Multiaxial Fatigue Life Assessment of Components of Forged Steel Ck 45 (SAE 1045) and of Sintered Steel Fe-1.5Cu by Integration of Damage Differentials (IDD)

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ABSTRACT. *IDD does not rest on the notions of cycles and damages per cycles but on loading differentials* d*s and damage differentials* d*D per* d*s. Thus, IDD proposes a completely different strategy of using S-N lines under cyclic proportional (in-phase) loadings for fatigue life assessment under any non-cyclic or/and non-proportional (out-of-phase) stress-time functions* $\sigma_x(t)$, $\sigma_y(t)$ *and* $\tau_{xy}(t)$. *Two empirical factors of loading non-proportionality,* f_c *and* f_τ, *are involved. For their correct selection, IDD needs a lot of verifications until a reliable* f_c *and* f_τ *data bank is accumulated. Preliminarily envisaged values of* f_c *and* f_τ *were 2 and 3. They had proved correct in five preceding verifications. One of them, with components of forged steel Ck 45 (SAE 1045), is represented in the paper. In addition, the same experimental tests of the same components but made of sintered steel Fe-1.5Cu were also used for a new IDD verification. It turned out that the sensitivity of that steel to the loading non-proportionality, including to the intensive rotation of the principal axes, was low: both* f_c *and* f_τ *had to be reduced to 1 in order to obtain satisfactory IDD lives. The paper is a contribution to development of the* f_c *and* f_τ *data bank and to the IDD techniques.*

INTRODUCTION

IDD, as a concept, seems to really have a potential: it proposes a completely different strategy of using input *S-N* lines under cyclic (constant-amplitude) proportional (in-phase) loadings for fatigue life assessment under any other uniaxial or multiaxial, proportional (in-phase in particular) or non-proportional (out-of-phase in particular) loading. The main difference from the previously known approaches is that IDD does not rest on the notions of cycles and damages per cycles but on the notions of loading differentials d*s* and damage differentials d*D* per the loading differentials d*s*. This allows the stress-time functions $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ of plane state of stress to be directly and universally processed as they are (with arbitrary variations), differential by differential.

Thus, there is no necessity of preliminary decomposition in cycles (the rain-flow procedure etc.), nor any need to apply some of the various fatigue life hypotheses.

This paper is supposed to be read as continuation from [1, 2]. The IDD concept and its current implementation, the present IDD method and software, are generally represented in [1]. Details and verification results are available in [2]. By the way, any detail is given in full on the IDD site: http://www.freewebs.com/fatigue-life-integral/.

Each differential ds is represented as a small finite element (finite difference) Δs (d ~ Δ). It is resolved into a proportional radial element Δs_r, a non-proportional circumferential element Δs_c and a non-proportional element $\Delta\tau$ in the IDD coordinate space σ'-σ''-$\Delta\tau$ [1, 2] where σ' and σ'' are the principal stresses. The elements Δs_r and Δs_c lie in the σ'-σ'' plane and $\Delta\tau$ is perpendicular to it. While Δs_r and Δs_c are described, the principal axes ' and " are immovable; $\Delta\tau$ only appears in case the axes ' and " rotate. The part of ΔD (~ dD) per Δs_r involves damage intensity R_r derived from the input S-N lines. The parts of ΔD per Δs_c and $\Delta\tau$ involve damage intensities $R_c = f_c R_r$ and $R_\tau = f_\tau R_r$. The life ends when the sum (the integral) of all the differentials dD ~ ΔD reaches 1. Hence, the currently used IDD life equation is deduced: Eq. (5) in [1].

The two factors of the loading non-proportionality, f_c and f_τ, are new empirical fatigue life characteristics. For their correct selection, IDD needs a lot of verifications until a reliable f_c and f_τ data bank is accumulated. Or, until some additional physical considerations may help in terms of sensitivity of different materials and structural components to the damaging influence of the non-proportional elements Δs_c and $\Delta\tau$.

