

Material Influence on Multiaxial Fatigue Response

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***ABSTRACT.** The multiaxial fatigue behaviour of components seems to depend mainly on the ductility of the material used. The ductility steers the damage mechanisms. While, in the case of low-ductility (brittle) materials, the normal stress (strain) is the decisive parameter, in the case of ductile materials, it is the shear stress (strain), and, for semi-ductile materials, a combination of normal and shear stresses (strains). Critical plane oriented hypotheses can consider these different parameters, but the difficulty lies in the definition of ductility and, based on this, the selection of the appropriate hypothesis. Therefore, especially for the evaluation of safety parts, experimental verifications are still necessary, because of the lack of a general multiaxial fatigue hypothesis.*

INTRODUCTION

Durability design against complex multiaxial loading is a very old challenge. The design of multiaxial loaded components in almost all industrial sectors like automotive, wind power plants, steel manufacturing, chemical processing, storage and handling, aeronautical, and many others still require large efforts in the numerical assessment, i.e. valid hypotheses.

However, to date all proposed fatigue strength hypotheses reveal limitations and possess no general validity. Thus, there is a continuing motivation for and process of development of so-called new hypotheses, always with observed limitations. These limitations seem mainly to arise from features steering the local deformations and from particular material characteristics, which are not properly taken into account; it is not sufficient to know only the fatigue behaviour under uniaxial loading, i.e. SN-curves, and to have data on crack propagation. Therefore, the present paper presents and contrasts examples to advise design engineers and scientists with regard to the influence of local deformations and of material ductility on the multiaxial fatigue behaviour.

EXPERIMENTAL PREREQUISITES

In developing multiaxial fatigue hypotheses, the experimental background is very decisive, because this delivers the verification basis. In this context, the conception of multiaxial tests requires the simulation of real local deformation conditions. In the past,

many multiaxial tests were carried out with unnotched specimens in obtaining the first material related data, without considering component related geometrical details like notches, which are of prime importance. Here, it must be mentioned that multiaxial load control is much easier to perform than multiaxial deformation control. However, it was noticed that load controlled multiaxial tests with unnotched and notched specimens from the same material did not result in the same tendencies with regard to the influence of constant or changing principal stress directions, simulated by in- and out-of-phase loading (proportional and non-proportional loading), on fatigue life [1]. For a long time, axial load controlled multiaxial tests with unnotched specimens were also carried out in other research works [e.g. 2-5], without being aware of the consequences of the selected loading mode on the fatigue life.

Namely, while axial load controlled multiaxial tests with unnotched specimens [1 to 5] under non-proportional (out-of-phase) loading deliver a prolongation of fatigue life ($N \geq 10^5$ cycles), Figs. 1 and 2, tests with notched specimens [1, 6] rendered a significant decrease of fatigue life, Figs. 3 and 4.

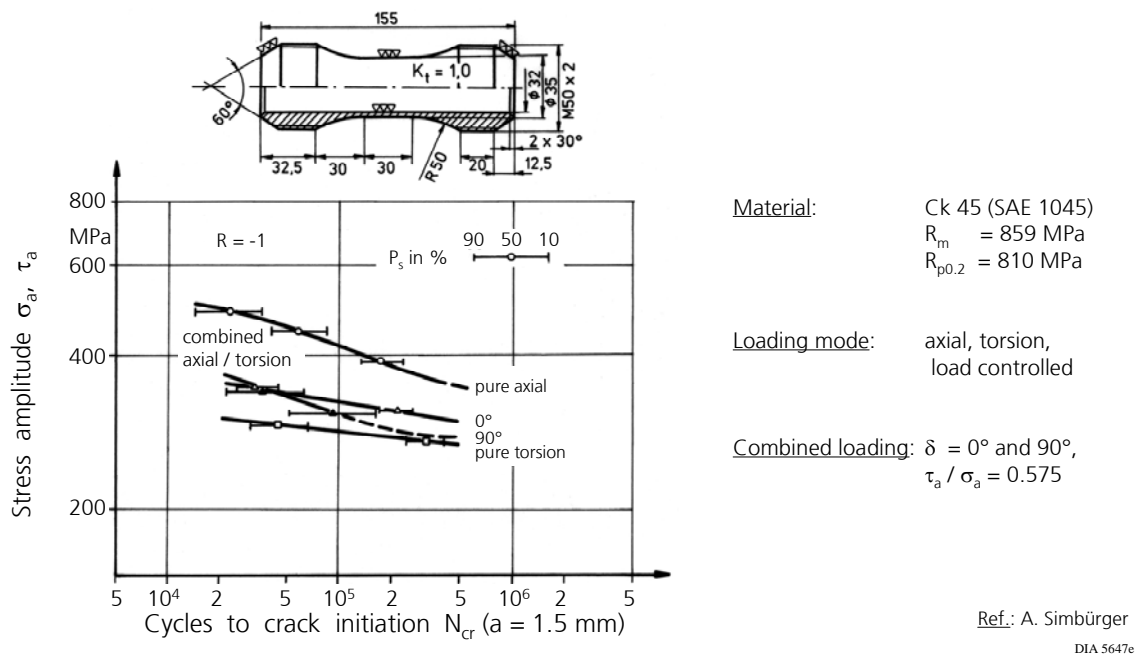
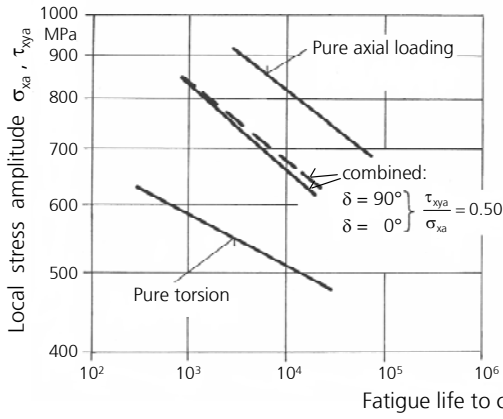
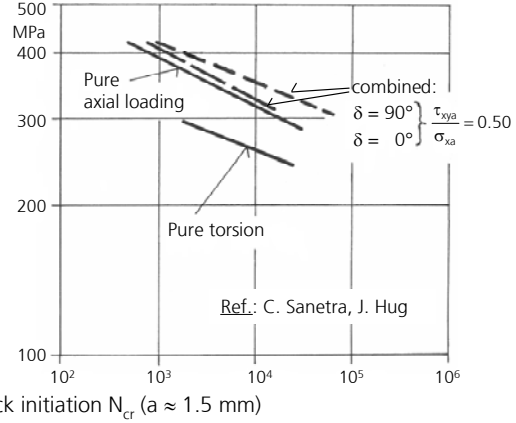


