Multiaxial HCF and LCF Constraints in Topology Optimization

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ABSTRACT. *Ease in application design methodology of lightweight and fatigue resistant structures is important for aircrafts and ground vehicles. Recently, very popular topology optimization can give an answer for the above demand if appropriate multiaxial fatigue constraints are accepted. In the paper investigation on the structure mass optimization with multiaxial high and low-cycle fatigue constraints is presented. Differences between Dang Van, total strain energy density [TSED] and von Mises criteria are clearly shown in a structural benchmark example. A very significant effect of shear stresses sensitivity occurs in the structure layouts. The topology optimization algorithm with fatigue constraints can easily be adapted to complex structural problems.*

INTRODUCTION

The development of structural optimization with fatigue constraints is critical for the product lifecycle design of airplanes and ground vehicles. Recently, many investigators work on this field with different approaches and goals. Predominate is the tendency of mass reducing with the assurance of structure durability [1-4]. In the literature, many examples of successful use of size and shape fatigue optimization can be found [5-8]. However, the proposals of the topology optimization with fatigue constraints are very seldom. Desmorat and Desmorat [9] presented a proposal of the topology optimization algorithm for optimization of fatigue resistance. The Lemaitre damage law was there used in maximization of a fatigue lifetime by optimizing shape of a structure in cyclic plasticity. Mrzyglod [10] proposes an algorithm of topology optimization designed for structures subjected to multiaxial high-cycle fatigue. The Dang Van's criterion in that paper was applied for fatigue damage estimation. Mrzyglod and Zielinski [11] used topology optimization with low-cycle fatigue constraints. The modified Neuber model and equivalent total energy for description of damage mechanisms were used.
In this article, a proposal of methodology of the fatigue topology optimization is presented. Multiaxial high-cycle and low-cycle fatigue criteria are examined in the investigation of structures subjected to complex loads.

ALGORITM OF TOPOLOGY OPTIMIZATION WITH STRESS CONSATRAINTS

The homogenization method [12], SIMP (solid isotropic material with penalization) [13] and EOS (evolutionary structural optimization) [14,15] are popular methods of topology optimization.

The algorithm described below can be attached to the ESO type. The ESO method foundation was first introduced by Mattheck [14]. He proposed method of shaping structures in a form of the surface of constant stresses. His idea was based on copying the mechanism of the growth of trees.

Let us assume the optimization problem formulated as follows:

$$\min f(\boldsymbol{\eta}) \tag{1}$$

the constraints are:

$$g_j(\boldsymbol{\eta}) \le \overline{g_j}, \; j = [\,1,2,\,\ldots,\,K\,] \tag{2}$$

where: $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots \eta_N]$ is a vector of N design variables; g_j is the j-th constraint (state parameter); $\overline{g_j}$ is the upper bound of constraints; $f(\boldsymbol{\eta})$ is the objective function (the volume of the structure), K is the number of constraints. The design variables N represents a pseudo-density of each finite element of the structures that varies between 0 and 1.

A following topology optimization algorithm is proposed (see Fig. 1) [16]:

1) calculation and record of stress values of for the M loading steps;

2) checking the constraint limits (e.g. von Mises eqv. stress); if the state parameter crosses its bound limit, a layer of finite elements is added to the structure boundary;

3) selection and removal of the groups of low stressed finite elements ($\sigma < \sigma_{MIN}$); the bound value σ_{MIN} is slowly increased at every iteration;

4) checking the number of interation '*loop*', if it exceeds the maximum number L_{MX} the optimization process is stopped, otherwise go to 1).

The finding of the optimum solution is assured by use of the mechanism of the changing algorithm direction. The algorithm in standard way starts from decreasing of the material. But, when the structure crosses the bound value of the state parameter (e.g. equivalent stress), it can switches to adding material. The process of material expansion is continued until stresses returns to admissible values [16]. This procedure can be compared to the algorithm of simulates annealing [17]. By repeating, the algorithm enlarging and decreasing the structure and goes to the best topology of the structure. As it was observed, for the definite value of loads, the obtained solution is accepted on the unique optimum layout.

