

# Mean Strain Effect on Multiaxial Fatigue Behavior of Ti-6Al-4V under Non-proportional Loading

Min WU<sup>1</sup>, Takamoto ITOH<sup>2</sup>, Yuuta SHIMIZU<sup>1</sup>,  
Hiroshi NAKAMURA<sup>3</sup> and Masahiro TAKANASHI<sup>3</sup>

<sup>1</sup> Graduate Student, Graduate School of Engineering, University of Fukui

<sup>2</sup> Division of Mechanical Engineering, Graduate School of Engineering,  
University of Fukui

9-1, Bunkyo 3-chome, Fukui-shi, Fukui 910-8507, Japan

Fax: +81-(0)776-27-8748,

E-mail: itoh@u-fukui.ac.jp

<sup>3</sup> Structural Strength Department, Research Laboratory, IHI Corporation

1, Shin-Nakahara-Cho, Isogo-ku, Yokohama-shi, Kanagawa 235-8501, Japan

Fax: +81-(0)45-759-2208

E-mail: hiroshi\_nakamura\_2@ihi.co.jp

**ABSTRACT.** *Multiaxial low cycle fatigue behavior of Ti-6Al-4V under non-proportional loading is studied. Strain controlled multiaxial fatigue tests at room temperature were carried out under uniaxial and non-proportional loadings using tubular specimen. The strain paths employed were three types of proportional and two types of non-proportional loadings. The former are a push-pull straining, a reversed torsion straining and a push-pull strain with mean axial strain. The latter are a circular straining where axial and shear strains have 90 degree phase difference and a reversed torsion straining with constant axial strain. In data correlation by Mises' equivalent strain range, failure lives in the circular straining test were about 1/10 of that in the push-pull test. The failure lives in the reversed torsion with constant axial strain were affected by not only non-proportionality but also mean/constant strain. This study discusses the fatigue property and evaluation of failure life under non-proportional loading with mean strain.*

## INTRODUCTION

Ti-6Al-4V alloy is frequently used as a material in rotating aero engine because this material has properties of high strength, light weight and corrosion resistance. The rotating aero engine receives cyclic loading under thermal and mechanical stresses which cause multiaxial low cycle fatigue (LCF). Under non-proportional loading in which principal directions of stress and strain are changed in a cycle, previous studies have reported a drastic reduction in failure life accompanies an additional cyclic hardening, which depends on material [1-8]. In addition, some studies in view of mean

stress and strain effect on fatigue property under proportional loading have been reported [9,10], however extensive studies of mean stress and strain under non-proportional loading have not been reported.

In this study, multiaxial LCF tests with mean strain under non-proportional loading using Ti-6Al-4V hollow cylinder specimen are performed to discuss the effect of mean strain on multiaxial LCF behavior. This study also shows parameter for life evaluation of Ti-6Al-4V under non-proportional loading with mean strain by applying a strain parameter and definition of stress and strain under non-proportional loading proposed by author into common criterion of Findley equation [11].

## **DEFINITIONS OF STRESSES, STRAINS AND NON-PROPORTIONAL FACTOR**

In non-proportional multiaxial fatigue, principal directions of stresses and strains change during a cycle. In such a case, strain and stress ranges, and mean strain and stress cannot be easily determined. An appropriate method of determining the ranges and mean values are needed in multiaxial fatigue under non-proportional loading. This section shows a method for calculating the stress and strain ranges and mean values in non-proportional multiaxial loading.

