On the use of the Modified Wöhler Curve Method applied along with the Reference Radius Concept to perform the multiaxial fatigue assessment of welded joints

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ABSTRACT. *The present paper summarises an attempt of using the Modified Wöhler Curve Method (MWCM) in conjunction with the reference radius concept (r_{ref}=1 mm) to estimate lifetime of welded joints subjected to multiaxial fatigue loading. The accuracy and reliability of the devised fatigue design methodology was checked through several data sets taken from the literature and generated by testing, under in-phase and out-of-phase biaxial loading, steel and aluminium welded samples. The results of such a systematic validation exercise seem to support the idea that the multiaxial fatigue life estimation technique summarised in the present paper can be considered as an interesting alternative method suitable for performing the fatigue assessment of those welded structures whose critical sites are damaged by in-service multiaxial fatigue loading.*

INTRODUCTION

The International Institute of Welding (IIW) [1] recommends three different linear-elastic strategies to calculate the stress quantities to be used to estimate fatigue strength of welded samples, that is, either nominal stresses, hot-spot stresses, or the use of the local stress fields determined by rounding the weld toe through a radius of 1mm length (where the latter method can be applied only if the thickness of the main plate is larger than, or equal to, 5 mm). Under the above circumstances, the reference principal stress range, $\Delta\sigma_{loc,FAT}$, at 2×10^6 cycles to failure is equal to 225 MPa for steel weldments, whereas it is equal to 71 MPa for aluminium joints (both reference ranges calculated for a probability of survival, P_S, equal to 97.7% and derived for a load ratio, R, equal to 0.5). Further, independently from the considered type of material, the uniaxial fatigue curve has its knee point, N_k, at 10^7 cycles to failure and the negative inverse slope, k, is equal to 3 for a number of cycles to failure, N_f, lower than N_k and equal to 22 for $N_f>N_k$ [2]. On the contrary, when torsional loadings are involved, two different strategies can be followed to determine the reference stress range, $\Delta\sigma_{loc,FAT}$, at 2×10^6 cycles to failure (P_S=97.7%, R=0.5) when r_{ref} is taken equal to 1mm [2]: the use of the maximum principal stress hypothesis results in a $\Delta\sigma_{loc,FAT}=\Delta\tau_{loc,FAT}$ value equal to 160 MPa for steel and to 63 MPa for aluminium, whereas, according to Von Mises, $\Delta\sigma_{loc,FAT}$ is equal to 280 MPa and to 90 MPa for steel and aluminium weldments, respectively; the negative inverse slope, k_0, under cyclic torsion is suggested as being taken equal to 5 for $N_f \leq N_k$ and to 22 for $N_f > N_k$, where $N_k=10^8$ cycles to failure.

Further, in a recent investigation [3] it has been observed that when moving from “thick and stiff” to “thin and flexible” welded structures the negative inverse slope increases from 3 to 5 under axial (or bending) fatigue loading and from 5 to 7 under torsional loading.

In light of the encouraging results obtained when using the reference radius concept to design welded joints not only against uniaxial, but also against multiaxial fatigue [4], in the present paper the notch stress concept is now attempted to be used in conjunction with the MWCM [5] in order to formalise a fatigue assessment technique suitable for estimating fatigue damage in welded connections subjected to multiaxial fatigue loading by adopting the same schematisation based on SN reference curves as the one recommended by the IIW.

To conclude, it is worth observing here that the MWCM postulates that fatigue damage is maximised on that material plane (i.e., the so-called critical plane) which experiences the maximum shear stress range. Moreover, according to Socie [6], the MWCM assumes that the fatigue damage extent depends also on the stress components perpendicular to the critical plane itself. In theory, since the critical plane is the one of maximum shear, the above fatigue damage model should be adopted only to estimate fatigue lifetime of those materials showing a ductile behaviour (like, for instance, structural steel) [7]. On the contrary, fatigue damage in semi-ductile materials (like, for instance, wrought aluminium and cast steel) should be estimated defining the orientation of the critical plane by simultaneously taking into account both the normal and tangential stress components [7]. In spite of the above fact, since to estimate fatigue lifetime the MWCM makes use not only of the maximum shear stress range but also of the range of the stress perpendicular to the critical plane, in the present investigation such an approach will be attempted to be employed to estimate fatigue damage in ductile as well as in semi-ductile welded materials.