Other more IDD parameters are the numbers N_r, N_c and N_τ [1, 2] which limit the zero R_r, R_c and R_τ areas in the σ'-σ'' plane: their borderlines are the lines of equal lives N_r, N_c and N_τ [2]. Techniques for correct selection of N_r, N_c and N_τ should also be acquired.

THE FACTORS f_c AND f_τ, ALSO THE PARAMETERS N_r, N_c AND N_τ, FROM THE PREVIOUS IDD VERIFICATIONS

In Fraunhofer LBF Institute, the present IDD method and software [1] was subjected to five verifications (1) – (5) in several cases of materials, components and non-proportional loadings: out-of-phase and of different frequencies (see the legend in Fig. 1). The preliminarily envisaged values of f_c and f_τ were 2 and 3 suggested from an initial adaptation (0) (see Fig. 1). They proved correct in the five verifications where the average of 20 ratios N_{cmp}/N_{exp} (of the IDD computed and experimental lives) turned out 1,03 with 0,45 standard deviation.

The whole N_{cmp}-N_{exp} diagram in Fig. 1 (including (0)) has 1,00 average of 38 ratios N_{cmp}/N_{exp} and 0,53 standard deviation from the average (the minimum of N_{cmp}/N_{exp} is 0,27 and the maximum is 2,86). These data speak very well for f_c = 2 and f_τ = 3 as values valid universally under: out-of-phase or different-frequency combined loadings, for steel or cast iron, notched or unnotched specimens, number of cycles to rapture or to a crack. Besides, both life increase (in verification (1)) and decrease (in all the other verifications) due to the out-of-phase angle were correctly evaluated.