### ***Definition of Stress and Strain***

Figure 1 illustrates three principal values,  $\mathbf{S}_i(t)$ , applied to a cube at time  $t$  together with xyz-coordinates (spatial coordinates), where “S” is the symbol denoting either stress ( $\sigma$ ) or strain ( $\epsilon$ ). This study defines the maximum principal value,  $SI(t)$ , as the maximum absolute value of the maximum or minimum principal value of  $\mathbf{S}_i(t)$  as,

$$SI(t) = \text{Max} [ |\mathbf{S}_1(t)|, |\mathbf{S}_3(t)| ] \quad (1)$$

In the equation,  $\mathbf{S}_1(t)$  and  $\mathbf{S}_3(t)$  are the maximum and minimum principal values at time  $t$  indicated in Fig.1, respectively. The “Max” denotes taking the maximum value from the two in the bracket. The maximum value of  $SI(t)$  during a cycle is taken as the maximum principal value ( $SI_{\text{max}}$ ) as follows,

$$SI_{\text{max}} = SI(t_0) = \text{Max} [ SI(t) ] \quad (2)$$

### ***Definition of Principal Stress and Strain Directions***

Figure 2 illustrates two rotation angles,  $\xi(t)/2$  and  $\zeta(t)$ , to express the direction change of principal value in XYZ-coordinates, where XYZ-coordinates are the material coordinates taking X-axis in the direction of  $SI_{\text{max}}$  and the other two axes in arbitrary directions. The rotation angle of  $\xi(t)/2$  is the angle between the  $SI_{\text{max}}$  and  $SI(t)$  directions and the rotation angle of  $\zeta(t)$  is the angle of  $SI(t)$  from the Y-axis in X-plane. The two angles of  $\xi(t)/2$  and  $\zeta(t)$  are equated as,

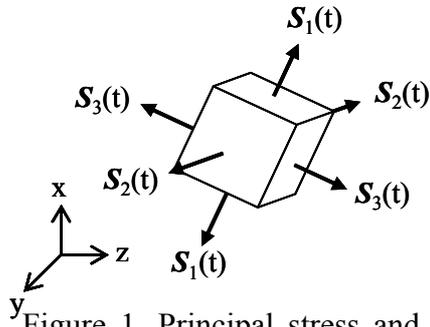


Figure 1. Principal stress and strain in xyz coordinates.

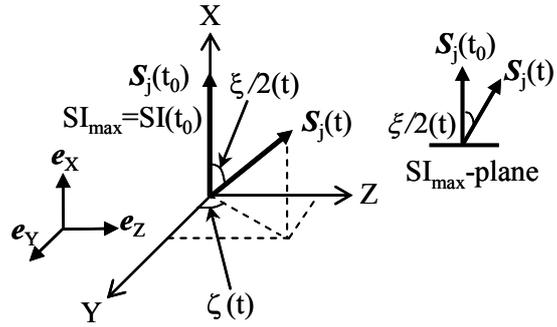


Figure 2. Definition of principal stress and strain directions in XYZ coordinates.

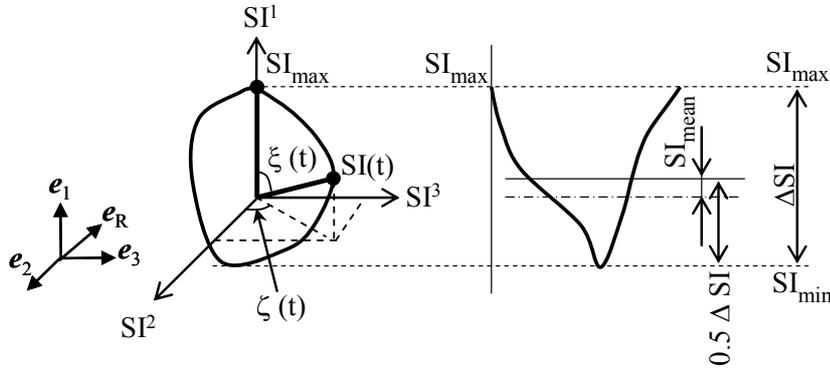


Figure 3. Definition of principal range and mean principal values.