STRESS ANALYSIS

In order to summarise the correct procedure to be used to determine the relevant local stress fields calculated according to the reference radius idea, consider the tube-to-plate welded connection sketched in Figure 1a. Such a joint is assumed to be subjected to combined tension and torsion, $\Delta\sigma_{nom}$ and $\Delta\tau_{nom}$ being the ranges of the applied uniaxial and torsional nominal stress, respectively. Initially the local linear-elastic stress field has to be determined by rounding the weld toe through a fictitious radius having length equal to 1mm (Fig. 1b) [4]. If $K_{t,n}$ is the stress concentration factor determined under axial loading (or bending), then the contribution due to the applied axial force/moment can be calculated as follows (where μ is Poisson’s ratio) [2]:

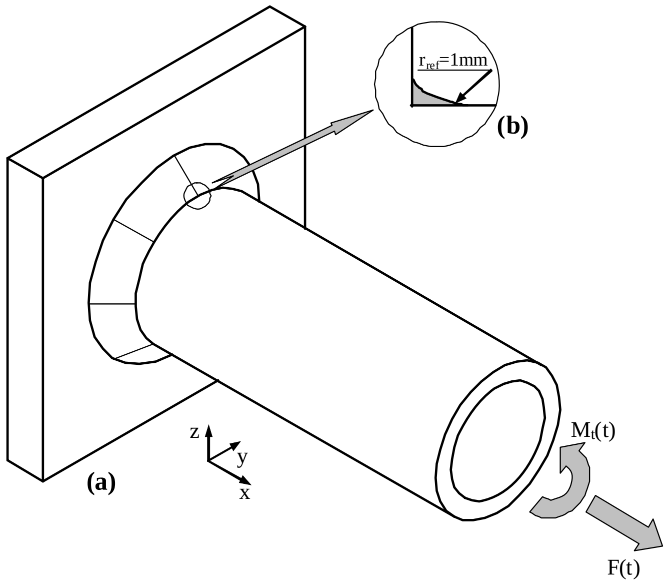

Figure 1. Tube-to-plate welded joint subjected to combined tension/torsion.

$$\Delta\sigma_x = K_{t,n} \cdot \Delta\sigma_{nom} \qquad (1)$$

$$\Delta\sigma_y = \mu \cdot \Delta\sigma_x \qquad (2)$$

Similarly, the range of the local shear stress damaging the weld bead under investigation takes on the following value [2]:

$$\Delta\tau_{xy} = K_{t,t} \cdot \Delta\tau_{nom} \qquad (3)$$

$K_{t,t}$ being the stress concentration factor under torsional loading.

Finally, even though examination of the state of the art [5] shows that many different strategies can be followed to determine the orientation of the plane experiencing the maximum range of the shear stress, to perform the validation exercise summarised below the critical plane was determined according to the so-called Maximum Variance Method [5, 8].

MODIFIED WÖHLER CURVE METHOD AND NOTCH STRESS CONCEPT

In order to reformulate the MWCM [5] to make it suitable for being used in conjunction with the notch stress concept, consider again the tube-to-plate welded joint sketched in Figure 1 and subjected to combined axial loading and torsion. The initial hypothesis is formed that the connection under investigation is in the as-welded condition.

Attention can be focused now on the simple uniaxial sub-case. According to Eqs (1) and (2), under axial (or bending) loading, the relevant local stress quantities relative to the critical plane are as follows [5]: $\Delta\tau=\Delta\sigma_x/2$ and $\Delta\sigma_n=\Delta\sigma_x/2$, where $\Delta\tau$ and $\Delta\sigma_n$ are the ranges of the local stress tangent and perpendicular to the critical plane, respectively. On the contrary, as far as torsional loadings are concerned, the local stress quantities relative to the critical plane take on the following values: $\Delta\tau=\Delta\tau_{xy}$ and $\Delta\sigma_n=0$.

At this point it is useful to recall that the most important peculiarity of the MWCM is that it directly accounts for the degree of multiaxiality and non-proportionality of the stress fields damaging the fatigue process zone through the so-called critical plane stress ratio, ρ_w, defined as [5]: $\rho_w=\Delta\sigma_n/\Delta\tau$. According to the above definition, ρ_w is equal to unity under uniaxial fatigue loading, whereas it equals zero under torsional loading.

