On determination of bifurcation angle in V-notched Brazilian disc specimen under mixed mode loading

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ABSTRACT. *The fracture initiation angle was first investigated experimentally in the V-notched Brazilian disc (V-BD) specimens made of graphite and soda-lime glass under mixed mode I/II loading for different notch opening angles and various notch tip radii. Then, the maximum tangential stress (MTS) criterion, frequently used for determining the mixed mode crack growth path in components containing a sharp crack, was utilized to determine the fracture initiation angle theoretically. The proposed approach is applicable to mixed mode brittle fracture of V-notched elements in whole domain between pure mode I and pure mode II loading. It was found that the experimental results could be well estimated by using the MTS criterion.*

INTRODUCTION

Notches are widely utilized in engineering components and structures among which the V-shaped ones are very popular. A V-notch introduces a discontinuity and acts as a stress raiser. The stress concentration makes the notched element vulnerable to mechanical loads and decreases dramatically the load bearing capacity of the V-notched component or structure. Notched structural components are sometimes made of brittle materials such as ceramics, brittle polymers, glasses and graphite. A crack may nucleate from the tip of V-notch and then grow suddenly due to the brittleness of these materials. The notch bifurcation angle under mixed mode loading is an important parameter in the design of structures because it plays a vital role in the overall damage in the structure. To date, several researchers have studied mixed mode brittle fracture for V-notched specimens theoretically or experimentally (e.g. [1-3]). In this study, first a new test sample, called V-notched Brazilian disc (V-BD) specimen, made of a commercial type of polycrystalline graphite and soda-lime glass is suggested and used to investigate the fracture initiation angle in the V-notched brittle components experimentally under mixed mode I/II loading. Then, a brittle fracture criterion, called V-MTS, is developed based on the maximum tangential stress (MTS) criterion [4] in order to estimate the fracture initiation angle of the tested specimens under mixed mode loading. It is shown that the experimental results can be well estimated by using the results of the V-MTS criterion.

EXPERIMENTS

It is well-known that the cracked and notched specimens can be subjected to three different types of in-plane loading namely pure mode I, pure mode II and mixed mode I/II loading. Several specimens have been earlier suggested in literature for the experimental investigation of brittle fracture in graphite, soda-lime glass and other brittle materials containing sharp cracks. One of the specimens frequently used in the past for mixed mode fracture experiments in cracked bodies is the centrally cracked Brazilian disc (CBD) specimen, see for example [5]. A modified specimen, called V-BD, was used in this research for conducting mixed mode fracture tests on V-notches. Fig. 1 shows the V-BD specimen in which β is the angle between the loading direction and the notch bisector line and the parameters D, $d/2$ and P are the disc diameter, the notch length and the applied compressive load, respectively.

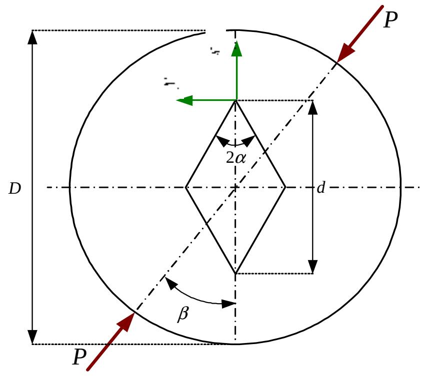


Fig. 1. The V-BD specimen.

When the direction of applied load P is along the notch bisector line (i.e. $\beta = 0$), the upper and the lower corners of the rhombic hole are subjected to pure mode I deformation. If the angle β is not zero, the notch is subjected to mixed mode I/II loading. When the angle β increases gradually, the loading conditions changes from pure mode I towards pure mode II. For a specific angle, called β_{II}, pure mode II deformation is achieved. The mode II loading angle β_{II} is always less than 90^{o} and depends on the notch length and its opening angle and also on the notch tip radius. The angles β_{II} were determined by using the finite element (FE) analysis. The materials used for fabricating V-BD specimens were a commercial grade of polycrystalline graphite and soda-lime glass that fail in a brittle manner at room temperature. The mechanical properties of the tested materials obtained from standard tensile and fracture toughness tests are presented in Table 1. For determining each parameter, five experiments were conducted. For the graphite specimens, the disc diameter (D), the notch length ($d/2$) and the thickness were 60 mm, 15 mm and 8 mm, respectively. These dimensions for the soda-lime glass specimens were 80 mm, 20 mm and 6 mm, respectively.