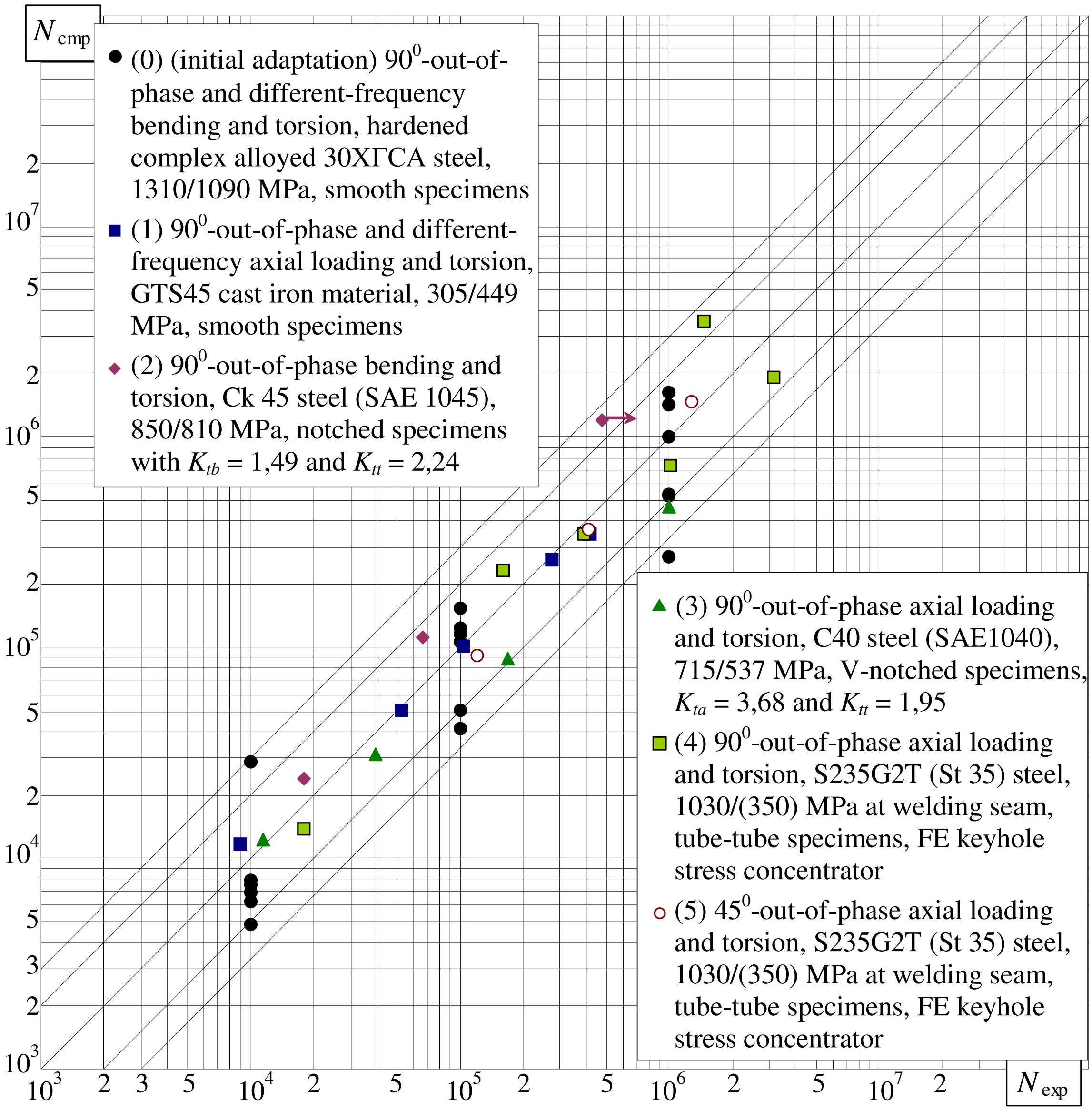

Figure 1. N_{cmp}-N_{exp} diagram from the previous verifications (0) – (5) [3, 2]

Despite the very good confirmation of f_c = 2 and f_τ = 3, there had been realistic expectations that, under non-proportional bending or axial loading and torsion, the sensitivity of all the materials and components to the loading non-proportionality would hardly appear always with the values 2 and 3 for the factors f_c and f_τ. Looking for exceptions, an additional "provocative" IDD verification was chosen to be done and represented in this paper. The occasion occurred when comparing experimental LBF results in [4] with the verification (2) [3]: the same tests of the same components but made of sintered steel Fe-1.5Cu [4] instead of forged steel Ck 45 (SAE 1045) [3], and available increase [4] in the life due to the 90^0 out-of-phase angle instead of the decrease in the verification (2) [3].

THE IDD VERIFICATION (2) AND THE RESULTS FROM IT

Figure 2 was taken from [3] and rearranged. It contains as many data as necessary for this paper; more details can be found in [3]. The ◆ points turned out shifted to the left from the ▲ points: there is life decrease due to the 90^0 out-of-phase angle.

The three averaged experimental "90^0-out-of-phase" lives are 17862, 66288 and >474500 cycles. The corresponding IDD computed lives proved 23930, 111100 and 1212000, i.e. the N_{exp}/N_{cmp} values are 1,34, 1,67 and <2,55. The respective IDD computed "90^0-out-of-phase" *S-N* line proved to be shifted to the left from the "in-phase" *S-N* line: IDD correctly evaluated the life decrease. These verification results were considered as confirming the settings $f_c = 2$, $f_\tau = 3$ and $N_c = N_\tau = N_r$ (what had been found as $1{,}4.10^6$).

Additional IDD computation was done in [3] setting also $N_c = N_\tau > N_r$. Then the computed "90^0-out-of-phase" *S-N* line agrees better with the ◆ points. On this occasion it was remarked [3] that the IDD user may always prefer, in favor of the safety, greater values of N_c and N_τ than N_r. Thus, the zero R_c and R_τ areas are compressed closer to the coordinate origin in the σ'-σ'' plane.