Turning back to the formalisation of the MWCM, consider now the modified Wöhler diagram sketched in Figure 2: such a log-log chart plots the range of the shear stress relative to the critical plane, $\Delta\tau$, against the number of cycles to failure, N_f. Owing to the way it is built, the above diagram allows the axial (or bending) and the torsional fatigue curves to be plotted together. In more detail, the torsional curve, characterised by a ρ_w value equal to zero, has negative inverse slope equal to $k(\rho_w=0)=k_0$ and range of the reference shear stress extrapolated at N_{FAT} cycles to failure, $\Delta\tau_{Ref}(\rho_w=0)$, equal to $\Delta\tau_{loc,FAT}$; the uniaxial curve ($\rho_w=1$) instead has negative inverse slope equal to $k(\rho_w=1)=k$ and range of the reference shear stress at N_{FAT} cycles to failure, $\Delta\tau_{Ref}(\rho_w=1)$, equal to $\Delta\sigma_{loc,FAT}/2$.

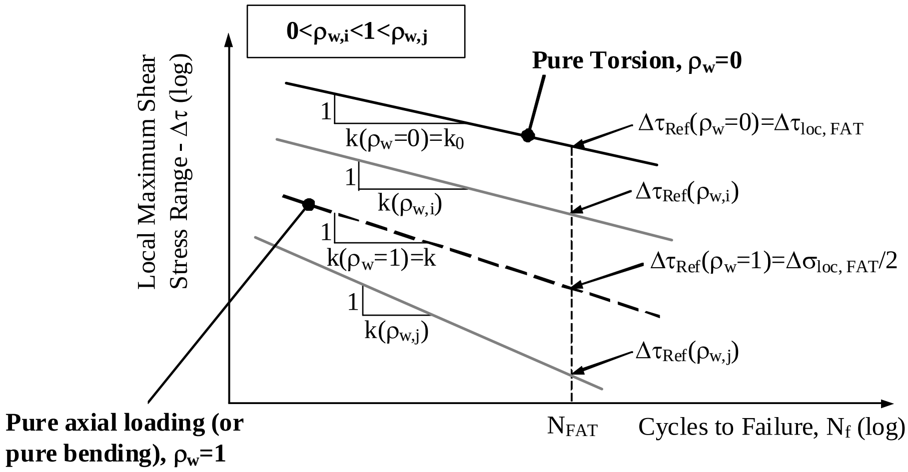

Figure 2. Modified Wöhler Curve Diagram.

According to the above schematisation, the hypothesis can be formed that modified Wöhler curves shift downwards in the diagram as ratio ρ_w increases [5]. This implies that the number of cycles of failure under multiaxial fatigue loading can be estimate, provided that the curve corresponding to the ρ_w value relative to the critical plane of the welded joint being assessed is positioned correctly. By performing a systematic validation exercise based on a large number of experimental data, it was proven that simple linear laws are enough accurate to correctly define the $\Delta\tau_{Ref}$ vs. ρ_w as well as the k vs. ρ_w relationships [5]. In particular, if the ranges of the uniaxial, $\Delta\sigma_{loc,FAT}$, and torsional, $\Delta\tau_{loc,FAT}$, reference shear stresses are used as calibration information, the $\Delta\tau_{Ref}$ vs. ρ_w linear function takes on the following form, the reference range of the shear stress, $\Delta\tau_{Ref}(\rho_w)$, being defined at $N_{FAT}=2\times10^6$ cycles to failure:

$$\Delta\tau_{Ref}\left(\rho_w\right)=\left(\frac{\Delta\sigma_{loc,FAT}}{2}-\Delta\tau_{loc,FAT}\right)\cdot\rho_w+\Delta\tau_{loc,FAT}\quad \text{for } \rho_w\leq\rho_{w,lim} \tag{7}$$

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where, in the above definitions, the limit value of ratio ρ_w is calculated as follows [5]:

$$\rho_{w,lim} = \frac{\Delta\tau_{loc}}{2\Delta\tau_{loc} - \Delta\sigma_{loc}}. \tag{9}$$

The k vs. ρ_W relationship instead can be defined as follows:

$$k(\rho_w) = [k - k_0] \cdot \rho_w + k_0 \qquad \text{for } \rho_W \leq 1 \text{ and } N_f \leq 10^8 \text{ cycles to failure} \tag{10}$$

$$k(\rho_w) \equiv k \qquad \text{for } \rho_W > 1 \text{ and } N_f \leq 10^8 \text{ cycles to failure} \tag{11}$$

On the contrary, for $N_f > 10^8$ cycles to failure, the negative inverse slope of Modified Wöhler Curves is suggested, as recommended in Ref. [2] for the torsional case, to be taken equal to 22, and it holds true independently from the value of the $\Delta\sigma_n$ to $\Delta\tau$ ratio characterising the time-variable stress state at critical locations.

