Notch optimization under multiaxial loading

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ABSTRACT. *Shape optimization of components subjected to multiaxial fatigue is considered. In these cases the application of static optimization algorithms, generally based on Von Mises stress, is no more applicable. In this paper an optimization routine for multiaxial fatigue is developed on the basis of the CAO technique proposed by Mattheck. According to a criterion for fatigue strength estimation of notched specimens made of ductile materials and subjected to mutiaxial fatigue: Liu-Zenner. Abaqus 6.8-1is used as the commercial software to develop finite element simulation and Python 2.4and Matlab2007 are used as the subroutine programs.*

INTRODUCTION

Fatigue failure is the most experienced failure in many fields such as automobile and aerospace industry. It is possible to avoid fatigue failures by simple over dimensioning the dangerous notches which afflict the components, but the global weight and the performances of the components will results worst. Another, more attractive way to improve the fatigue behaviour of machine element is the definition of procedures able to lead toward the optimum design of notched parts. In the present study, computer aided optimization (CAO) [1] is used as an optimization method to increase the fatigue life in notched components.Previously, Peng et al[2] have tried to optimize the notches based on other method for uniaxial loading, Wilczynski[3] has tried to optimize the shape under multi axial fatigue loading for crack propogation.

The present approach is composed of five steps. Roughly speaking, it is based on simulating the stress field in the notch zone namely “Growth Zone” with temperature and decreases the elastic module in that zone simultaneously. Abaqus 6.8-1[4] is used as commercial software to apply this method. In the original CAO method which is developed by Mattheck [1], the Von-Mises stress is the criterion for calculating the stress concentration factors and also the stress which should be transformed to temperature. Liu-Zenner is the chosen criterion [2] due to the fact that based on the litreture [6, 7] the fatigue prediction of this method in different types of loading is reasonable. Liu-Zenner is an integral criterion based on the average value of the shear and normal stress acting on each material plain; thus the exact definition of shear stress on each plane is required. In this paper, Papadopoulos definition [8] of the amplitude and mean value of the shear stress acting on the plane is used. This definition is almost free from any ambiguity because they are based on the construction of a unique minimum-circumscribed circle to the load path described by the shear stress on the critical plane. As it is mentioned, in order to apply the criterion, it is necessary to calculate the integrals over the planes in Liu-Zenner method. Thus the space, obviously, should be divided into several planes. This division should be implementable on numerical software. The Webber [9] method which deals with the way to obtain a homogeneous distribution of planes having almost the same area and also the determination of the smallest circle surrounding the loading path is implemented. At the end, an example based on the developed method is represented. Results obtained from the numerical examples indicate that distribution of the Liu-Zenner [5] calculated stress on the notch area in comparison with the original shapes is significantly imporved leading to decrease and even in some cases to avoid stress concentration on the notch.

OPTIMIZATION PROCEDURE

The paper is based on a method introduced by Mattheck [1] which is derived from the natural phenomenon of adaptive growth in trees. The CAO method [1] is briefly described in the following steps. The flowchart is also illustrated in Figure 1.

1. A finite element model of the structure representing the desired appearance of the component is produced by Abaqus 6.8 [3].Fatigue loading is applied by introducing the stress amplitude and phases of the harmonic function; consequently, the history of stress tensor through the time will be obtained on each node.
2. Based on the FEM results the Liu-Zenner [5] stress for each node will be calculated by using Python 2.4 [10]. The strength of the specimen could be estimated based on the calculated fatigue limit.
3. The computed stresses are then substituted by a virtual temperature distribution. In this way the points which showed previously highest mechanical stresses would be the hottest points in the component. Moreover, the modulus of the elasticity in the upper layer is set to only 1/400 of the initial value. Thus there would be a fictitious soft layer with particularly high temperature at original overloaded zones and rather cold layers in the unloaded region. Before applying and changing the stress field into temperature field, it should be noted that without considering the ambient temperature the notched area will always be increased.
4. In the next FEM computation which considers just the thermal loads, the previous mechanical load (tension) is set to zero. Moreover, only the soft upper layer will have a thermal expansion factors $\alpha>0$. During this computational stage with only thermal loading, the 'pudding-soft' upper layer expands corresponding to its temperature distribution, and that is the Growth Zone which previously experienced the highest loads (in computation step 2) at this stage tolerates the highest

temperature and expands most clearly, i.e. it grows more. All the prodcedure is controlled by Matlab 2007 [11].

5. The structure already improved by growth in computation step 4 is already shape-optimized to some extents, and occasionally one such growth cycle is sufficient. This is checked by again setting the E-modules of the soft layer at the value of the basic material and starting at step 2 with a new FEM computation under purely mechanical loading, which will deliver a more homogeneous stress distribution with greatly reduced notched stresses. The computation loops 2-5 are run through repeatedly, till the stress concentration factor stops changing due to fact that construction conditions forbid further growth.