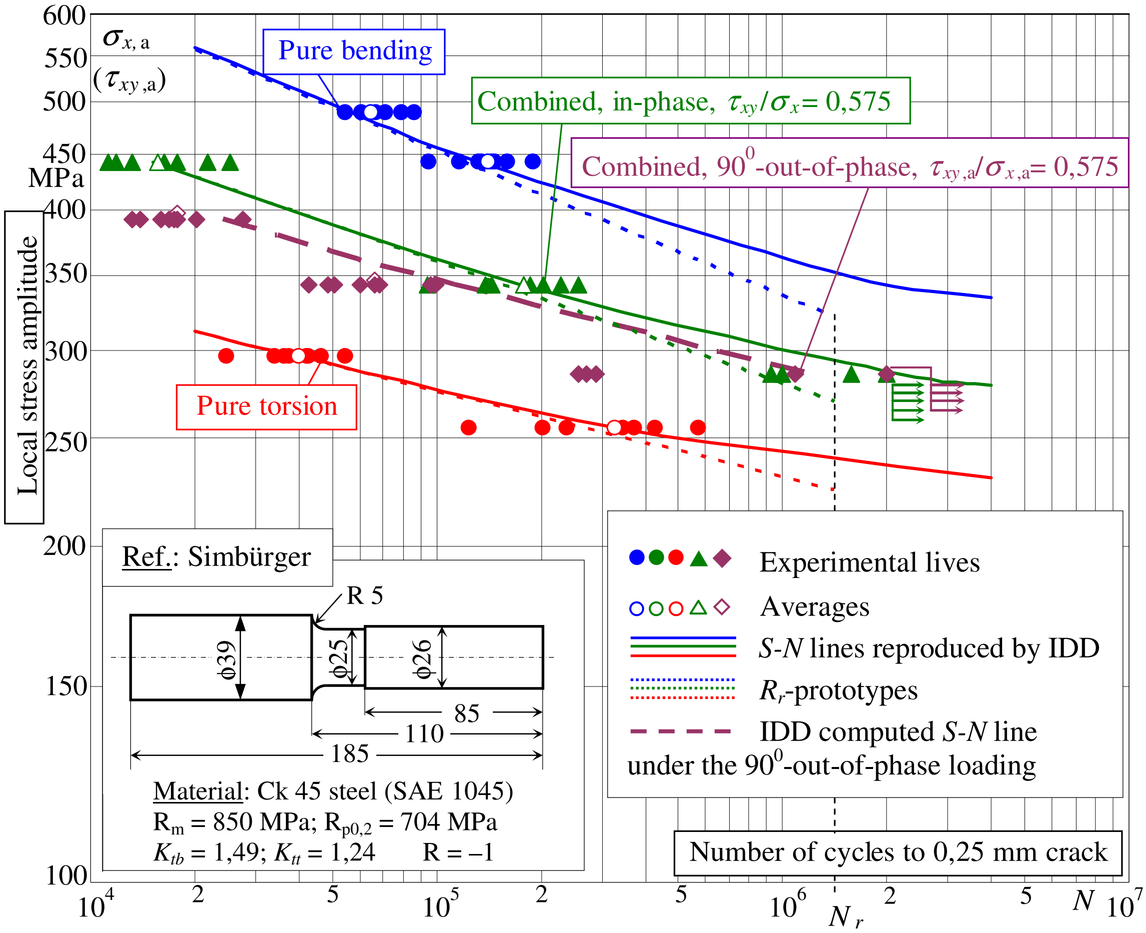

Figure 2. Illustration to the IDD verification (2) and the results from it

THE SIMILAR IDD VERIFICATION BUT WITH SINTERED STEEL

Experimental data [4, 5] and composing the R_r-prototypes

Figure 3 illustrates the experimental lives under pure bending, pure torsion and combined in-phase (proportional) bending and torsion with τ_{xy}/σ_x = 0,575. They can serve for composing the input R_r-prototypes' equations of the kind $s^m N = s_r^m N_r = A$ = constant [1, 2, 3]. As already known [1, 2], from the R_r-prototypes IDD reproduces corresponding smoothly bending *S-N* lines in the so-called smooth mode. They have equations of the kind $(s^m - s_r^m) = A$. The *S-N* lines should agree well enough with the scattered experimental lives and their averages. Besides, N_r should be obtained the same [1, 2] for all the R_r-prototypes.