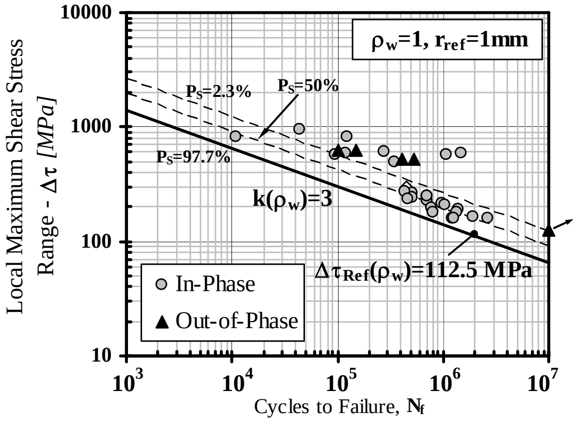
Table 1. Parameters of the reference fatigue curves used to calibrate the MWCM [2, 3].

Material	$\Delta\sigma_{loc,FAT}$ [a] *[MPa]*	$\Delta\tau_{loc,FAT}$ [a] *[MPa]*	**Stiff Structures** **k**	**k_0**	**Flexible Structures** **k**	**k_0**	$\rho_{w,lim}$
Steel	225	160	3	5	5	7	1.7
Aluminium	71	63	3	5	5	7	1.45

[a]Principal stress ranges, $N_{FAT}=2\times10^6$ cycles to failure, $P_S=97.7\%$, R=0.5

To conclude the reasoning summarised in the previous paragraphs, Table 1 reports, for steel and aluminium weldments, the reference values to be used to determine the constants in the MWCM's governing equations as well as the corresponding value of $\rho_{W,lim}$.

So far, the formalisation of the MWCM in terms of the reference radius concept has been based on the assumption that the considered welded joints were in the as-welded condition. This resulted in a great simplification of the problem because the mean stress effect could be neglected without any significant loss of accuracy [4]. On the contrary, when residual stresses are relieved through appropriate post-welding treatments, if, on one hand, the overall fatigue strength of welded joints increases, on the other hand, they become more and more sensitive to the presence of non-zero mean stresses [4]. Another important issue which deserves to be considered here in great detail is the effect of superimposed static shear stresses on the overall fatigue strength of weldments loaded in torsion. As far as the authors are aware, this tricky aspect is still under discussion, so that, there exists no universally accepted rule suitable for efficiently taking into account the presence of shear load ratios larger than -1 [2]. In any case, according to what experimentally observed in several un-welded metallic materials [9], the hypothesis can be formed that the presence of non-zero mean shear stresses can be neglected with little loss of accuracy also when stress-relieved welded connections are involved.



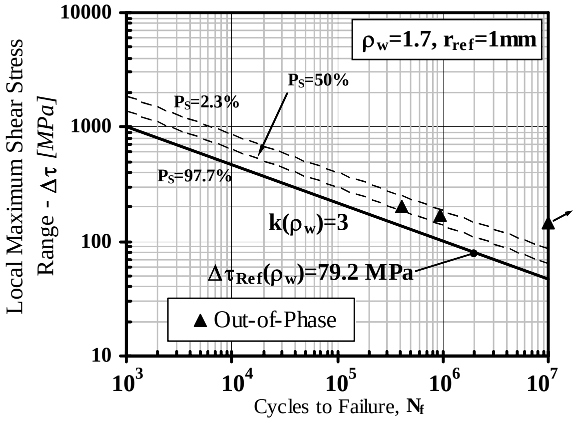


Figure 3. Comparison between the design curves supplied by the MWCM applied along with the reference radius concept and the considered experimental results generated by testing "thick and stiff" as-welded samples made of steel.