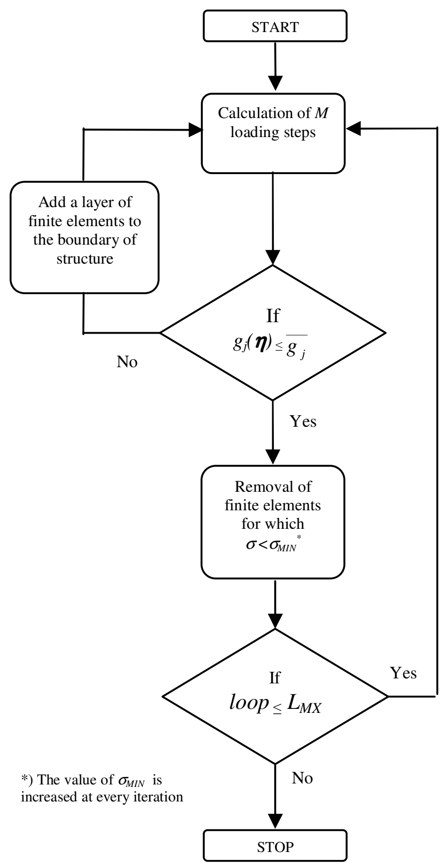

Figure 1. The topology optimization algorithm

For the fatigue topology optimization, the damage matrix should be constructed for every iteration consisting of several load steps. The stress matrix values for load cases are added according to Rainflow Cycle Counting rules [18]. It means that only maximum values between two compared matrices are transferred to the resulting damage matrix . The final matrix represents the cumulative damage matrix of the whole loading sequence.

FATIGUE CRITERIA FOR STRUCTURAL OPTIMIZATION

The computational time is very important from the point of view of structural optimization effectiveness. The complicated and non-linear numeric analyses make the optimization process of real technical objects very difficult.

The review of numerical convenient criteria of the multiaxial high-cycle fatigue was made by Ballard et al. [19] . Singh et al. [20], Desmorad [21], Mrzyglod and Zielinski [11] proposed fast damage estimation methods for the multiaxial low-cycle fatigue.

The Dang Van criterion was chosen for the high-cycle optimization investigation [19,22]. This MHCF formula joins a high accuracy with computational effectivness. The successful examples of use Dang Van's criterion to size and topology optimization is shown in Mrzyglod and Zielinski [6,7], Mrzyglod [10].
The hypothesis is based on average stresses in an elementary volume *V*, it takes into consideration the average value of shear and normal stresses in this volume. Dang Van formulated his hypothesis observing local plastic deformations on a microscopic scale, on the level of crystallites. They can initiate micro-cracks even then, when a studied structure remains in macroscopic scale in a range of elastic strains. According to Dang Van the fatigue damage appears in a definite time, when the combination of local shear stresses $\tau(t)$ and a hydrostatic stress $\sigma_H(t)$ cuts the borders of an admissible fatigue area.
The numerical convenient form of criterion is as follows [19]:

$$\max_A \left[\tau(t) + \kappa_1 \sigma_H(t)\right] \leq \lambda \tag{3}$$

where: A is the area of studied object,

$$\tau(t) = \frac{\sigma_1(t) - \sigma_3(t)}{2},$$

$$\sigma_H(t) = \frac{1}{3}\left(\sigma_1(t) + \sigma_2(t) + \sigma_3(t)\right).$$

The material parameters can be expressed by data from two high-cycle fatigue tests: reversed bending (fatigue limit f_{-1}) and reversed torsion (fatigue limit t_{-1}). For the criterion $\lambda = t_{-1}$, $\kappa_1 = 3\, t_{-1}/f_{-1} - 3/2$.

The low-cycle fatigue analysis deal with the non-linear calculations. This has clear influence on the efficiency of the structural optimization. The obstacle can be avoiding by an application of the Neuber formula [23], the method of an approximate estimation of non-linear deformations [11,20,21].
The low-cycle fatigue criterion, which allows for quick estimation of low-cycle damage is based on the Ramberg-Osgood material defined by the formula:

$$\varepsilon_{aeq} = \frac{\sigma_{aeq}}{E} + \left(\frac{\sigma_{aeq}}{\kappa}\right)^{\frac{1}{n}} \tag{4}$$

where σ_{aeq}, ε_{aeq} are equivalent stress and strain, respectively, E is the Young modulus and κ, n are material constants.
Moreover, the modified Manson-Coffin-Basquin model of low-cycle fatigue:

$$\varepsilon_{aeq} = \varepsilon^{e}_{aeq} + \varepsilon^{p}_{aeq} = \frac{\sigma'_f}{E}\left(2N_f\right)^{b} + \varepsilon'_f \left(2N_f\right)^{c} \tag{5}$$

where $\varepsilon^{e}{}_{aeq}$, $\varepsilon^{p}{}_{aeq}$ are elastic and plastic equivalent strains, respectively, N_f is a number of cycles and σ'_f , ε'_f , b, c are material constants. Between these constants and κ; in Eq. 4 are the following relation. The admissible stress σ_{aeq}^{adm} in case of low-cycle fatigue constitutive Eq. 5 takes the form:

$$\sigma_{aeq}^{adm} = \sigma'_f \left(2N_f\right)^b \tag{6}$$

The modified Neuber model:

$$\sigma_{ij}^{e}\varepsilon_{ij}^{e} = \sigma_{ij}^{e-p}\varepsilon_{ij}^{e-p} \tag{7a}$$