$$\frac{\xi(t)}{2} = \cos^{-1} \left( \frac{\mathbf{S}_j(t_0) \cdot \mathbf{S}_j(t)}{|\mathbf{S}_j(t_0)| |\mathbf{S}_j(t)|} \right) \quad (0 \leq \frac{\xi(t)}{2} \leq \frac{\pi}{2}) \quad (3)$$

$$\zeta(t) = \tan^{-1} \left( \frac{\mathbf{S}_j(t) \cdot \mathbf{e}_Z}{\mathbf{S}_j(t) \cdot \mathbf{e}_Y} \right) \quad (0 \leq \zeta(t) \leq 2\pi) \quad (4)$$

where dot in Eqs 3 and 4 denotes the inner product,  $t_0$  is the time to take  $SI_{\max}$  and  $\mathbf{e}_Y$  and  $\mathbf{e}_Z$  are the unit vectors in Y and Z directions, respectively.  $\mathbf{S}_j(t)$  is a vector of principal value and the subscript  $j$  takes 1 or 3, e.g.,  $j$  takes 3 when  $SI_{\max} = |\mathbf{S}_3(t_0)|$ .

### Definitions of Stress and Strain in Polar Figure

Figure 3 shows the trajectory of  $SI(t)$  in a 3D polar figure in a cycle where the radius is taken as the value of  $SI(t)$ , and the angles of  $\xi(t)$  and  $\zeta(t)$  are the angles shown in the figure. The rotation angle  $\xi(t)$  has double amplitude compared with that in the specimen shown in Fig. 2. The principal range,  $\Delta SI$ , is determined as the maximum range of projection of  $SI(t)$  on the  $SI^1$ -axis. The  $SI^1$ -axis is the axis directing to the principal direction of  $SI_{\max}$ . The mean value of the principal value,  $SI_{\text{mean}}$ , is given as the middle value of the range.  $\Delta SI$  and  $SI_{\text{mean}}$  are equated as,

$$\Delta SI = \text{Max}[SI_{\text{max}} - \cos \xi(t) SI(t)] = SI_{\text{max}} - SI_{\text{min}} \quad (5)$$

$$SI_{\text{mean}} = \frac{1}{2} (SI_{\text{max}} + SI_{\text{min}}) \quad (6)$$

where  $SI_{\text{min}}$  is the  $SI(t)$  to maximize the value of the bracket in Eq. 5. The value of  $SI(t)$  in the polar figure have no negative value, so the sign of  $SI_{\text{min}}$  in the figure is set as follows for calculation. The sign of  $SI_{\text{min}}$  is set to be positive if it does not cross the  $SI^2$  or  $SI^3$  axis and the sign should set minus if it crosses the axis. The physical meanings of the  $\Delta SI$  and  $SI_{\text{mean}}$  given by Eqs 5 and 6 are the maximum range and the mean value of the principal value on the  $SI_{\text{max}}$ -plane.

### ***Non-proportional Strain and Non-proportionality of Loading Path***

The authors proposed the following non-proportional strain range for correlating LCF lives under non-proportional loading [4,5,7,8].

$$\Delta \varepsilon I_{\text{NP}} = (1 + \alpha f_{\text{NP}}) \Delta \varepsilon I \quad (9)$$

In the equation,  $\Delta \varepsilon I$  is the principal strain range stated previously and  $\alpha$  is a material parameter expressing the amount of additional hardening by non-proportional loading [4,5]. Another method for determining  $\alpha$  is to take the same lives between in the push-pull and circular strainings at the same  $\Delta \varepsilon I$  [5].

$f_{\text{NP}}$  is the non-proportional factor that expresses the severity of non-proportional loading equated as using the vector product of unit vectors,

$$f_{\text{NP}} = \frac{1}{4 (SI_{\text{max}})^2} \int_C |\mathbf{e}_1 \times \mathbf{e}_R SI(t)| ds \quad (10)$$

## **EXPERIMENTAL PROCEDURE**

Material tested was Ti-6Al-4V which received solution treatment at 960°C for 1 hour followed by water cooling and then annealing at 705°C for 2 hours followed by air cooling. Micro structure consists of alpha phase (Hexagonal close-packed crystal structure) and the two-phase mixture of alpha and beta phase (Body-centered cubic crystal structure). Shape and dimensions of specimen employed was a hollow cylinder specimen with 9 mm I.D., 11 mm O.D., and 6.8 mm gage length.

