Multiaxial Fatigue Assessment for Notched Bars under Non-proportional Loading Using Integral Stress Approach

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ABSTRACT. *In this theoretical study an integral approach using shear stress intensity hypothesis (SIH) is evaluated on notched bars subjected to proportional as well as non-proportional multiaxial loading, i.e. on shafts. The calculation of SIH is successfully implemented into a finite element analysis and automated within commercial FE software Abaqus. In addition, approved fatigue directives (DIN 743 and FKM-Guideline) as well as critical plane approach using Smith-Watson-Topper (SWT) and Fatemi-Socie (FS) parameters are applied to estimate the fatigue strength of diverse notches and loading combinations and compared with SIH results. Observed differences among the approaches are discussed and consequently limitations of particular parameters can be enunciated.*

INTRODUCTION

Shafts and axles are typical power train components with high human and industrial safety importance. Thus reliable design and calculation methods are essential for a secure service of numerous engineering systems. In consequence of service loading (torque and bending moments) shafts are subjected to multiaxial stress state with a certain grade of non-proportionality. Furthermore, press and shrink fits as well as bearing seats have an essential effect on the structural design of the bar components resulting in shoulders, undercuts and other notches. These geometric flaws affect and contribute also to the existing stress state multiaxiality.

The design and fatigue calculation for shafts and axles is established in German standard DIN 743 [1]. This work is based on a nominal stress approach and focused on typical shaft geometries and notches as well as loadings and materials, i.e. steels. For a fatigue life assessment on more complex volumetric parts FKM-Guideline “Analytical stress assessment” [2] was developed as a standard for industrial application. Nominal as well as local stress approaches are included and for fatigue assessment under synchronous proportional loading. In case of high multiaxiality of the stress tensor components numerous fatigue theories can be applied [3]. The most of suggested fatigue parameters are based upon stresses, strains, stress invariants and strain energy density respectively. Very common is the use of the critical plane approach, which uses to be implemented also in commercial fatigue codes. Here, the fatigue failure is predicted on one certain critical plane. More accurate especially in case of high non-proportionality of the applied loading is the integral approach [4]. This theory considers dynamic stress components in all intersection planes of a volume material element. Besides of earlier presented accuracy of the particular fatigue criterions this theoretical study should demonstrate the comparability of the standardised fatigue calculation algorithms with the contemporary fatigue theories. Typical shaft geometries with notches under diverse constant amplitude loadings were used as demonstrators in this work.

FATIGUE CALCULATION STANDARDS

Nominal Stress Method

The nominal stress method is represented by the German standard DIN 743 for fatigue assessment of shafts and axles. It is valid only for steels (normalised, tempered as well as hardened) and considers solely proportional multiaxial loading combinations. The safety factor S_{DIN} is calculated using the occurring nominal tensile, bending and torque (shear) stress amplitudes σ_{zda}, σ_{ba}, τ_{ta} and the endurable amplitudes of the notched shaft σ_{zdADK}, σ_{bADK}, τ_{tADK}, Eq. 1. These follow from the alternating endurable amplitudes of the material σ_{iW} (i = zd, b and t) reduced by the construction notch factors K and the influence (ψ_{iK}) of constant mean stresses σ_{mv}, Eq. 2. $K_1(d_{eff})$ as well as $K_2(d)$ are so called size effect factors and depend on the diameter of the shaft or bar stock. Besides the both component surface factors $K_{F\sigma}$ and K_V the most important value is the fatigue notch factor β_σ. This factor is defined as a fatigue limit ratio of smooth and notched specimen, but it can be calculated also using the stress concentration factor α_σ and the geometrical size effects in notch root Eqs. 3. In n the positive effect of stress gradient G is considered. The authors of this fatigue standard specify the uncertainty of the assessment of maximal 20%. Thus, the minimal safety factor S_{DIN} is 1.2. Recommended values for practical use are between 1.5 and 2.5. More detail information about the evolution and enhancements of this algorithm can be found in [5].