Thus, some calculation trials are necessary for obtaining N_r together with s_r and m for each R_r-prototype. After doing these trials, the R_r-prototypes' expressions $s_r^m N_r = A$ under pure bending and pure torsion were obtained as $211^{8{,}432}3.10^6$ and $125^{6{,}212}3.10^6$, with $N_r = 3.10^6$. Their straight lines are shown in Fig. 3. The corresponding reproduced *S-N* lines (Fig. 3) have the equations $(\sigma_{x,a}^{8{,}432} - 211^{8{,}432})N = 211^{8{,}432}3.10^6$ and $(\tau_{xy,a}^{6{,}212} - 125^{6{,}212})N = 125^{6{,}212}3.10^6$ providing good agreement with the points ● and ●.

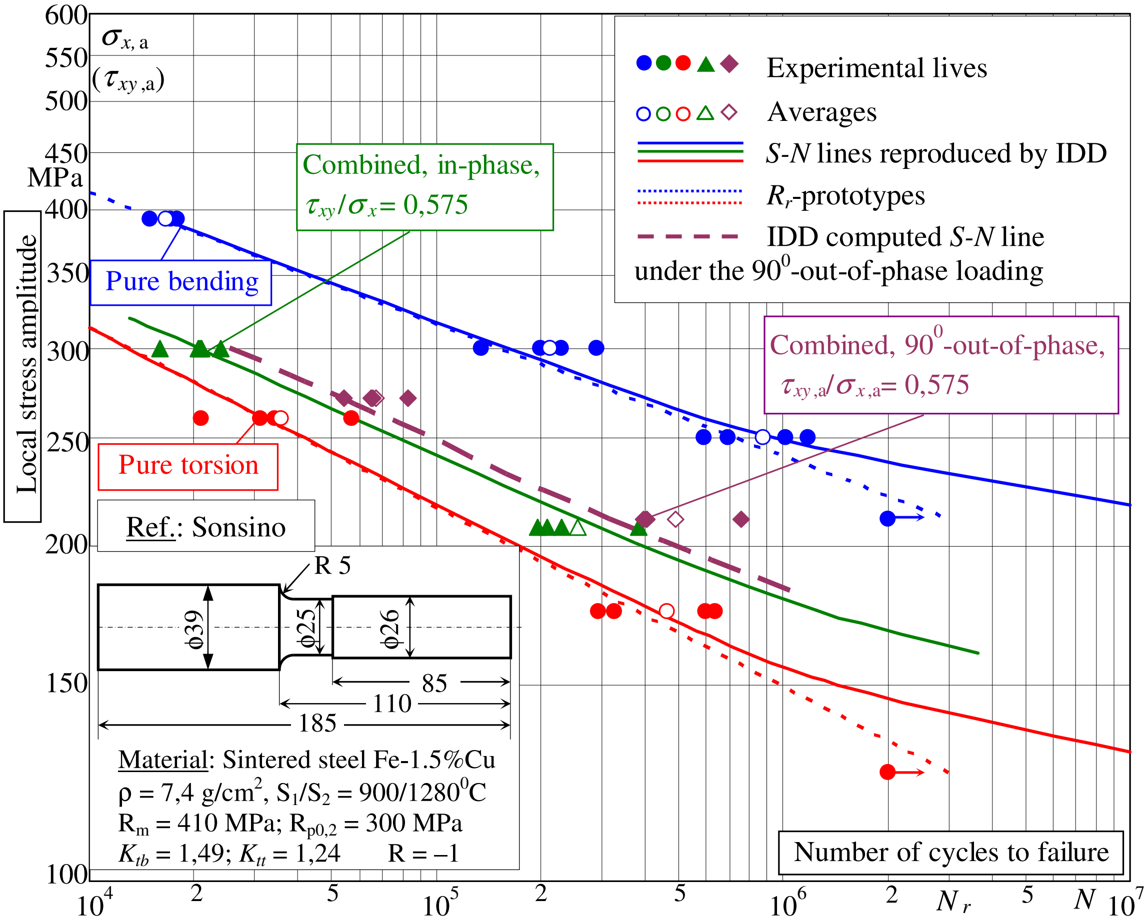

Figure 3. Illustration to the IDD verification with the sintered steel

By the way, the Excel file relating to Fig. 3, as well as the other files mentioned next, will be available on the IDD site after ICMFF9 is held, or can be provided in person.

IDD computation of the "in-phase" S-N line

At least two input R_r-prototypes are necessary to use the IDD computer program *EllipseT* [1, 2, 3]: the two ones above, under pure bending and pure torsion. They already form elliptic lines of equal lives [2] (which can be preliminarily seen in Fig. 4). If more than two R_r-prototypes are provided, they are definitely necessary only in case they do not agree well with the same elliptic lines. Thus, in the previous verification (2), the third R_r-prototype, under the combined in-phase loading, proved to be definitely necessary.

But now, using only the two above R_r-prototypes, it turned out that the "in-phase" *S-N* line, computed by IDD, is in good agreement with the ▲ points (Fig. 3). Hence, this line does not need its own R_r-prototype. For IDD computation of this line, the following levels of $\sigma_{x,\mathrm{a}}$ were picked out: 320, 300, 208, 175 and 160 MPa. Corresponding input C-files [1, 2] were composed by *EllipseT* and named C320-0, C300-0, C208-0 and C160-0 ("-0" means zero out-of-phase angle). Each of them contains the ordinates of in-phase sinusoids $\sigma_x(t) = \sigma_{x,\mathrm{a}}\sin\omega t$, $\sigma_y(t) = 0{,}21\sigma_{x,\mathrm{a}}\sin\omega t$ and $\tau_{xy}(t) = 0{,}575\sigma_{x,\mathrm{a}}\sin\omega t$.

Meanwhile it is to note that the stress-concentration factor $K_{tb} = 1{,}49$ (Fig. 3) leads to the ratio $\sigma_y/\sigma_x = 0{,}21$, as cleared in [3] (Neuber diagram is used).

Together with those four input C-files, an L-file [1] was also entered in *EllipseT* (any of the two L-files discussed below). *EllipseT* computed the respective four lives and, based on them, the "in-phase" *S-N* line was drawn in Fig. 3.

While processing any of the above C-files, a radial straight-line trajectory is displayed in the IDD graph mode [1]. For example, from C320-0, the radial straight line appears as shown in Fig. 4 (that line was copied from a different *EllipseT* window and overlapped in Fig. 4). It has the ratio $k = \sigma''/\sigma' = -0{,}071$ which can be derived [3] from $\tau_{xy}/\sigma_x = 0{,}575$ and $\sigma_y/\sigma_x = 0{,}21$. The rest three straight-line trajectories lie on the same radial line. By the way, in Fig. 4, two other radial lines are labeled with $k = 0{,}21$ and $k = -1$. The two input R_r-prototypes are valid for them.

IDD computation of the "90^0-out-of-phase" lives

As shown in Fig. 3, there are two averaged experimental "90^0-out-of-phase" lives. They are $6{,}7.10^4$ and $4{,}9.10^5$ at the levels 271 and 211 MPa. Two C-files were created by means of *EllipseT*: C271-90 and C211-90. In addition, for obtaining two more points of a longer IDD computed *S-N* line under the 90^0-out-of-phase loading (Fig. 3), files C300-90 and C180-90 were also composed. The τ_{xy} sinusoid is 90^0 out of phase in these files.