Total strain controlled multiaxial LCF tests were conducted at room temperature under five types of strain path. The strain rates were 0.1 or 0.5%/s on Mises' equivalent total strain base. Figure 4 shows the strain paths on  $\varepsilon-\gamma/\sqrt{3}$  plot and strain waveforms employed in the test, where  $\varepsilon$  and  $\gamma$  are axial and shear strains. The strain paths employed are three types of proportional strainings and two types of non-proportional

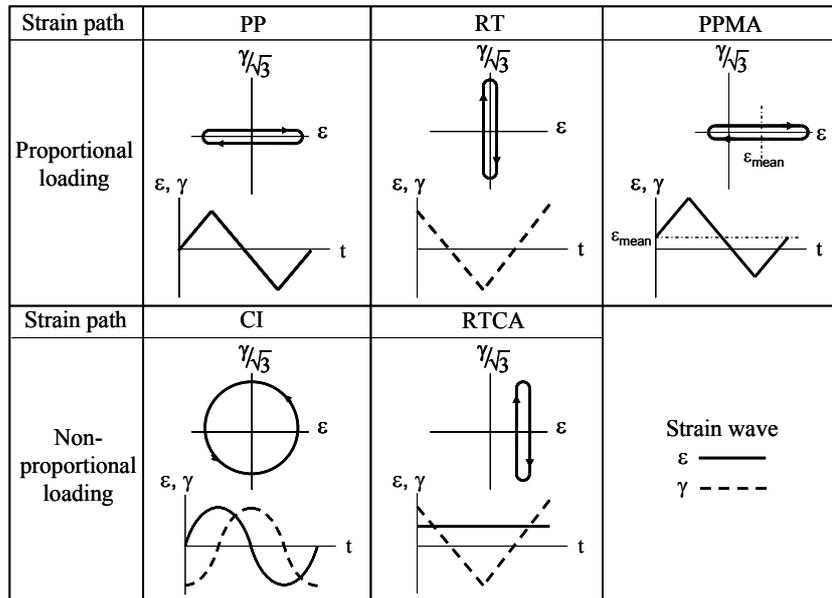


Figure 4. Strain paths and strain waveform employed.

strainings. The former are a push-pull straining (PP), a reversed torsion straining (RT) and a push-pull straining with constant mean axial strain (PPMA). The latter are a circular straining where axial and shear strains have 90 degree phase difference (CI) and a reversed torsion straining with constant tensile axial straining (RTCA). The number of cycles to failure ( $N_f$ ) was defined as the cycle at which the axial or shear stress amplitude decreased to 3/4 from that in stable state.

## EXPERIMENTAL RESULTS AND DISCUSSION

Figure 5 shows a cyclic stress and strain relationship obtained in step-up test (multiple step test) under PP and CI strainings where strain range based is increased by 0.2% at

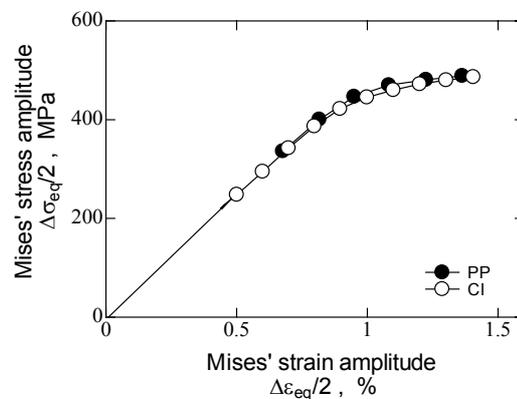


Figure 5. Cyclic stress-strain relations in PP and CI strainings.

Table 1. Summary of test parameters.