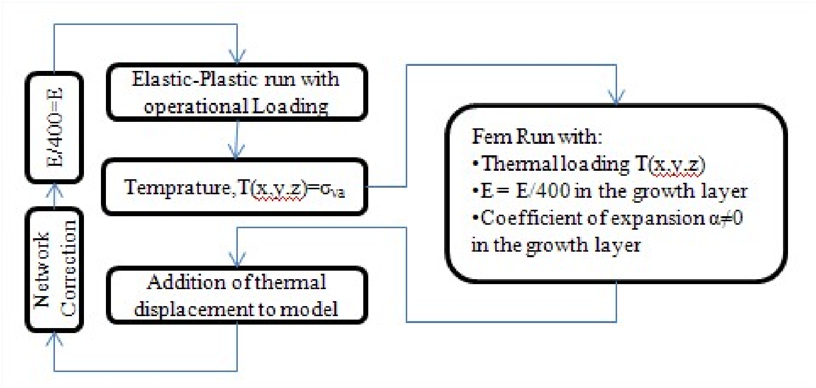


Figure 1: Flowchart of proposed CAO method

MULTI AXIAL FATIGUE CRITERIA

Finding fatigue limit under multi axial loading has been one of the most controversial issues in the last century; due to the fact that, without omitting and simplifying the problem condition, most of the structures in real life, which are under the cyclic load, suffer from the multi axial fatigue damages. There are many different criteria from different categories which have been proposed to find the fatigue limits. The Liu-Zenner is the chosen criterion which is described briefly in the following part.

Liu-Zenner criterion [2]

The Liu-Zenner [5] multi-axial criteria of integral approach and of the critical plane approach can be derived as special cases from the general fatigue criteria. Based on the literature [6, 7] the estimated life time according to this criterion shows appropriate results in different loading condition. The Liu-Zenner multi-axial criteria of integral approach and of the critical plane approach can be derived as special cases from the general fatigue criterion. The Eqs (2) and (3) are based on Figure 2. In the following equations The $\sigma_{\upsilon a,\sigma}$ and $\sigma_{\upsilon a,\tau}$, stress amplitudes, are calculated in each cutting plane from the time function of the stress components.

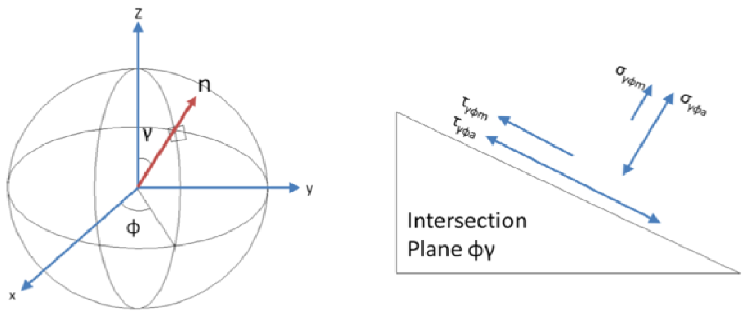


Figure 2: Integration domain of the SIH and stress components in the intersection plane γφ [5]

$$\sigma_{va,\tau} = \left\{ \frac{15}{8\pi} \int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi a}^{\mu_1} Sin\gamma d\varphi d\gamma \right\}^{1/\mu_1} \quad (2)$$

$$\sigma_{va,\sigma} = \left\{ \frac{15}{8\pi} \int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \sigma_{\gamma\varphi a}^{\mu_2} Sin\gamma d\varphi d\gamma \right\}^{1/\mu_2} \quad (3)$$

$$\sigma_{va} = \left[a\sigma_{va,\tau}^2 + b\sigma_{va,\sigma}^2 \right]^{1/2} \quad (4)$$

$$a = \frac{1}{5}\left[3\left(\frac{\sigma_W}{\tau_W}\right)^2 - 4 \right], \quad b = \frac{1}{5}\left[6 - 2\left(\frac{\sigma_W}{\tau_W}\right)^2 \right]$$

In these equations the exponents µ1 and µ2 can be varied between 2 and infinity. More information is available in [3]. In order to simplify the calculation, the exponents are selected as *µ1= µ2=2*. The coefficients *a* and *b* are determined for pure alternating tension-compression and pure alternating torsion.
This equation should be valid only for a, b>0. Therefore, this method, with µ1= µ2=2, is only applicable for the materials which are compatible in the following Eqs3:

$$\frac{2}{\sqrt{3}} \le \left(\frac{\sigma_W}{\tau_W}\right) \le \sqrt{3} \quad (5)$$