$$S_{DIN} = \frac{1}{\sqrt{\left(\frac{\sigma_{zda}}{\sigma_{zdADK}} + \frac{\sigma_{ba}}{\sigma_{bADK}}\right)^2 + \left(\frac{\tau_{ta}}{\tau_{tADK}}\right)^2}} \tag{1}$$

$$\sigma_{iADK} = \frac{\sigma_{iW}(d_B) \cdot K_1(d_{eff})}{\left(\frac{\beta_\sigma}{K_2(d)} + \frac{1}{K_{F\sigma}} - 1\right) \cdot \frac{1}{K_V}} - \psi_{iK} \cdot \sigma_{mv} \tag{2}$$

$$\beta_\sigma = \frac{\alpha_\sigma}{n} \tag{3}$$

Local Stress Method

For more complex component geometry and loading cases the more universal and sophisticated FKM-Guideline "Analytical Stress Assessment" is commonly used. Besides the nominal stress method the fatigue strength calculation can be performed also using local principal stresses derived e.g. from a finite element analysis (FEA). The algorithm leads to a present cyclic grade of utilisation $a_{BK\sigma v}$ or safety factor S_{FKM} as reciprocal value which bases on the von Mises fatigue criterion, Eq. 4. The stress factors $s_{a,p}$ ($p = 1, 2$ and 3) are computed as ratio of the occurring principal stress amplitudes σ_{pa} and component fatigue strength multiplied by a total safety factor j_{ges}, Eq. 5. The minimal value of j_{ges} is set analogously with DIN 743 to 1.2. Mean stress factor $K_{AK,\sigma p}$ and construction factor $K_{WK,\sigma p}$ reduce the alternating tensile fatigue strength of the material σ_{Wzd}. The formula for the construction notch factor (Eq. 6) is consistent with Eqs. 2 and 3 except for the stress concentration (α_σ), which is already included in the local stress components σ_{pa}. The geometrical notch size factor $n_{\sigma p}$ considers similarly the stress gradient effect.

$$S_{FKM} = \frac{1}{a_{BK\sigma v}} = \frac{1}{\sqrt{\frac{1}{2} \cdot \left[(s_{a,1} - s_{a,2})^2 + (s_{a,2} - s_{a,3})^2 + (s_{a,3} - s_{a,1})^2 \right]}} \tag{4}$$

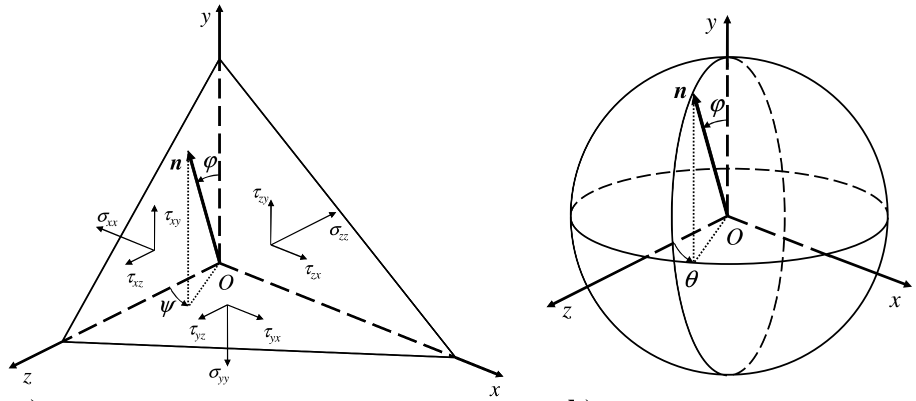
$$s_{a,p} = \frac{\sigma_{pa} \cdot j_{ges}}{K_{AK\sigma p} \cdot \dfrac{\sigma_{Wzd}}{K_{WK\sigma p}}} \tag{5}$$

$$K_{WK\sigma p} = \frac{1}{n_{\sigma p}} \cdot \left[1 + \frac{1}{K_F} \cdot \left(\frac{1}{K_{R\sigma}} - 1 \right) \right] \tag{6}$$

In general, the algorithm of the FKM-Guideline is valid for steel, cast iron as well as aluminium materials and is verified for synchronous loadings. In [6] application and evaluation of the local stress approach is presented.