Figure 1. Woehler-lines for load controlled pure and combined axial loading and torsion

a. 30 CrNiMo 8, quenched and tempered



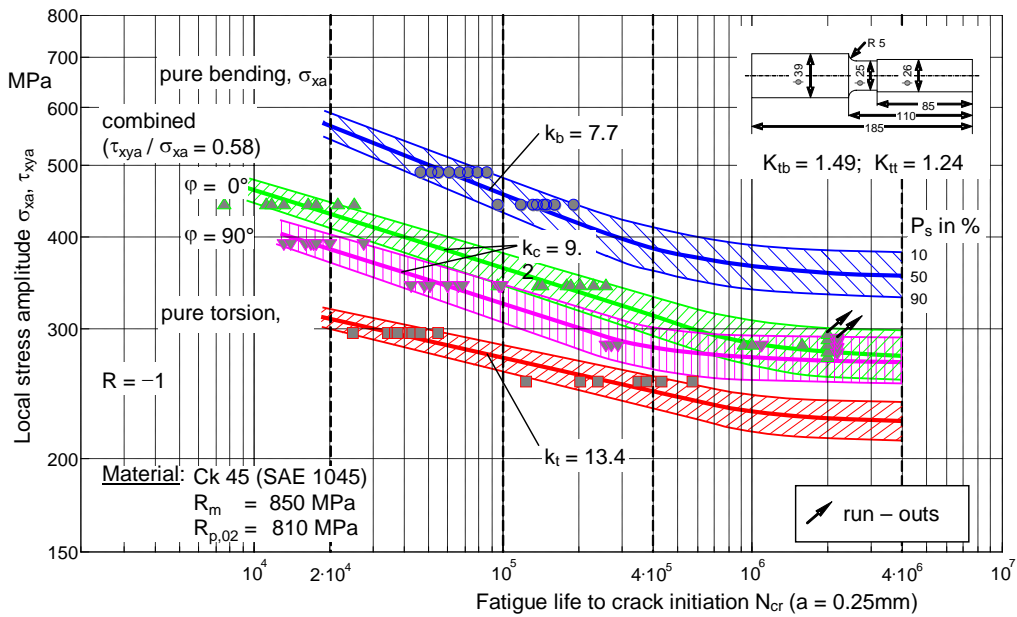
b. X 6 CrNiTi 18 10 (AISI 304)



Specimen: Cylindrical hollow, $D = 22$ mm, $t = 1.5$ mm, $K_t = 1.0$ $R = -1$, $P_s = 50\%$

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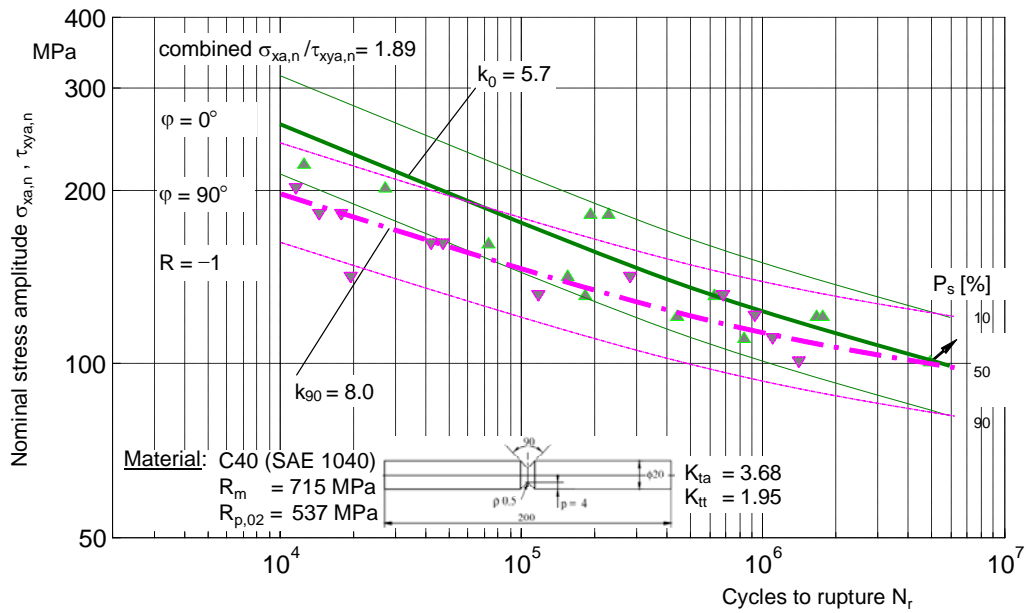
Figure 2. Results of load controlled tests under pure and combined axial and torsional loading