The equivalent mean stresses, and the mean normal stresses are calculated as follows:

$$\sigma_{vm,\tau} = \left\{ \frac{\int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi a}^{\mu_1} \tau_{\gamma\varphi m}^{v_1} Sin\gamma d\varphi d\gamma}{\int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi a}^{\mu_1} Sin\gamma d\varphi d\gamma} \right\}^{1/v_1} \quad (6)$$

$$\sigma_{vm,\sigma} = \left\{ \frac{\int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi a}^{\mu_2} \tau_{\gamma\varphi m}^{v_2} Sin\gamma d\varphi d\gamma}{\int_{\gamma=0}^{\pi} \int_{\varphi=0}^{2\pi} \tau_{\gamma\varphi a}^{\mu_2} Sin\gamma d\varphi d\gamma} \right\}^{1/v_2} \quad (7)$$

And finally, the failure condition is formulated directly by a combination of the all equivalent stresses from Eqs (3), (5), and (6):

For calculating the failure condition the values of fatigue limit for pure bending σ_W, fatigue limit for pure torsion loading τ_W, fatigue limit $\sigma_{W(R=0)}$, $\tau_{W(R=0)}$ are necessary. The fatigue limit under pulsating tensile stresses $\sigma_{W(R=0)}$ is evaluated by assuming a mean stress sensitivity factor equal to 0.2 and for fatigue limit under pulsating torsion stress the same assumption in [7] is used.

$$\sigma_{va}^2 + m\sigma_{vm,\tau}^2 + n\sigma_{vm,\sigma} = \sigma_W^2 \tag{8}$$

$$m = \frac{\sigma_W^2 - \left(\frac{\sigma_W}{\tau_W}\right)^2 \left(\frac{\tau_{W(R=0)}}{2}\right)^2}{\frac{4}{7}\left(\frac{\tau_{W(R=0)}}{2}\right)^2}, \; n = \frac{\sigma_W^2 - \left(\frac{\sigma_{W(R=0)}}{2}\right)^2 - \frac{4m}{21}\left(\frac{\sigma_{W(R=0)}}{2}\right)^2}{\frac{5}{7}\left(\frac{\sigma_{W(R=0)}}{2}\right)^2}$$

$$M = \frac{\sigma_W - \sigma_{W(R=0)}}{\sigma_{W(R=0)}} = 0.2 \;, \quad \tau_{W(R=0)} = \frac{4\tau_W}{1 + \frac{2\sigma_W}{\sigma_{W(R=0)}}} \tag{9}$$

Definition of Shear Stress

In integral type approaches the accurate definition of shear stress seems the most important part. The evaluation of the amplitude and mean value of the shear stress acting on the critical plane should be resolved satisfactorily for proportional cyclic loading conditions. The situation is much more complex regarding the definition of the amplitude and mean value of the shear stress for non proportional loading. The complexities arise from the fact that, unlike the normal stress vector which conserves its direction, the shear stress vector τ changes in magnitude and direction inside each load cycle. The minimum-circumscribed circle (MCC) to a plane polygon P is: either one of the circles drawn with a diameter equal to a line segment joining any two vertices of P or one of the circum circles of all the triangles generated from every three vertices of P. This method is presented for the first time by Papadopoulos [8].

APLICATION PROCEDURE:

In order to apply the novel CAO procedure, three types of commercial software have been used. The algorithm of implementing the mentioned steps is as follows:

1. The model with all the boundary condition, define loads, etc is solved in ABAQUS 6.8 [4].
2. For each integration point stress tensor in each time increment is read; for example if the overall time is one second and time increment is 0.01 for 200 elements; 2000 stress tensor will be read from FEM results.
3. the following steps, which have been done with Python 2.4[10], should be done for each elements:
    a. Set of the normal vector **n** which is calculated based on Weber division[9] method is calculated

b. By multiplying the stress tensor for that element in every single time increment and each single vector in **n,** calculate the normal and shear stresses on each plane in space.
c. Find the MCC based on the modified randomized method [12] for the load path which was calculated in the last step.
d. Calculate the mean and alternative normal and shear stressed for each normal vector in **n.**
e. Calculate the Liu-Zenner integral based formula number 1 to 7

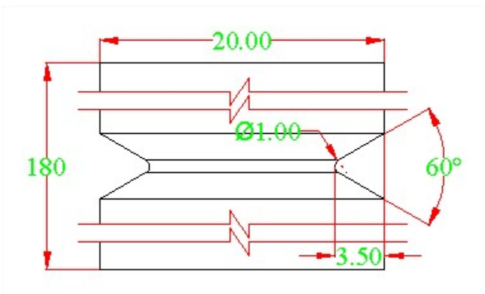
4. Step number 3 should be continued for each element.
5. The Liu-Zenner [5] stress, calculated in the previous steps, should be calculated on each node; this step is done by Python [10] attached to Abaqus [4].
6. Apply the node stress as the temperature. And calculate the displacement due to the applied temperature. Before this analysis, the elastic modules should be reduces for example into 1/400th of the main elastic modules.
7. Make a new a model by applying the obtained displacement on the nodes; continue these steps in order to reach the optimum results based on the design specification. This step is done with Matlab 2007 [11].