CRITICAL PLANE APPROACH

Fatigue strength calculations working with the critical plane theory identify fatigue fracture plane orientation (Fig. 1) and quantify the expected fatigue life on the basis of transformed stress and strain tensors. Depending on the dominating crack type normal or shear stress components are assumed to contribute to the fatigue damage. In general, this method is applicable for proportional and non-proportional multiaxial loadings. The quality of the fatigue assessment depends essentially on the choice of the correct parameter [3].



a) **b)**

Figure 1. a) Critical plane approach, b) integral approach, after [7].

Within the aspired comparison with fatigue standards and the integral approach two of the most common parameters representing normal (Eq. 7) and shear (Eq. 8) strain dominated fatigue failures [8, 9].

$$SWT = \sigma_{N,\max} \cdot \varepsilon_{N,a} \tag{7}$$

$$FS = \gamma_{N,a} \cdot \left(1 + k_{FS} \cdot \frac{\sigma_{N,\max}}{R_{p,c}} \right) \tag{8}$$

INTEGRAL APPROACH

The integral approach acts on the assumption that stresses not only on a certain critical plane but in all planes contribute to the fatigue damage of the material. Basing on this theory Zenner et al. [10] developed shear stress intensity hypothesis (SIH) which bases on a root-mean-square of the shear stresses $\tau_{\theta\varphi}$, Eq. 9. In this hypothesis the influence of mean stress is included using alternating (W) and pulsating (Sch) fatigue strength for normal (σ) and shear (τ) stress, Eqs. 10 and 11. Analogous to the industrial fatigue standards a safety factor S_{SIH} can be expressed according to Eq. 12. In [11] Liu showed a very good agreement of the SIH with experimental result under diverse multiaxial loadings. Mean stress effect is also described as shown in [12].

$$\sigma_{va} = \left\{ \frac{15}{8\pi} \int_{\vartheta=0}^{\pi} \int_{\varphi=0}^{2\pi} \left[a\tau_{\vartheta\varphi a}^2 (1 + m\tau_{\vartheta\varphi m}^2) + b\sigma_{\vartheta\varphi a}^2 (1 + n\sigma_{\vartheta\varphi m}) \right] \sin\vartheta d\vartheta d\varphi \right\}^{\frac{1}{2}} \tag{9}$$

$$a = \frac{1}{5}\left[3\left(\frac{\tau_W}{\sigma_W}\right)^2 - 4 \right] \qquad b = \frac{1}{5}\left[6 - 2\left(\frac{\tau_W}{\sigma_W}\right)^2 \right] \tag{10}$$

$$a \cdot m = \frac{\sigma_W^2 - \left(\frac{\tau_W}{\sigma_W}\right)^2 \cdot \left(\frac{\tau_{Sch}}{2}\right)^2}{\frac{12}{7}\left(\frac{\tau_{Sch}}{2}\right)^4} \qquad b \cdot n = \frac{\sigma_W^2 - \left(\frac{\sigma_{Sch}}{2}\right)^2 - \frac{4}{21} a \cdot m \left(\frac{\sigma_{Sch}}{2}\right)^4}{\frac{15}{14}\left(\frac{\sigma_{Sch}}{2}\right)^3} \tag{11}$$

$$S_{SIH} = \frac{\sigma_W}{\sigma_{va}} \tag{12}$$

MODEL GEOMETRIES

The model geometry was chosen according to earlier analyses of shrink-fitted couplings. The nominal diameter D of the shaft was 40 mm. Three diverse notch geometries (shoulder, round groove and relief groove) including two radii modifications (Table 1) were selected as representative geometric demonstrators for the fatigue assessment. The smaller diameter d of the shaft was always 36 mm. The chosen five geometries are explicit included in the DIN 743 standard and so a precise fatigue strength calculation without extrapolation can be performed. The different notch radii should demonstrate the influence of the geometrical notch size effect, see Eq. 3 and 6.