Ref.: A. Simbürger (LBF)

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Figure 3. Fatigue testing results obtained with notched specimens under combined bending and torsion ($\varphi = 0^\circ$ and 90°)

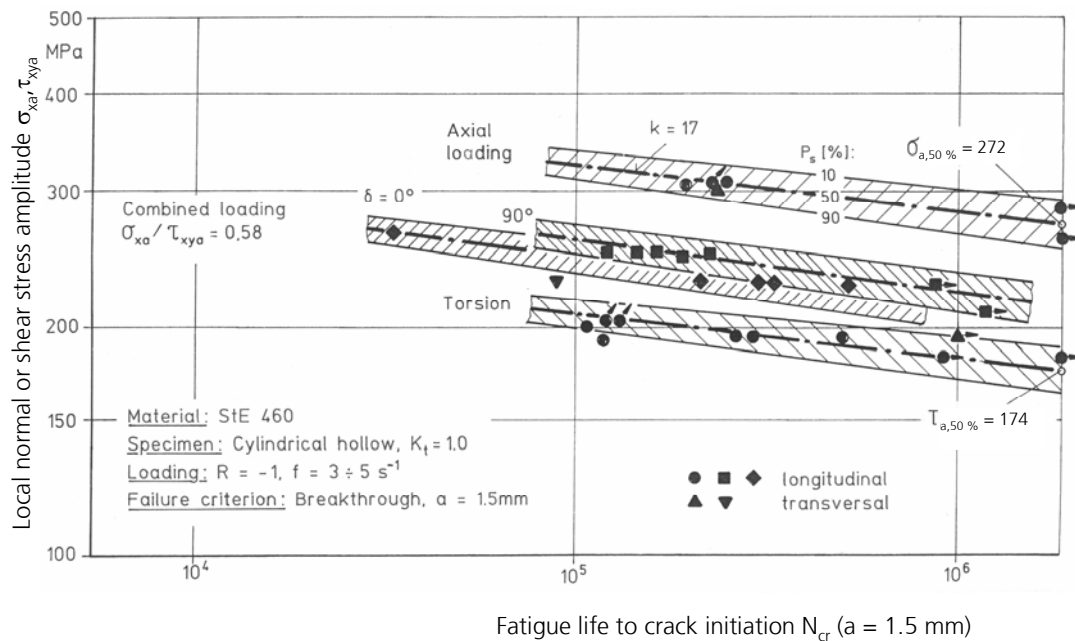


Ref.: B. Atzori et al.

DIA 8124e

Figure 4. Fatigue testing results obtained with notched specimens under combined axial loading and torsion ($\varphi = 0^\circ$ and 90°)

The reason for this contradictory behaviour was clarified by load and deformation controlled tests with unnotched specimens [7], Figs. 5 and 6.



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Figure 5. SN-curves for uniaxial and combined load controlled loading of unnotched hollow specimens