or

$$\sigma_{aeq}^{e}\varepsilon_{aeq}^{e} = \sigma_{aeq}^{e-p}\varepsilon_{aeq}^{e-p} \tag{7b}$$

where the indices *e* , *e-p* mean elastic and elastic-plastic quantities, respectively, while σ_{aeq}^{e} and ε_{aeq}^{e} are equivalent stress and strain understood in sense of the Beltrami [24] total strain energy density hypothesis (TSED), which includes the whole energy.
The left side of Eq. 5 can be expressed by Eq. 7b as

$$\varepsilon_{aeq}^{e-p} = \frac{\left(\sigma_{aeq}^{e}\right)^2}{E\sigma_{aeq}^{e-p}} \tag{8}$$

hence, we obtain the final form of the energetic constraint:

$$\frac{\left(\sigma_{aeq}^{e}\right)^2}{2E} \leq \frac{\left(\sigma'_f\right)^2}{2E}\left(2N_f\right)^{2b} + \frac{1}{2}\sigma'_f\,\varepsilon'_f\left(2N_f\right)^{c+b} \tag{6}$$

In this constraint the left side contains only elastic results and the right-hand side depends only on the material constants and number of cycles. Obviously, we can express in this way the elastic-plastic strains using in Eq. 8 the adequate, purely elastic results and the material constants. This way of proceeding evidently shortens the computer time of the optimization procedure.

TOPOLOGY OPTIMIZATION EXAMPLE

The Michell's problem [25] of optimizing truss topology for stress constraints under a variable load condition was selected as an optimization example. The example is based on the Lewiński-Rozvany analytical benchmarks for topological optimization IV [26]. The FE model of the example structure with boundary conditions is shown in Fig. 2, the design space dimensions are proportional to length L = 16a and breadth B = 10a. For the realization of the example as well as the topology optimization procedure the ANSYS® APDL script language was used [27].

Two load cases were applied for the chosen structure, pulsating (R = 0) and symmetrical (R = -1). The conducted test's results for criteria Dang Van, TSED and von Mises were presented in Figures 3-6.

Consideration that the criteria of low and high-cycle fatigue are characterized by different damage limits, in computational examples load values were artificially set on the levels that give the similar volumes of structures.

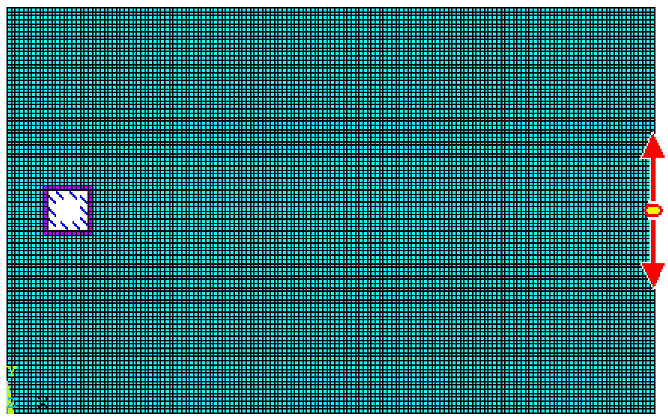

Figure 2. 2D FE model with displacement boundary conditions (along the small square) and pulsating (R = 0) and symmetrical (R = -1) loads.

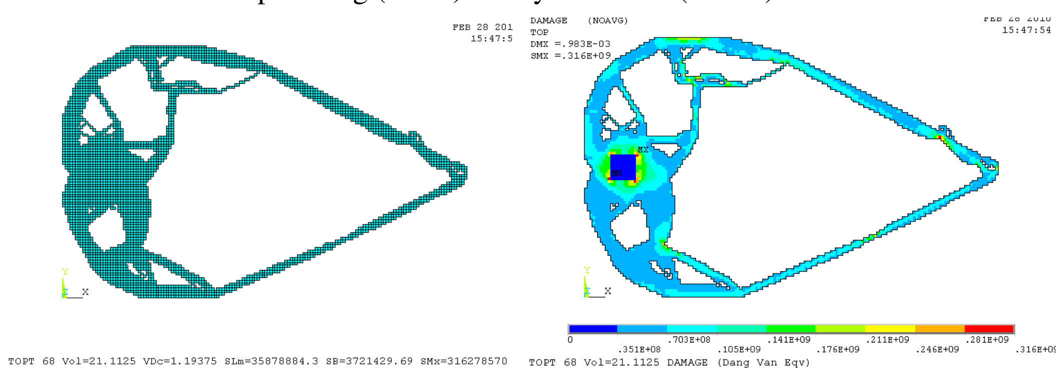

Figure 3. Results for the Dang Van criterion, structure layout for pulsating load