Table 1. Mechanical properties of the tested materials.

Material	E (GPa)	σ_t (MPa)	K_{Ic} (MPa m$^{0.5}$)
Graphite	8.05	27.5	1.0
Soda-lime glass	72	14	0.6

To study the effects of the notch opening angle on the fracture behavior of the V-BD specimens, three values of notch opening angle 2α = 30, 60, 90 (deg.) were considered. Also, the notch tip radii ρ = 0, 1, 2 mm and ρ = 1, 2, 4 mm were considered for preparing the graphite and soda-lime glass samples, respectively. The specimens were accurately fabricated by using a high-precision 2-D water jet cutting-machine. For each geometry shape and loading angle, three fracture tests were performed by using a universal tension-compression test machine under displacement-control condition with loading rate of 0.05 mm/min. A total number of 297 mixed mode fracture tests were performed for various notch geometry parameters and different loading angles β from 0 (pure mode I) to β_{II} (pure mode II). Figs. 2 and 3 show two examples of the graphite and soda-lime glass specimens subjected to mode I and mixed mode fracture tests. Fig. 4 shows sample graphite specimens broken after mixed mode fracture tests.

Fig. 2. The V-BD graphite specimen in a mode I fracture test.

Fig. 3. The V-BD soda-lime glass specimen in a mixed mode I/II fracture test.

In the next section, a mixed mode failure criterion is suggested to estimate the fracture initiation angles of the tested V-notched specimens. For this purpose, first, the elastic stress distribution around a rounded-tip V-notch is presented under general in-plane loading condition. Then, the phenomena of the maximum tangential stress (MTS) criterion [4] are applied to the tangential component of the stress field in order to achieve a set of theoretical curves for estimating the fracture initiation angles.

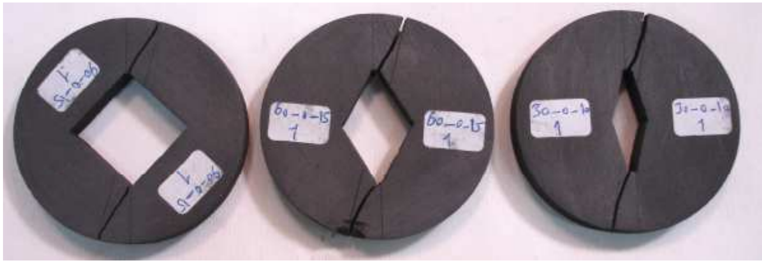


Fig. 4. The graphite specimens broken after mixed mode I/II fracture tests.

FRACTURE CRITERION

The classic MTS criterion is a well-known failure criterion frequently used for investigating mixed mode brittle fracture for sharp cracks [1]. According to the MTS hypothesis, fracture occurs along the direction perpendicular to the direction of maximum tangential stress θ_0^1. In this paper, the MTS criterion is extended to V-notched domains and a V-MTS criterion is proposed to estimate the fracture initiation angle in V-notched brittle components.

According to the requirements of the MTS criterion, the first and the second derivatives of the tangential stress $(\sigma_{\theta\theta})$ [6] with respect to θ should be zero and negative, respectively. The V-MTS criterion suggests that brittle fracture initiates radially from a point on the notch border along which the tangential stress at a critical distance r_c is a maximum. The direction θ corresponding to this point is called here the fracture initiation angle θ_0^1.