To conclude, according the above considerations, the practical rule proposed in Ref. [2] is suggested here as being directly extended, in terms of fatigue enhancement factor [1], to multiaxial fatigue situations by simply rewriting it as a function of the load ratio, $R_{CP}=\sigma_{n,min}/\sigma_{n,max}$ [10], calculated considering the stress perpendicular to the critical plane, that is

steel welded joints:
$f(R_{CP})=1.32$ for $R_{CP}<-1$
$f(R_{CP})=-0.22\times R_{CP}+1.1$ for $-1\leq R_{CP}\leq 0$
$f(R_{CP})=-0.2\times R_{CP}+1.1$ for $0<R_{CP}<1$

aluminium welded joints:
$f(R_{CP})=1.88$ for $R_{CP}<-1$
$f(R_{CP})=-0.55\times R_{CP}+1.33$ for $-1\leq R_{CP}\leq 0$
$f(R_{CP})=-0.66\times R_{CP}+1.33$ for $0\leq R_{CP}<1$

VALIDATION EXERCISE

In order to check the reliability of the design methodology formalised in the previous section, several data sets generated by testing steel and aluminium welded samples were selected from the technical literature (See Ref. [5], Appendix B, for a detailed description of the analysed experimental results). In more detail, the following geometries were considered: tube-to-plate welded joints, rectangular hollow section specimens with longitudinal fillet welded gussets, square hollow section tube-to-plate welded connections and, finally, cruciform samples with longitudinal attachments. Such a welded specimens were tested under a variatey of in-phase and out-of-phase biaxial loading paths. In Figures 3 to 5 the design curves calculated according to the MWCM applied along with the reference radius concept are compared to the considered experimental results: the above charts seem to support the idea that the MWCM applied along the r_{ref}=1mm concept is successful in estimating the allowable fatigue lifetime of both steel and aluminium welded structures subjected to proportional as well as to non-proportional multiaxial fatigue loading.