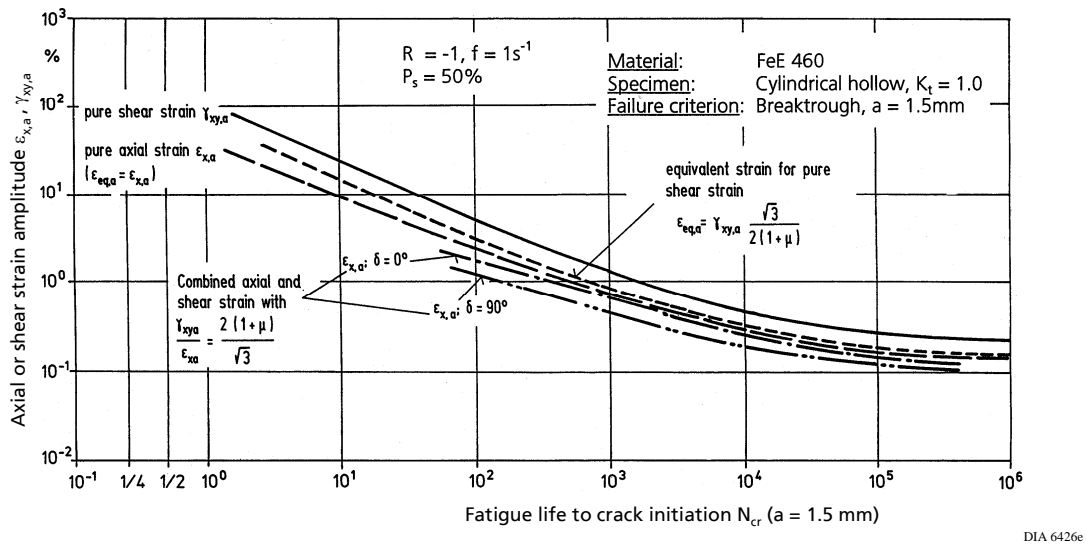


Figure 6. Strain controlled determined SN-curves with unnotched hollow specimens

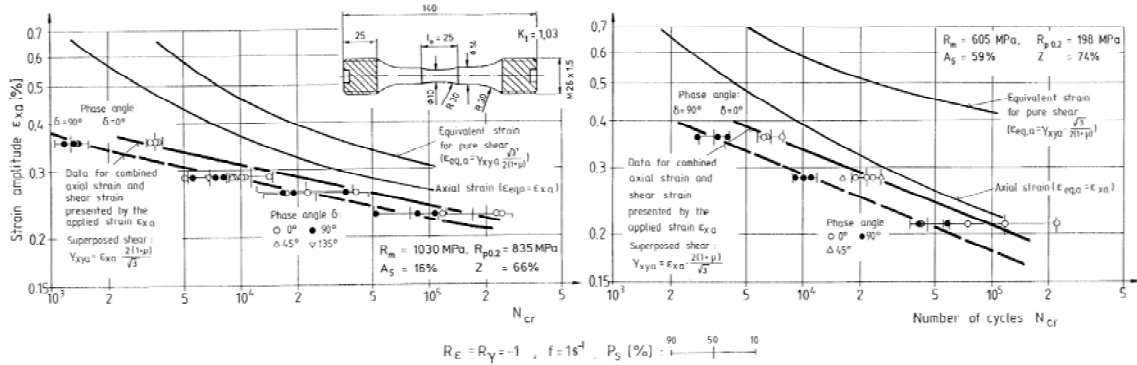
The deformation control of the unnotched specimens resulted clearly in the reduction of fatigue life under out-of-phase (non-proportional) loading, Fig. 6, as observed with notched specimens, Figs. 3 and 4, because in notches the deformation is also controlled by the stress gradients, as long as the structural yield point is not exceeded [8] or a ratcheting or a cyclic creep is not activated. The decrease of fatigue life under non-proportional loading arises from a multiaxial cyclic hardening, in comparison to proportional loading [7, 8]. This multiaxial cyclic hardening is a consequence of the forcing of energy into the material under deformation control and causes an earlier damage than under in-phase loading [7, 8]. The multiaxial cyclic hardening under deformation control is not related to the cyclic softening or hardening under uniaxial pure axial straining or pure torsional shear straining [7, 8].

In Figs. 2 and 7, left, a tempered cyclic softening high strength, quenched and tempered ductile steel and in Figs. 2 and 7, right, two similar austenitic cyclic hardening ductile steels are presented. Despite the differences in the uniaxial cyclic behaviour, the response to multiaxial loading, either under multiaxial load control of unnotched specimens [2, 3], Fig. 2, or under notch imposed multiaxial deformation control of unnotched specimens [7, 8], Fig. 7, reveals the same tendencies, i.e. an increase of fatigue life under non-proportional load control, Figs. 1, 2, 5, or the component related decrease of fatigue life under non-proportional deformation control, Figs. 6 and 7.

Until this point, the multiaxial fatigue response of mainly cyclic softening quenched and tempered and of cyclic hardening austenitic ductile steels, determined with unnotched specimens, has been discussed. The non-component related multiaxial behaviour of load controlled unnotched specimens is also related to plasticity effects depending on the applied load/deformation level, which are not controlled by stress/strain gradients, as they are in the notches of components. As soon as the

a. 30 CrNiMo 8, quenched and tempered

b. X 10 CrNiTi 18 9 (AISI 316)



DIA 6206e

Figure 7. SN-curves for controlled and combined axial and shear strain (Failure criterion: initial crack $a \leq 1$ mm)

multiaxial load/deformation level is above the branching point of the cyclic and monotonic stress-strain curves, a multiaxial load control will cause an uncontrolled increase of the deformations (axial strain and shear strain) under in-phase (proportional) loading, which is higher than the out-of-phase (non-proportional) loading [7, 8]. This causes the observed higher fatigue life for out-of-phase loading of the unnotched specimens of the displayed ductile steels, Figs. 1 and 2. However, if the deformations are below the branching point, they remain globally elastic and, in this case, load control of unnotched specimens corresponds to the deformation control. The consequence is that, at such load levels, the multiaxial fatigue behaviour corresponds to the behaviour of the notched specimens, i.e. the decrease of fatigue life under non-proportional loading of unnotched specimens, Fig. 1.

In the case of low-ductility (or non-ductile (brittle)) materials, the relation between the branching point and the load levels from 10^4 cycles upwards is much more favourable with regard to global elastic behaviour. Therefore, for such materials, like cast steels or cast iron [9, 10], the multiaxial fatigue behaviour may be also investigated using unnotched specimens under load control and result in component like tendencies.

If a recommendation has to be made here with regard to specimen geometry and loading mode, then it can be stated that multiaxial load controlled tests with unnotched specimens should be avoided. Instead, multiaxial deformation controlled tests with unnotched specimens should be carried out, in order to determine a more component related material response. However, industrial reality will be approached in a more realistic way if the tests are carried out with components or component like specimens containing the fatigue critical areas, i.e. notches, in which the local multiaxial deformation is deformation controlled, as long as the structural yield point is not exceeded and a cyclic creep or ratcheting does not occur.

MULTIAXIAL LOADING AND CRACK BEHAVIOUR

Most of the results presented in the previous section were determined for the failure criterion of initiation of a technically detectable crack with a defined depth ($a \approx 0.25$ to 1.50 mm) or surface length ($l \approx 0.50 - 3.00$ mm). This criterion is important for components for which a crack propagation is not allowed, e.g. automotive safety components like stub axles, hubs, brakes etc. However, there are also applications where the crack formation and propagation have to be considered, from stand points of material development as well as of safety aspects (damage tolerance) or of determining inspection intervals.