Considering the MTS criterion, one can write:

$$\frac{\partial \sigma_{\theta\theta}(r,\theta)}{\partial \theta} = 0 \tag{1.a}$$

$$\frac{\partial^2 \sigma_{\theta\theta}(r,\theta)}{\partial \theta^2} \langle 0 \tag{1.b}$$

By substituting $(\sigma_{\theta\theta})$ from [6] into Eq. 1.a and replacing r and θ by $r_{c,V}$ and θ_0^1, one can derive an equation for determining the angle θ_0^1 in terms of mode I and mode II notch stress intensity factors (NSIFs) $K_I^{V,\rho}$ and $K_{II}^{V,\rho}$. In pure mode I, $K_{II}^{V,\rho}$ is zero and θ_0^1 is zero because of symmetry in geometry and loading. In pure mode II, $K_I^{V,\rho}$ is zero and fracture initiates along an angle denoted by $\theta_{0,II}^{1}$ where the value of $\theta_{0,II}^{1}$ is determined by solving Eq. 2.

$$\frac{\partial f_{\theta\theta}(\theta_0^{1})^{(II)}}{\partial\theta_0} + \left(\frac{r_{c,V}}{r_0}\right)^{\mu_2-\lambda_2} \frac{\partial g_{\theta\theta}(\theta_0^{1})^{(II)}}{\partial\theta_0} = 0 \qquad \Rightarrow \qquad \theta_0^{1} = \theta_{0,II}^{1} \tag{2}$$

where $f_{\theta\theta}(\theta_0^{1})^{(II)}$ and $g_{\theta\theta}(\theta_0^{1})^{(II)}$ are functions of θ reported in [6]. The parameter r_0 is the distance between the coordinate origin and the notch tip that depends on the notch opening angle and the notch tip radius [6]. According to Eq. 2, the parameter $\theta_{0,II}^{1}$ depends on the notch critical distance $r_{c,V}$, the notch opening angle and the notch tip radius.
The critical distance $r_{c,V}$ can be determined from:

$$r_{c,V} - \left[\frac{K_{Ic}^{V,\rho}\left[f_{\theta\theta}(0)^{(I)} + \left(\frac{r_{c,V}}{r_0}\right)^{\mu_1-\lambda_1} g_{\theta\theta}(0)^{(I)}\right]}{\sqrt{2\pi}\,(\sigma_{\theta\theta})_c}\right]^{\frac{1}{1-\lambda_1}} = 0 \tag{3}$$

To estimate $r_{c,V}$ for a V-notch, first, the values of notch fracture toughness $K_{Ic}^{V,\rho}$ and $(\sigma_{\theta\theta})_c$ should be known. $K_{Ic}^{V,\rho}$ can be obtained from a mode I fracture test on an appropriate V-notched specimen. The parameter $(\sigma_{\theta\theta})_c$ is a material property and is commonly considered to be the tensile strength σ_t for brittle and quasi-brittle materials.
To obtain the curves of fracture initiation angles, a parameter called the notch mode mixity parameter ($M^e{}_V$) is defined as:

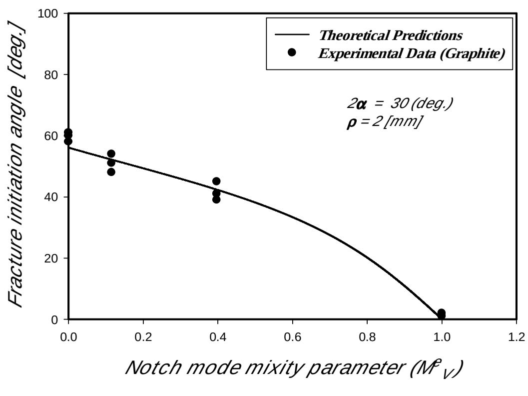
$$M^e{}_V = \frac{2}{\pi}\tan^{-1}\left(\frac{K_I^{V,\rho}}{K_{II}^{V,\rho}}\, r_{c,V}^{\;(\lambda_1-\lambda_2)}\right) \tag{4}$$

The value of $M^e{}_V$ varies from zero (for pure mode II) to one (for pure mode I). In order to plot the curves of fracture initiation angle, the below steps should be followed:

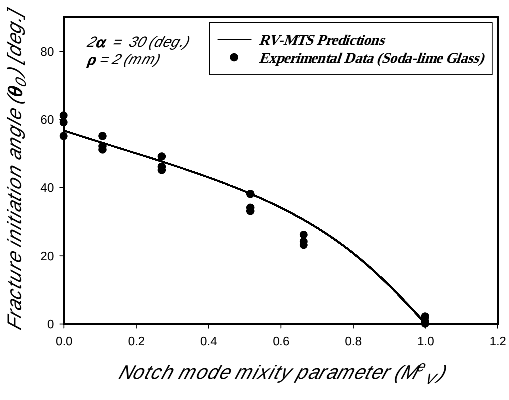
1- Choose an arbitrary value for $M^e{}_V$ between 0 and 1.
2- Substitute $M^e{}_V$ into Eq. 4.
3- Solve Eq. 4 and find the fracture initiation angle θ_0^{1}.
4- Repeat the steps 1 to 3 for other values of $M^e{}_V$.
5- Draw the parameter θ_0^{1} versus $M^e{}_V$.

RESULTS AND DISCUSSION

In this section, the results of the V-MTS criterion in estimating the mixed mode fracture initiation angle of V-notched specimens made of graphite and soda-lime glass are compared with the experimental results. Figs. 5 (a) and 5 (b) show the theoretical curves of fracture initiation angle for V-notches in graphite and soda-lime glass with 30^o opening angle together with the experimental results. These curves are related to the notch tip radius equal to 2 mm. According to these figures, as M_V^e decreases from 1 (mode I) to 0 (mode II), the fracture initiation angle (θ_0^1) is increased from 0^o up to a mode II fracture initiation angle $\theta_{0,II}^1$ which depends on the notch opening angle and the notch tip radius. A very good agreement is observed between the theoretical and the experimental values of the fracture initiation angle as presented in Figs. 5 (a) and 5 (b) for 2α = 30^o. Similar good agreements were found for 2α = 60^o and 90^o and for other notch tip radii. The mean discrepancies between the theoretical and the experimental values of fracture initiation angle (θ_0^1) for graphite and soda-lime glass specimens are approximately 4% and 5%, respectively that demonstrate the effectiveness of the V-MTS criterion. While the plotted curves are limited only to graphite and soda-lime glass, similar curves can be plotted for other brittle materials by using the procedure described in the previous section. The numerical computations (not shown here) indicated that for ρ > 5 *(mm)*, all the curves are unified and an increase in the notch tip radius does not affect the curve considerably. This may be due to the fact that the sensitivity of the notch to the stress concentration is decreased as the notch tip radius increases. It is noteworthy that the notch bifurcation angle can be simply measured in experiments by plotting a straight line from the center of curvature of the notch tip to the fracture location on the notch border. The fracture location on the notch border is recognized by the angle θ_0^1 which is measured from the coordinate origin located at the distance r_0 from the notch tip. It is clear that the plotted line is perpendicular to the notch border which is consistent with the experimental observations.



(a)



(b)

Fig. 5. Curves of mixed mode fracture initiation angles for V-notches in (a) graphite and (b) soda-lime glass, with notch opening angle of 30^o and notch tip radius of 2 mm.

CONCLUSIONS

Because the notch bifurcation angle is an important parameter in the evaluation of overall damage in the notched structure, this parameter was investigated in the V-notched graphite and soda-lime glass specimens both experimentally and theoretically under mixed mode I/II loading. A new test sample called the V-BD specimen was used for an experimental investigation of brittle fracture in graphite and soda-lime glass components containing V-shaped notches in the whole domain from pure mode I to pure mode II loading conditions. A new criterion, called V-MTS, was also utilized to estimate the experimental results of the fracture initiation angle. The existence of a very good agreement between the theoretical estimations and the experimental results demonstrates that the experimental results can be successfully estimated by using the proposed criterion. It is noteworthy that the use of the V-MTS criterion in practical cases is convenient because it can be presented as a set of fracture initiation angle curves for direct estimation of notch bifurcation angle in V-notched components.

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