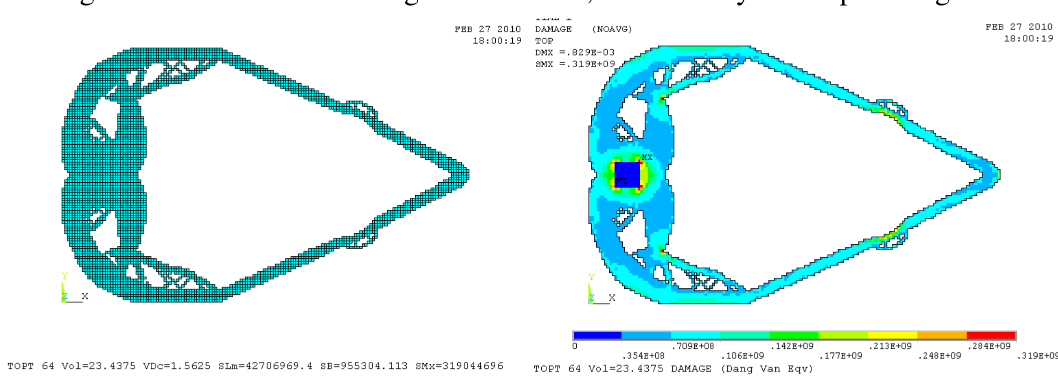

Figure 4. Results for the Dang Van criterion, structure layout for symmetrical load

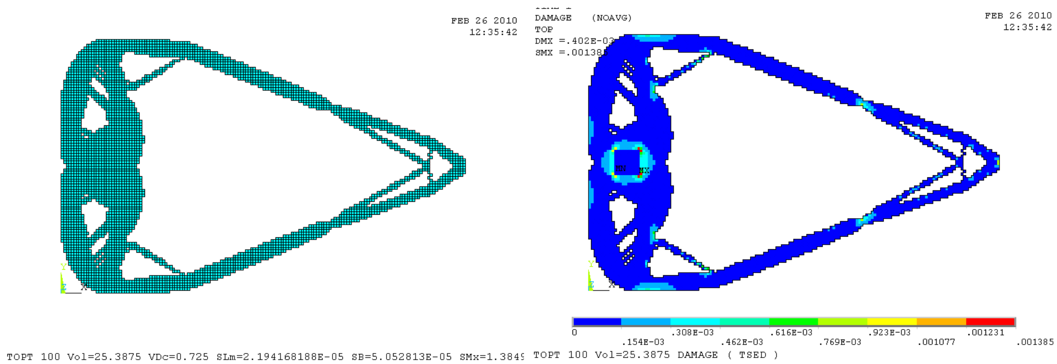

Figure 5. Results for the TSED criterion, structure layout for symmetrical load

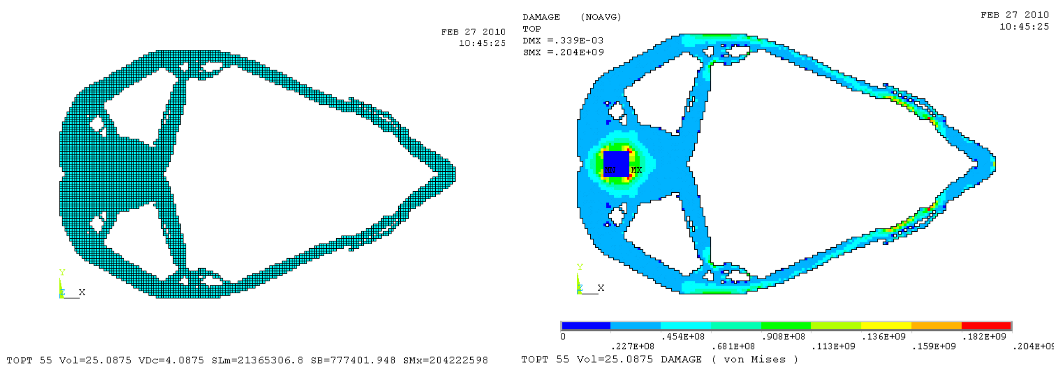

Figure 6. Results for the von Mises criterion, structure layout for symmetrical load

CONCLUSIONS

The presented methodology of topology optimization consist in application of low and high-cycle constraints to automatic design of the structural layout. One can conclude from presented experiments, that low and high-cycle constraints give sometimes different results in optimized structures. It can be clearly seen that the Dang Van criterion discloses the effect of shear stresses in the structure. The optimization algorithm in pulsating and symmetrical load conditions forms entirely different topologies for the high-cycle constraints. On the contrary, the von Mises and TSED low-cycle criteria give identical structure layouts. Both, low and high-cycle criteria caused similar results of topology optimization for oscillating load conditions. However, the von Mises criterion leads to a layout similar to Michell's analytic solutions [26].

In opinion of the author, the method of topology optimization with multiaxial HCF constraints can be recommended for design of fatigue resistant structures.

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