With regard to crack initiation and propagation, the microstructure plays an important role. A banded microstructure of a structural fine grained ductile ferritic steel, as displayed in Fig. 8, influences the direction of early crack propagation especially significantly. The early cracks do not lie in the expected direction when the plane of maximum axial or shear strains are calculated [7].

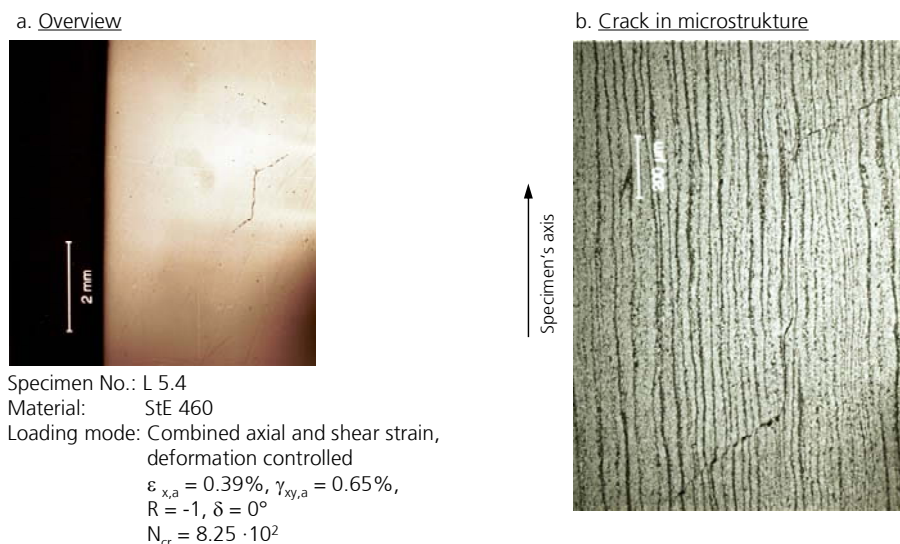


Figure 8. Position of a crack on an unnotched cylindrical hollow specimen under deformation controlled in-phase loading

Certainly, the formation direction of early cracks is determined not only by the interaction between the grain orientation and the microstructure itself (austenitic, ferritic, tempered, grain structure, i.e. body- or face-centred cubic, hexagonal etc.), Fig. 9, and the ductility but also by the load level [11].

Investigations with unnotched specimens certainly make sense for understanding the material behaviour, for drawing conclusions on the consequences for possible microstructural improvements and for recognizing tendencies with regard to the influence of multiaxial stress/strain states on the material behaviour [12]. However, they

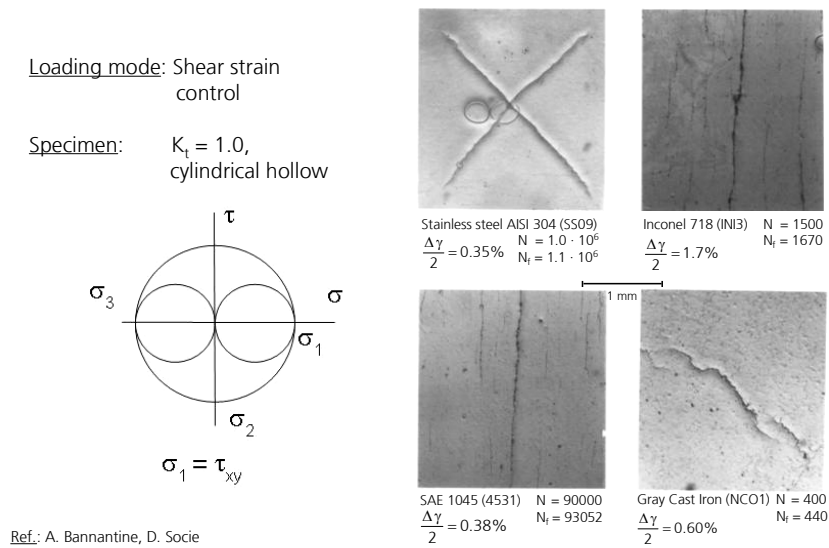


Figure 9. Early crack formation under pure torsion of thin hollow specimens

will not give an indication of the component related influence of the material on the crack propagation behaviour, because of the missing stress/strain gradients when unnotched specimens are used. For this, propagation investigations with multiaxial loaded components or component like specimens are necessary.

The results in Fig. 3 with the ductile quenched and tempered steel SAE 1045 were presented for the failure criterion of a first technical crack, but the tests were continued until the final rupture of the specimens. So, the crack propagation phases for in- and out-of-phase loadings were also determined. The fatigue lives for crack initiation, propagation and total failure are displayed in Fig. 10, and also the ratios between the lives for crack propagation and total failure. It can be seen clearly that not only the fatigue life to crack initiation but also the crack propagation life is reduced significantly by the non-proportional (out-of-phase) loading with the changing principal stress directions. At the lower load level, the crack-propagation phase decreases compared to the higher.

The results in Fig. 4 [6] also confirm that the crack propagation phase under out-of-phase loading is reduced compared to in-phase loading. These tests were carried out with specimens having a higher stress concentration than the specimens displayed in Fig. 3. Therefore, it can be assumed that the major parts of the determined fatigue lives consist of crack propagation.