Strain path	Strain range $\Delta\varepsilon$ or $\Delta\gamma/\sqrt{3}$ , %	Mean or constant axial strain		Principal strain range $\Delta\varepsilon_I$ , %	Non-proportional strain range		Non-prop. factor $f_{NP}$
		$\varepsilon_{mean}$ OR $\varepsilon_{const}$ , %	$\varepsilon_{I,mean}$ , %		$\Delta\varepsilon_{NP}$ , %	$\Delta\varepsilon_{NP}^*$ , %	
PP	1.00	0	0	1.00	1.00	1.00	0
	1.20	0	0	1.20	1.20	1.20	0
	1.37	0	0	1.37	1.37	1.37	0
	1.70	0	0	1.70	1.70	1.70	0
RT	1.00	0	0	0.87	0.87	0.87	0
	1.20	0	0	1.04	1.04	1.04	0
	1.50	0	0	1.30	1.30	1.30	0
	1.70	0	0	1.47	1.47	1.47	0
PPMA	1.00	0.20	0.20	1.00	1.00	1.10	0
	1.20	0.60	0.60	1.20	1.20	1.50	0
	1.20	-0.20	-0.20	1.20	1.20	1.10	0
	1.20	-0.60	-0.60	1.20	1.20	0.90	0
CI	0.70	0	0	0.70	1.02	1.02	1.00
	1.00	0	0	1.00	1.45	1.45	1.00
	1.20	0	0	1.20	1.74	1.74	1.00
	1.70	0	0	1.70	2.47	2.47	1.00
RTCA	0.8	0.2	0.07	0.72	0.84	0.88	0.38
	0.8	0.4	0.24	0.64	0.76	0.88	0.43
	0.9	0.4	0.22	0.74	0.89	1.00	0.44
	1	0.4	0.2	0.85	1.02	1.12	0.44
	1	0.8	0.62	0.64	0.73	1.04	0.32
	1.2	0.4	0.18	1.05	1.25	1.34	0.43
	1.2	-0.4	-0.18	1.05	1.25	1.16	0.43
	1.6	0.4	0.13	1.44	1.69	1.75	0.38
	1.6	0.8	0.48	1.28	1.53	1.77	0.43

each 10 cycles from 1% to 3% on Mises' base. These results show no additional hardening due to non -proportional straining.

Table 1 summarizes test parameters in experiments. In the table,  $\Delta\varepsilon_I$ ,  $\varepsilon_{mean}$ ,  $\Delta\varepsilon_{NP}$  and  $f_{NP}$  were equated by equations in the previous section.  $\Delta\varepsilon_{NP}^*$  will be defined later in this section. Figure 6 shows the relationship between Mises' equivalent strain range ( $\Delta\varepsilon_{eq}$ ) and failure life ( $N_f$ ). In the figure, the thick solid line is drawn based on the data of PP test and the two thin lines show a factor of 2 bands.  $N_f$  in RT test are correlated within the band, but  $N_f$  in CI test decreased down to 1/10 comparing to that in PP test.  $N_f$  in PPMA and RTCA reduced by mean strains and constant strain and are correlated unconservatively out of the band.

In order to evaluate the lives taking account of non-proportional loading,  $N_f$  are correlated by  $\Delta\varepsilon_{NP}$  in Fig. 7, where  $\alpha$  employed is  $\alpha=0.45$  determined by the degree of life reduction due to non-proportional straining because large reduction in life is shown with no additional hardening due to non-proportional straining.  $N_f$  in CI test are

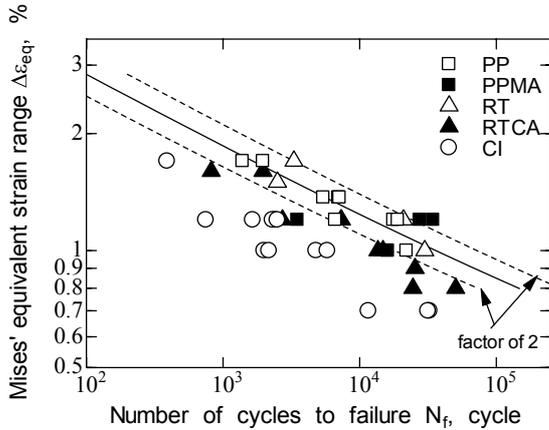


Figure 6. Relationship between  $\Delta\varepsilon_{eq}$  and  $N_f$ .