Table 1. Analysed notch geometries.

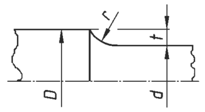
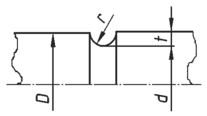
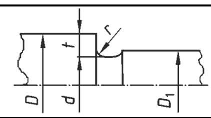
Name	Geometry parameter		Drawing
Shoulder **S10**	r = 10 mm	t = 2 mm	
Shoulder **S2**	r = 2 mm	t = 2 mm	
Groove **R10**	r = 10 mm	t = 2 mm	
Groove **R2**	r = 2 mm	t = 2 mm	
Relief groove DIN 509-E1x0.3 **RG**	r = 0.8 mm	t = 2 mm D_1 = 36.6 mm	

Figure 2 shows a finite element model of the shaft with applied boundary conditions, here in version with shoulder and radius S10. Commercial FE code Abaqus 6.8-2 was used for the numerical simulation of the loadings. Solely second order elements (C3D20R) were used within the analyses. Linear elastic material behaviour was assumed for the steel (E = 210,000, ν = 0.3).

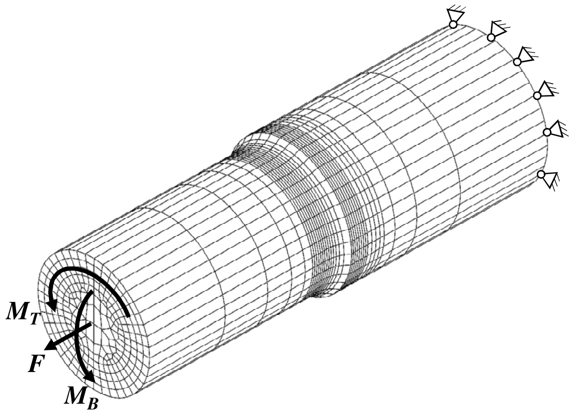


Figure 2. Finite element model with applied boundary conditions (S10).

APPLIED LOADINGS

The applied multiaxial loadings can be divided in following two categories, Table 2.

Table 2. Analysed multiaxial loadings.

Load case	Geo-metry	Tension		Bending		Torque		Phase
		F [kN]	R_F	M_B [Nm]	R_B	M_T [Nm]	R_T	φ_L [°]
LC11	S10	-	-	678	-1	678	1	-
	S2			531		531		
τ_{ta}/σ_{ba} =	R10			573		573		
0.5	R2			417		417		
	RG			324		324		
LC12	S10	64	1	-	-	971	-1	-
	S2	51				847		
	R10	56				879		
	R2	39				687		
	RG	30				583		
LC13	S10	-	-	438	1	944	-1	-
	S2			322		820		
	R10			356		861		
	R2			249		673		
	RG			186		579		
LC21	S10	-	-	678	-1	339	0	0
	R10			573		286		
	RG			324		162		
LC22	S10	-	-	678	-1	339	0	90
	R10			573		286		
	RG			324		162		
LC23	S10	-	-	678	-1	678	-1	0
	R10			573		573		
	RG			324		324		
LC24	S10	-	-	678	-1	678	-1	90
	R10			573		573		
	RG			324		324		

Load cases LC11-13 have only one dynamic stress component and were computed for infinite life (N > 5 Mio.) basing on the so called "endurance limit" of the normalised steel 1.0503 (C45E) estimated by the shaft standard DIN 743 with the safety factor $S_{DIN} = 1.2$. Loading combinations LC21-24 base on LC11 (typical industrial application) and have dynamic torque with reduced moment or phase difference φ_L (bending time leading). Here, non-proportional loading conditions were investigated.