AN EXAMPLE OF IMPLEMENTATION

The following notch has been chosen for an example to implement the method. In Figure 3a the main geometry of the notch is presented. The applied forces are presented in Eqs 10. The loading is 90 degree out of phase.

$$T = 75\ Sin8t\ (Nm) \qquad F = 35\ Cos8t (kN) \tag{10}$$

In Figure 3b the FEM model in Abaqus is shown. The Quadratic 20 nodes elements are used for notch and tetrahedral 10 node elements are used for the rest. Material properties are limited to elastic module and Possion ratio for the simple steel.



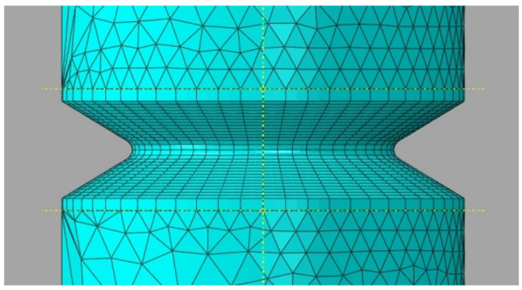


Figure 3: a)The original geometry of the selected Notch, b) The FEM model with the described mesh

In Figure 4 the optimization evolution in the notch profile after 5, 10, 15, and 20 iterations is presented. In Figure 9 the changes in calculated stress based on Liu-Zenner method through the notch profile for each node is demonstrated. Based on the results for this example it can be concluded by implementing this approach the equivalent stress after 20 iterations from 900 MPa is reduced to 400 MPa whereas the profile based on the Figure 5 did not change dramatically.

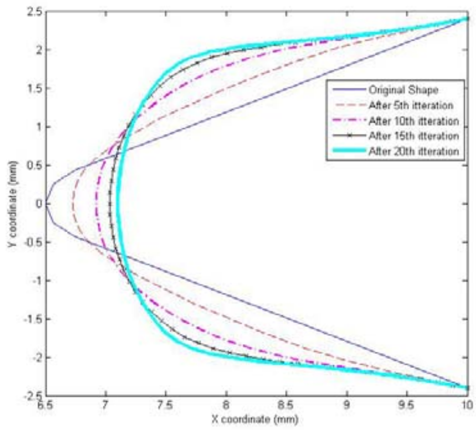


Figure 4: The Optimization evolution of notch geometry

The iteration loop stops as soon as the difference of the obtained results of two successive iterations is less than the defined tolerance. In the presented example, based on the results, the differences between iteration number 19 and 20 is less than the defined tolerance; thus the optimization can be stopped.

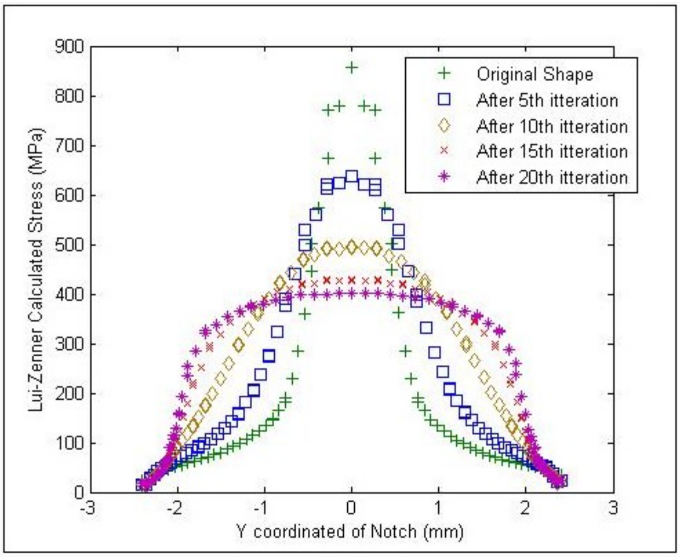


Figure 5: the Liu-Zenner stress changing through the iteration numbers

CONCLUSION

A novel optimizing method has been presented for modifying the notched geometry under multi axial fatigue loading. Based on the obtained results it can be inferred that this method could be effective also for different geometries with different kind of loading. The method has been applied to many different geometries and loading conditions just one of which has been presented in this article. According to the obtained results the method was found to be applicable to a wide range of loading and geometry combinations. Furthermore it was very quick with respect to number of iterations.

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