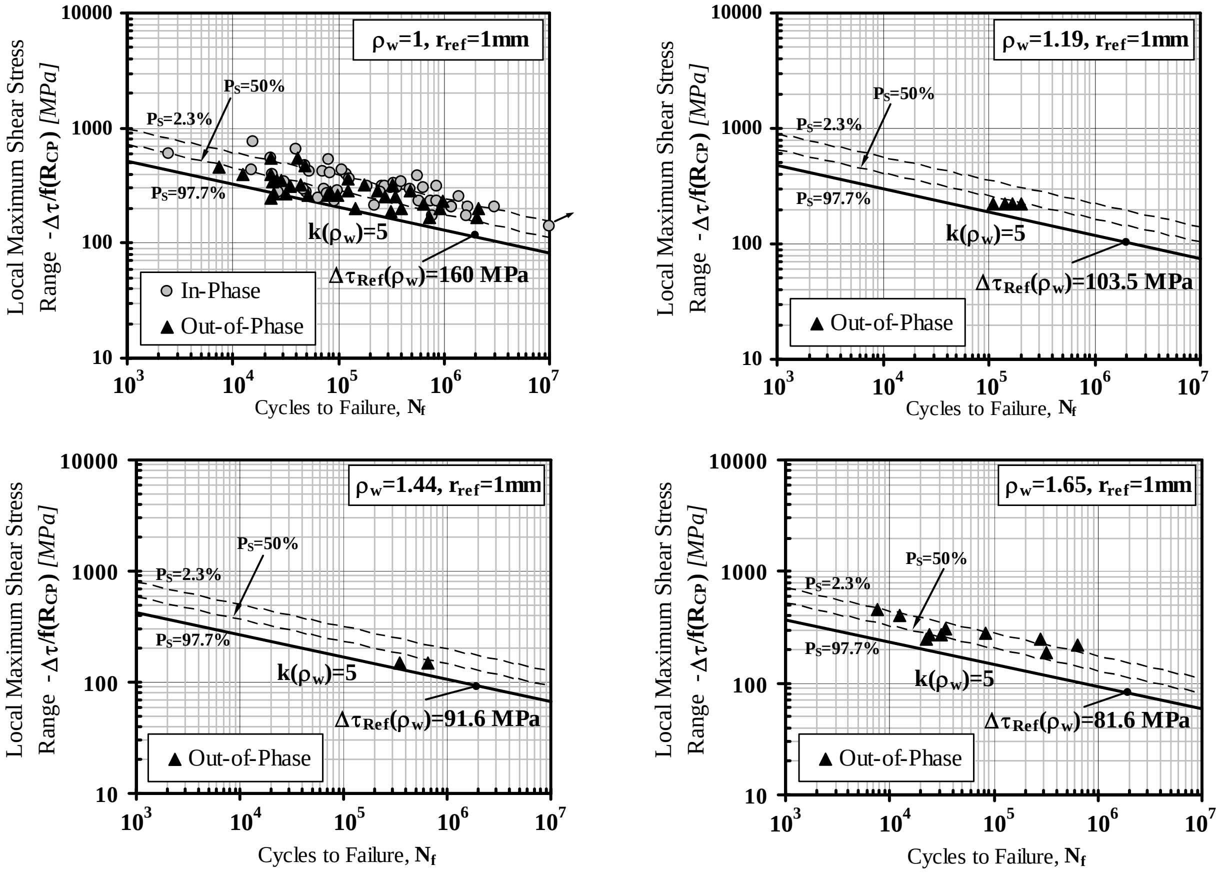


Figure 4. Comparison between the design curves supplied by the MWCM, applied along with the reference radius concept, and the considered experimental results generated by testing "thin and flexible" samples made of steel.

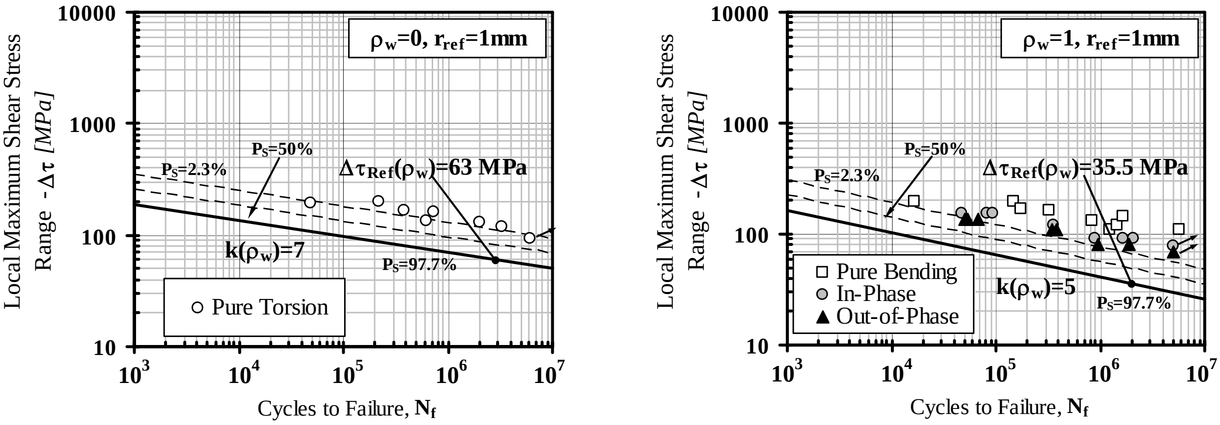


Figure 5. Comparison between the design curves supplied by the MWCM, applied along with the reference radius concept, and the considered experimental results generated by testing "thin and flexible" as-welded samples made of aluminium.

As to the diagrams reported in Figure 4, it is worth noticing here that, in order to summarise all the experimental results in a reduced number of charts, instead of correcting the modified Wöhler curves through the multiaxial fatigue enhancement factor, the shear stress range relative to the critical plane, $\Delta\tau$, was simply divided by $f(R_{CP})$ when stress-relived samples were involved. To conclude, the diagrams of Figure 5 make it evident that the application of the proposed approach to aluminium weldments resulted in allowable fatigue lifetimes characterised by an evident degree of conservatism. This may be ascribed to the fact that the weld toe radius in the FE models was taken equal to 1mm, whereas the average length of the measured radius was equal to 17mm [11]. Further, it is worth observing that the fatigue response under multiaxial loading of the above tube-to-plate samples was seen to be independent of the out-of-phase angle: this particular situation is a consequence of the fact that the tested aluminium was characterised by a semi-ductile behaviour [11].

CONCLUSIONS

1) The MWCM can be applied to design real welded components against multiaxial fatigue by post-processing the relevant stress states expressed in terms of either nominal, hot-spot, or notch stresses.
2) More work needs to be done in this area to extend the use of such an approach to those situations involving variable amplitude multiaxial fatigue loading.

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