First, an L-file [1, 2] named L23 was made. In it, $f_c = 2$ and $f_\tau = 3$ were set, as well as $N_c = N_\tau = N_r$ (= 3.10^6), in accordance to the previous verifications (0) – (5). Against the experimental lives $6{,}7.10^4$ and $4{,}9.10^5$, the computed ones proved $2{,}13.10^4$ and $1{,}289.10^5$, i.e. $N_{\mathrm{cmp}}/N_{\mathrm{exp}} = 0{,}32 = 3{,}1^{-1}$ and $N_{\mathrm{cmp}}/N_{\mathrm{exp}} = 0{,}26 = 3{,}8^{-1}$. In reference to the N_{cmp}-N_{exp} diagram (Fig. 1), these results would be below the line of the 3 error factor. They could hardly be interpreted as scattered confirmation of $f_c = 2$ and $f_\tau = 3$. Besides,

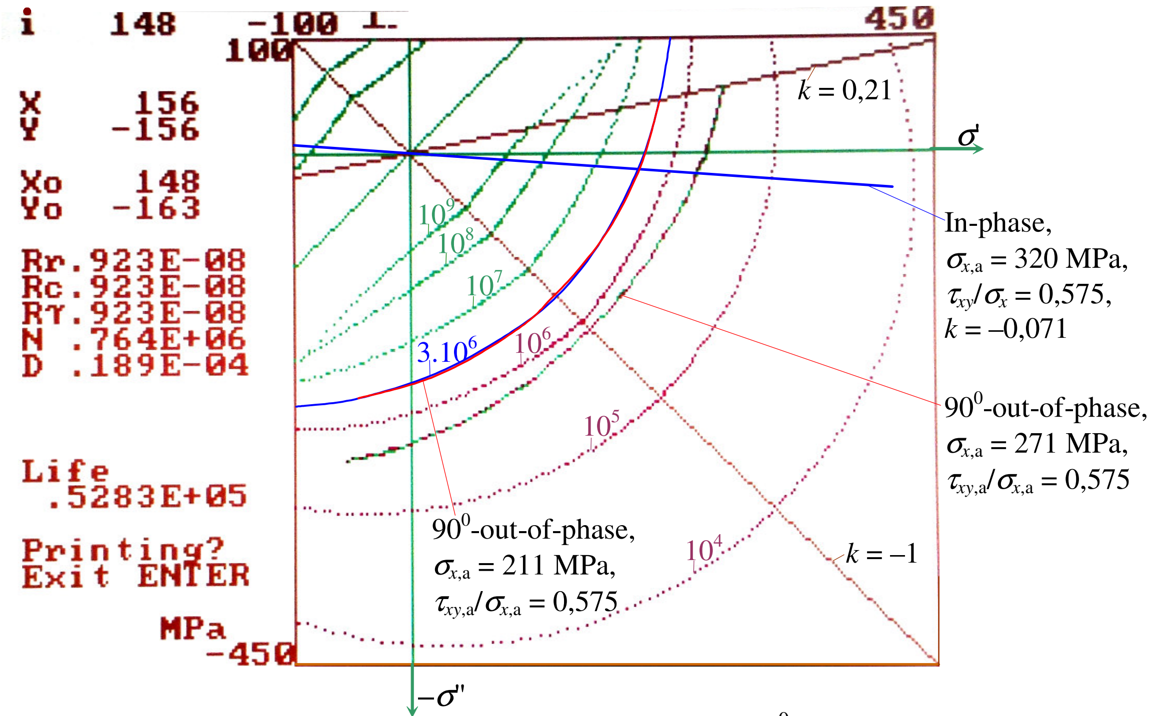

Figure 4. *EllipseT* graph window for computing the "90^0-out-of-phase" life with $\sigma_{x,a}$ = 271 MPa, and additional graph items overlapped

instead of the experimental "90^0-out-of-phase" life increase, a decrease is assessed.