These results possess a relevance because they were obtained on component like notched specimens, Figs. 3 and 4. Unfortunately, crack propagation results obtained with comparable loading conditions and geometries were not available for other materials, especially for less ductile materials, for which neutral behaviour or an increase of fatigue life under non-proportional loading is observed (see next chapter). It will remain an open question if both the crack initiation and the crack propagation life will be influenced with the same tendency.

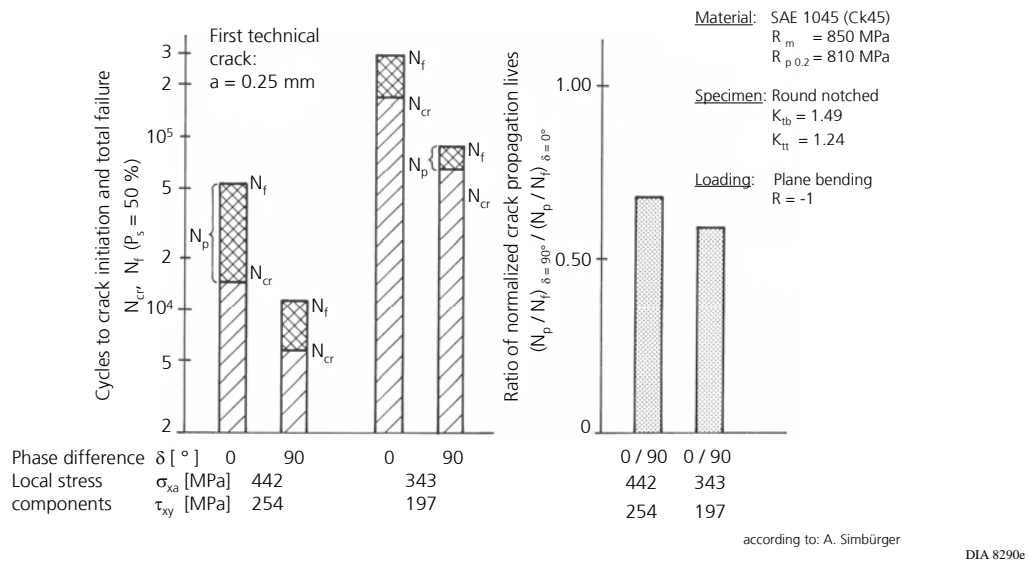


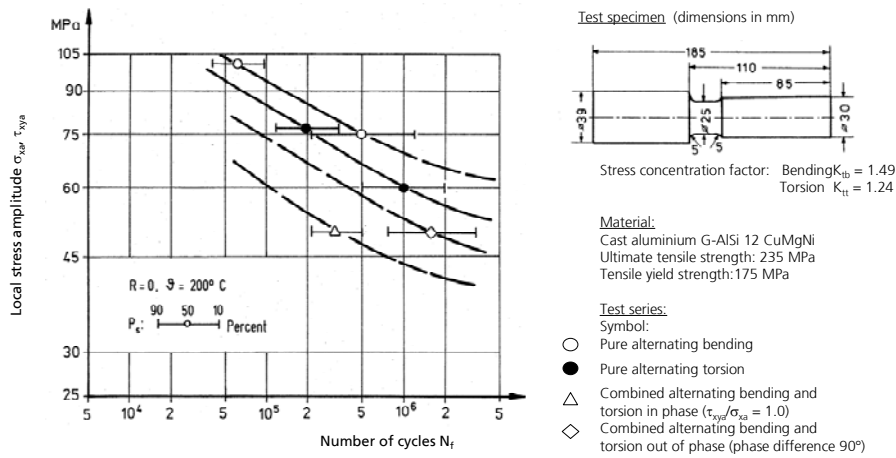
Figure 10. Influence of phase difference on crack propagation of a ductile steel

In relation to the crack propagation behaviour, multiaxial load controlled tests with fillet notched solid specimens, Fig. 3, and unnotched hollow specimens [13] do not show the same tendency. Tests carried out with thin cylindrical hollow specimens of the ductile fine grained steel StE 460 show a prolongation of the crack propagation life for non-proportional loading in comparison to proportional axial loading and torsion [13, 14]. This example underlines again that specimen geometry related stress gradients in radial (core) direction and different friction effects on crack propagation surfaces especially under torsion may lead to opposing results.