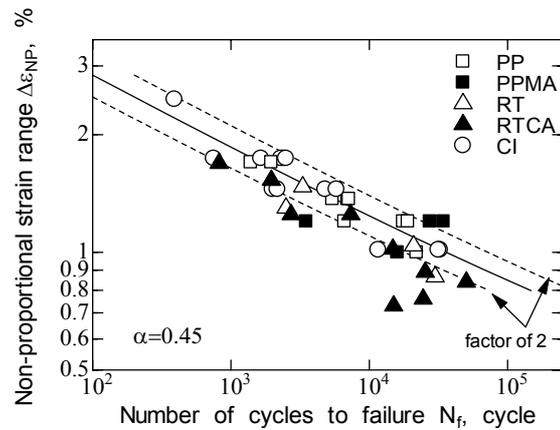


Figure 7. Relationship between  $\Delta\varepsilon_{NP}$  and  $N_f$ .

evaluated within the band, but  $N_f$  in RTCA test are still correlated out of the band unconservatively, which suggests that life evaluation requires taking account of not only strain non-proportionality but also mean strain effect.

Common used criterion for life evaluation under uniaxial loading condition with mean stress and strain is shown by Findley [11], which is equated by

$$F = \frac{\Delta\tau}{2} + k \sigma_n \quad (11)$$

where  $\Delta\tau$  is shear stress ranges.  $\sigma_n$  is normal stress on a plane.  $k$  is parameters which can be determined as lives in PPMA test takes equivalent to those in PP test. Eq. 11 is modified to strain parameter as it can be applicable to multiaxial fatigue under non-proportional loading, which is equated as

$$\Delta\varepsilon_{NP}^* = \Delta\varepsilon_{NP} + 2k\varepsilon I_{mean} \quad (12)$$

where the value of  $k$  is put as  $k=0.22$  based on the  $N_f$  in PP and PPMA tests. Although mean stress relaxation had occurred with increasing cycle in experiment, the mean stress remained until fatigue life. Thus, the mean strain could be introduced into Eq. 12 instead of mean stress.

Figures 8 shows correlations of  $N_f$  with  $\Delta\varepsilon_{NP}^*$ . It suggests that the parameter given by Eq. 12 becomes suitable parameter for life evaluation of Ti-6Al-4V under non-proportional straining with mean axial strain.

## CONCLUSION

Strain controlled multiaxial low cycle fatigue test using Ti-6Al-4V hollow cylinder specimen under non-proportional straining with mean/constant tensile strain. Failure life

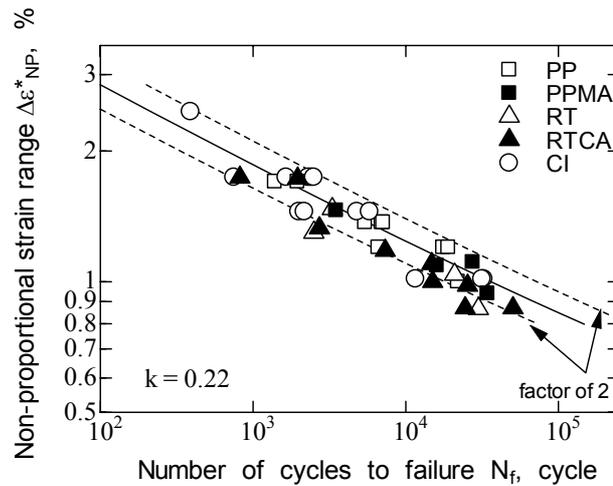


Figure 8. Relationship between  $\Delta\varepsilon^*_{NP}$  and  $N_f$ .

is affected by both non-proportional straining and mean/constant tensile strain. Degree of reduction in failure life depends on strain path. The proposed strain parameter equated with non-proportional strain range and mean strain based on the definition of strains proposed for multiaxial fatigue under non-proportional straining were applicable for the life evaluation of Ti-6Al-4V.

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