Table 1 at σ_a = 740 and τ/σ_a = 0,3.

Table 1. The parameters m and s_r of the R_r-prototypes for k = 0; $N_r = 2{,}2.10^6$

σ_a [MPa]	740	650	600	510	740	650	600	510
τ/σ_a	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,3
m	6,571	6,471	6,410	6,084	7,345	6,818	6,757	6,624
s_r [MPa]	643,3	627,1	617,8	599,4	710,2	677,2	667,2	648,0

In a similar way, the parameters of the rest 7 R_r-prototypes in Table 1 were obtained. They are all valid for $k = \sigma''/\sigma' = 0$ where σ' and σ'' are the principal stresses (k = 0 means uniaxial state of stress). Eight more R_r-prototypes, valid for k = –1 (pure shear), also participate in the L-files. They less influence the IDD computed lives under the loadings considered. They were assumed to have the same slopes m in Table 1. Their parameters s_r were obtained from those ones in Table one after dividing by the above ratio 1,3 of equivalence of the uniaxial state of stress and the pure shear.

In the L-files, the IDD factors of loading non-proportionality f_c and f_τ [6] were set as 2 and 3 like in [7, 8]. The IDD parameters N_c and N_τ [6] were set equal to 3.10^6: a bit greater than N_r according to the considerations in [7, 8]. Using the *EllipseT* computer program [6], the 8 IDD lives were computed and the two *S-N* lines for τ/σ_a = 0,2 and τ/σ_a = 0,3 were drawn as shown in Fig. 4.

In the IDD graph mode [6, 7, 8], the loading trajectory looks like a parabola in IV quadrant of the σ'-σ'' plane. Its pick is the point (τ, $-\tau$) and its two branches approach the σ' and $-\sigma''$ axes (the parabola is available on the IDD site). The ratios t_r, t_c and t_τ[6] are 0,72, 0,42 and 0,42 in case τ/σ_a = 0,3, and the greatest damage ratio is d_τ [6] (d_τ = 0,73). In case τ/σ_a = 0,2, t_r, t_c and t_τ are 0,81, 0,31 and 0,35; d_τ = 0,6 while d_r = 0,68.

CONCLUSIONS

The paper is a contribution to the fatigue knowledge and research under the multiaxial non-proportional loading of rotating or rotated bending combined with constant or varying torsion.

For rotating bending and constant torsion, the proposed statically indeterminate two-shaft system is a good technical solution.

The second technical solution proposed, with rotated bending of an immovable specimen, seems to be more resourceful. In the version with constant torsion, it would be no problem to provide τ/σ_a greater than 0,3. Besides, by means of the additional cam mechanism, varying torsion with different options can also be produced. As well, axial compression can be exerted. Thus, testing conditions are implemented that cover loading conditions of real machine shafts.

The effect of life increase was experimentally revealed in case constant torsion is added to rotating bending with τ/σ_a up to 0,3. This effect is more apparent at higher σ_a levels. At a σ_a level which could be considered as a (conditional) fatigue limit, the effect is transformed into keeping the same fatigue limit despite the added torsion.

Other fatigue life assessment methods would hardly reveal that effect. The IDD method did this correctly. Indeed, the two computed lines in Fig. 4 agree fairly well with the experimental lives. There was a series of hypothetical settings necessary for the IDD computation (no experimental *S-N* lines valid for different static stresses were available). However, if checking different settings, the IDD computed lines will not be shifted too much to the left or right: the effect in question will be again assessed, anyway. It is so thanks to a decrease in the IDD damage intensities (to a certain extent) when adding (equivalent) static (mean) stress.

The IDD application done can be considered as confirming the settings [7, 8] $f_c = 2$, $f_\tau = 3$ (except for sintered steel [8]) and $N_c = N_\tau \geq N_r$ (see recommendations in [7, 8]).

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