FATIGUE STRENGTH CALCULATION

As mentioned above the fatigue assessment was performed for steel 1.0503 using following material data: R_p = 340 MPa, R_m = 620 MPa, σ_W = 260 MPa, τ_W = 170 MPa, σ_{Sch} = 470 MPa, τ_{Sch} = 323 MPa. The calculation of SWT, FS and SIH was automated within the FE-Software using C++ and FORTRAN subroutines. Finally, safety factors according to Eqs. 4 and 12 were computed. Both critical plane factors SWT and FS were normalised, i.e. related to the mean value obtained from all calculations. The results for load cases LC11-13 are shown in Fig. 3. Very good fatigue correlation was achieved using FKM algorithm. Critical plane approach was also in accordance with DIN standard, especially using FS parameter. The integral approach showed an absolute deviation, the mean value of S_{SIH} was about 2.0. An "outlier" value for R10 under LC11 was observed, which is not coincident with [13]. A possible reason for this behaviour could be the missing consideration of geometry notch and stress gradient effects (cp. Eqs. 3 and 6) within the integral approach.

Fig. 4 shows the comparison of integral and critical plane approach for non-proportional load cases LC21-24. Both fatigue parameter *SWT* and *FS* are lightly diverging whereas the *SIH* results to a certain value (approx. 1.7). At the same time the applied phase difference seems to be overrated by *SWT* and *FS*. Other authors report a positive or negligible effect of phase shift within fatigue tests [13, 14].

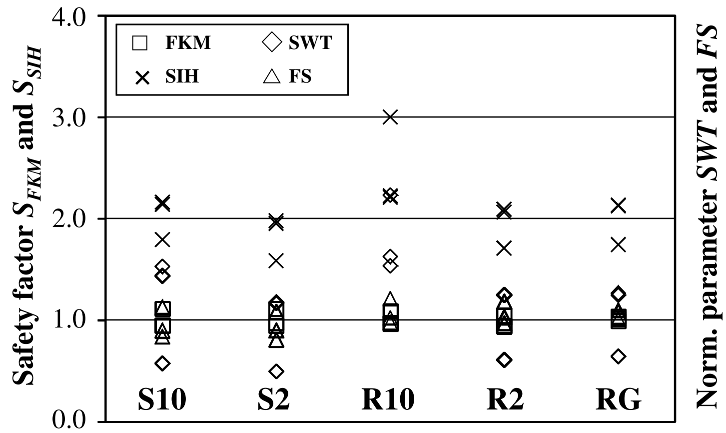


Figure 3. Comparison of safety factors and normalised fatigue parameters for LC11-13.

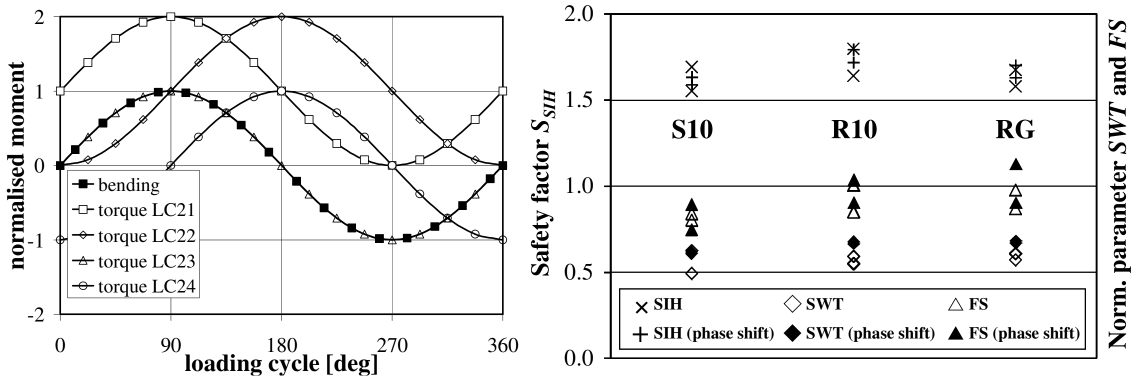


Figure 4. Comparison of integral and critical plane approach for non-proportional load cases LC21-24.

CONCLUSIONS

A general accordance of the integral approach with fatigue standards including a light overrating of the calculated fatigue strength was observed. The *SIH* does not consider the geometric notch size effects, thus differences among the notches were observed. The studied phase difference effect coincides with other experimental results.

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