Therefore, L11 was made with $f_c = 1$ and $f_\tau = 1$. According to the considerations in the IDD site, f_c and f_τ are not expected to be less than 1: $f_c = 1$ and $f_\tau = 1$ are considered minima. The computed life from C271-90 is $5{,}283.10^4$ now (see Fig. 4) i.e. $N_{cmp}/N_{exp} = 0{,}79 = 1{,}25^{-1}$: fairly well. The additional computed life from C300-90 is $2{,}527.10^4$.

However, if keeping $N_c = N_\tau = N_r$ (= 3.10^6) again, then computed "90^0-out-of-phase" lives below the level 211 MPa will immediately become infinity (i.e. no damage). Indeed, as shown in Fig. 4, the "90^0-out-of-phase" trajectory with $\sigma_{x,a}$ = 211 MPa (from C211-90) practically coincides with the borderline between the zero and non-zero damage intensity areas: the line of equal life $N_r = 3.10^6$.

That is why, as already recommended in [3], $N_c = N_\tau$ were set in L11 greater than $N_r = 3.10^6$: $N_c = N_\tau = 10^7$, just to make sure that the trajectory from C180-90 will also be in non-zero R_c and R_τ areas. Then, from C211-90, the computed life is $3{,}13.10^5$ (and from C180-90 it is $1{,}141.10^6$). Against the experimental lives $6{,}7.10^4$ and $4{,}9.10^5$, the averaged ratio N_{cmp}/N_{exp} from C271-90 and C211-90 is $0{,}71 = 1{,}4^{-1}$. The IDD computed *S-N* line under the 90^0-out-of-phase loading (Fig. 3) is shifted to the right from the "in-phase" line, i.e. the experimental "90^0-out-of-phase" life increase is correctly assessed.

DISCUSSION AND CONCLUSIONS

It turned out that the sensitivity of the sintered steel Fe-1.5Cu steel to loading non-proportionality is low: both f_c and f_τ had to be reduced to 1 in order to obtain satisfactory IDD lives. Thus, so far, the empirical data bank for f_c and f_τ, being developed, seems so: $f_c = 1$ and $f_\tau = 1$ for sintered steels, and $f_c = 2$ and $f_\tau = 3$ for other steels. As well, $f_c = 2$ and $f_\tau = 3$ for cast iron materials: in the verification (1), IDD exactly assessed the available experimental "90^0-out-of-phase" life increase.

The values $f_c = 2$ and $f_\tau = 3$ could be still universally set for every non-proportional loading. Then, at least for sintered steels, the life assessment error will be on the safety side. By the way, until the f_c and f_τ data bank is not fully reliable, the IDD user will be advised to use values of f_c and f_τ greater than 2 and 3 in favor of the safety.

The IDD concept is worth recommending next joint-effort development and large-scale application for enlarging and précising the f_c and f_τ data as a new kind of characteristics of material's fatigue behavior. Especially f_τ data of sintered steels under pure $d\tau$-loading [1] (constant principal stresses in rotating principal directions) would be very indicative and interesting. This emphasizes once again that researchers have to be engaged for solving the tough problem of experimental implementation of the pure $d\tau$-loading (see the IDD site). The interest to pure c-loading [1] should also be increased, now for testing sintered steels under such loading, as well.

By the way, the Von Mises ratio $\tau_{xy,a}/\sigma_{x,a} = 0{,}577$ provides the most intensive rotation of the principal axes under the 90^0 out-of-phase angle [3]: the trajectory ratio t_τ [1] is 0,61. And, any other fatigue life assessment method needs essential revision in the case of sintered steel [4, 5]. Maybe appropriate physical material considerations would explain and evaluate the low sensitivity of this steel to loading non-proportionality.

As to the (extrapolated) numbers N_c and N_τ, it became more apparent that they should be selected greater than N_r, especially if f_c and f_τ are set lower than 2 and 3. In general, contracting the zero R_r, R_c and R_τ areas closer to the coordinate origin is in favor of safety and in agreement with the opinion that there should be no fatigue limit under realistic conditions [6]. Instead, fatigue strength at a specified number of loading cycles should be considered.

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