INFLUENCE OF MATERIAL DUCTILITY ON MULTIAXIAL FATIGUE RESPONSE AND DAMAGE CAUSES

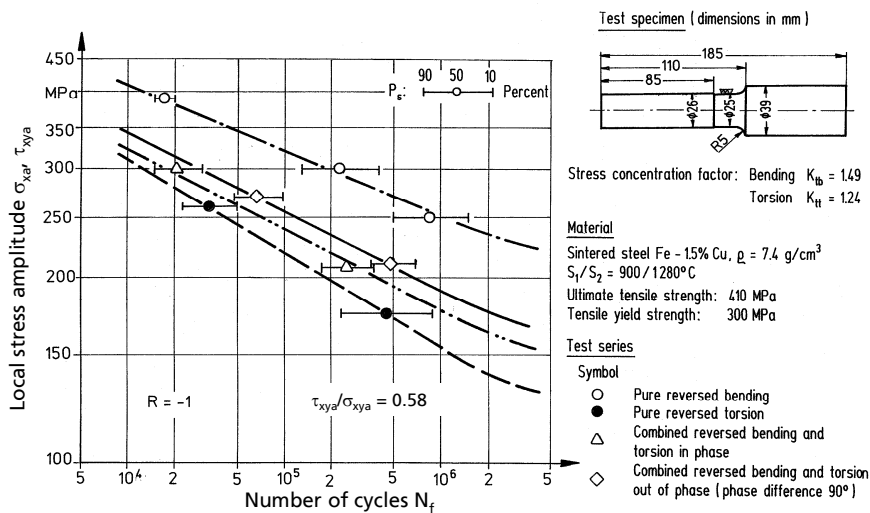
Non-welded material states

The initial examples of the influence of material ductility on crack formation, Figs. 9 and 10, have already been presented. In this section, this issue will be addressed more from the view point of component behaviour, but first for non-welded material states, i.e. wrought, sintered and cast materials. Figs. 3 and 4 have already demonstrated that non-proportional loading significantly decreases the fatigue life of ductile steels, here SAE1045 ($e = 22\%$) [1] and SAE1040 ($e = 32\%$) [6], up to 10^6 cycles; at higher fatigue lives, the SN-curves intersect with each other [16]. In contrary to this, materials with low ductility, like the cast aluminium piston alloy ($e < 2\%$), Fig. 11 [15], or sintered steels even at elevated densities ($e = 13\%$), Fig. 12 [15, 16], display a significant increase of fatigue life under non-proportional (out-of-phase) loading. These investigations were carried out with the same component like geometries as in Fig. 3 under load control.



DIA 3804e

Figure 11. SN-curves for pure and combined bending and torsion



DIA 3394

Figure 12. SN-curves for pure and combined bending and torsion

The fatigue life increasing influence of low ductility materials is also displayed in Fig. 13. The ferritic cast nodular iron EN-GJS-400 ($e \leq 11\%$) shows a significant increase of fatigue life under out-of-phase fully reversed as well as under pulsating multiaxial loading[9]. This is observed under both constant and variable amplitude loadings.

Nevertheless, for low-ductility materials like the mentioned cast aluminium piston alloy, the sintered steels or cast nodular iron, it can be assumed that both the crack initiation and the crack propagation lives are larger for out-of-phase loading compared to in-phase loading. A few deformation controlled tests with unnotched hollow cast magnesium AZ 91 (G-MgAl9Zn1) specimens with low ductility ($e \leq 2.5\%$) have also shown an increase of fatigue life under non-proportional loading [17].

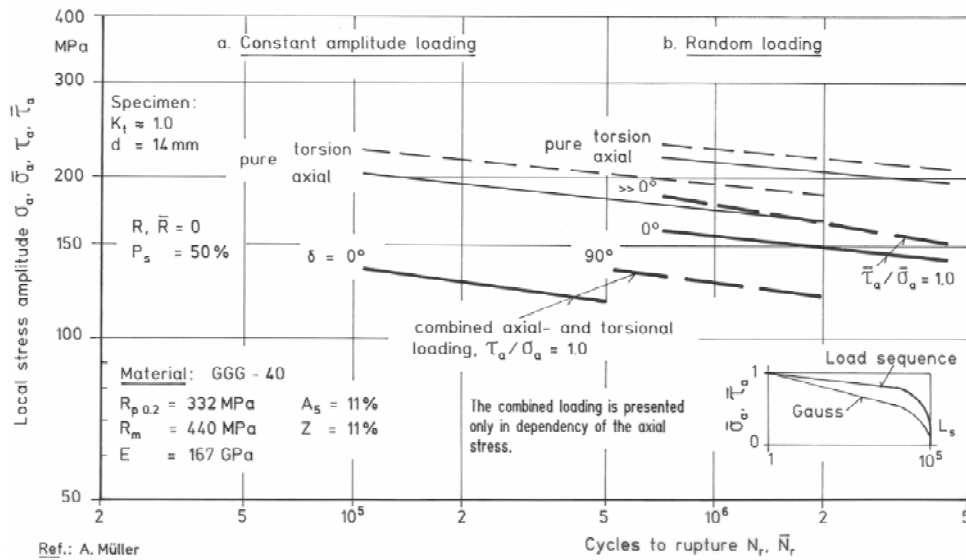


Figure 13. Influence of multiaxial pulsating loading on the fatigue behaviour of the nodular cast iron GGG-40, pulsating loading ($R = 0$)

Last but not least, the behaviour of semi-ductile materials is also addressed by the example of the cast steel G-X 5 CrNi 13 4 (1.4313) with a martensitic-ferritic microstructure ($e \leq 20\%$) [10], Fig. 14. This material behaves in the same way under both proportional and non-proportional loadings; the changing of principal stress directions by the out-of-phase loading does not affect the behaviour obtained under in-phase loading.

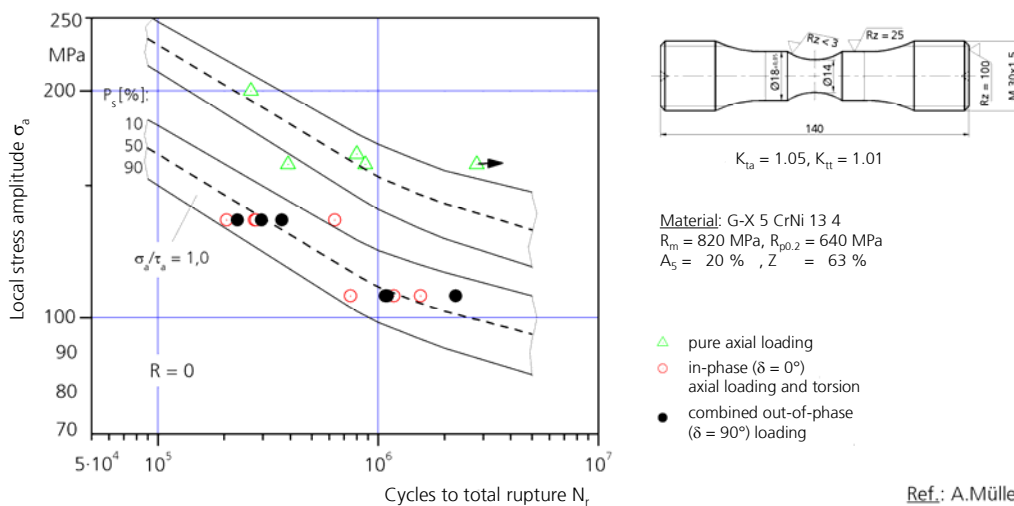
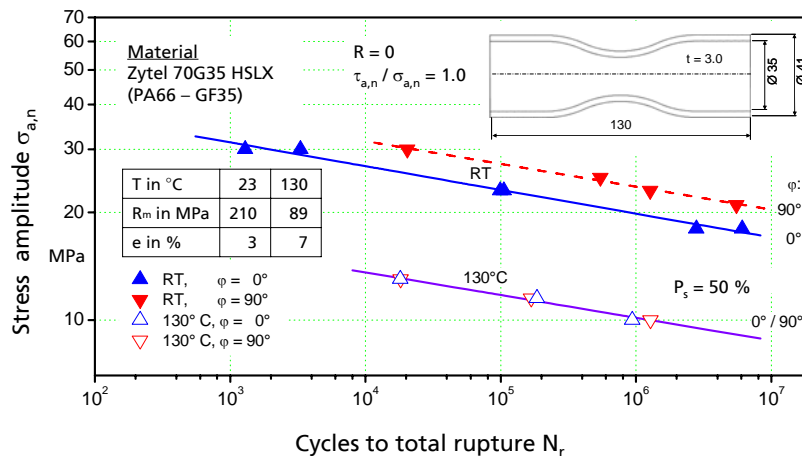


Figure 14. SN-curves of a cast steel under uni- and multi-axial loading

Beside the metallic materials discussed, some preliminary information on the multiaxial fatigue behaviour of fibre reinforced materials will also be presented in the following discussion.

The assumption that ductility is the decisive parameter with regard to the material response under multiaxial loading also seems to be valid for the short-fibre reinforced polyamide PA 66-GF35 with 35 % content of glass fibres [18]. Load controlled tests with unnotched thin tubular specimens [19] at room temperature ($e = 3\%$) reveal an increase of fatigue life under out-of-phase loading in contrast to in-phase loading [19], Fig. 15, i.e. the same behaviour as observed for low-ductility (brittle) metals. However, the tests carried out at 130°C ($e = 7\%$), result in a neutral behaviour, similar to the behaviour of semi-ductile metals. The increase of temperature augments the ductility significantly from low- to semi-ductile and this leads to a change of the material response.



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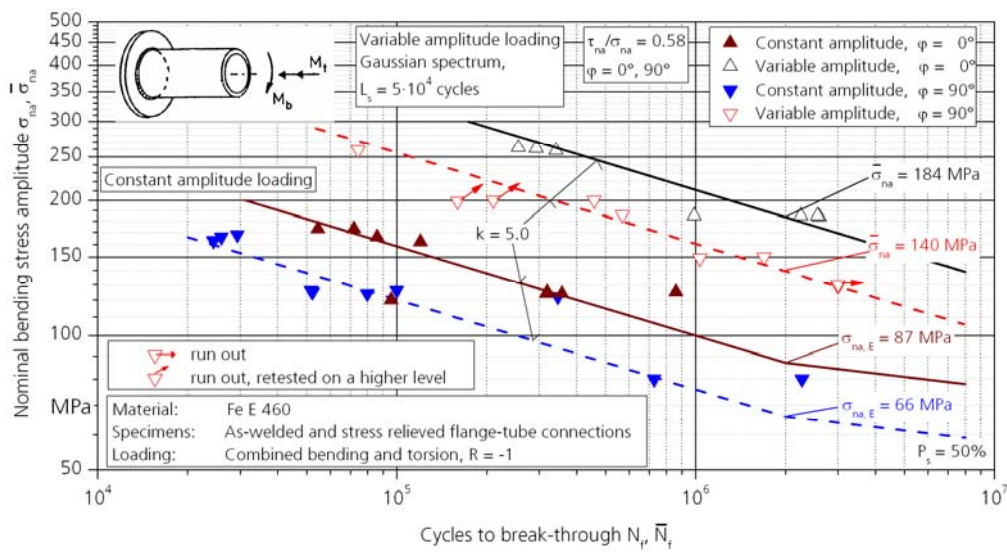
Figure 15. Woehler-lines for short-fibre reinforced polyamide under combined axial and torsional loading, pulsating ($R = 0$)

Welded material states

The multiaxial fatigue behaviour of welded joints under proportional and non-proportional loading is also determined by material ductility.

Flange-tube connections ($t = 10\text{ mm}$) of seam welded ferritic fine grained steel FeE 460 ($e = 25\%$) reveal a significant decrease of fatigue life under both constant and variable amplitude out-of-phase loading in contrast to in-phase loading [20, 21], Fig. 16. Also, after $N = 10^6$ cycles, the fatigue life curves do not intersect, as is observed for the non welded quenched and tempered steel SAE 1045, Fig. 3.

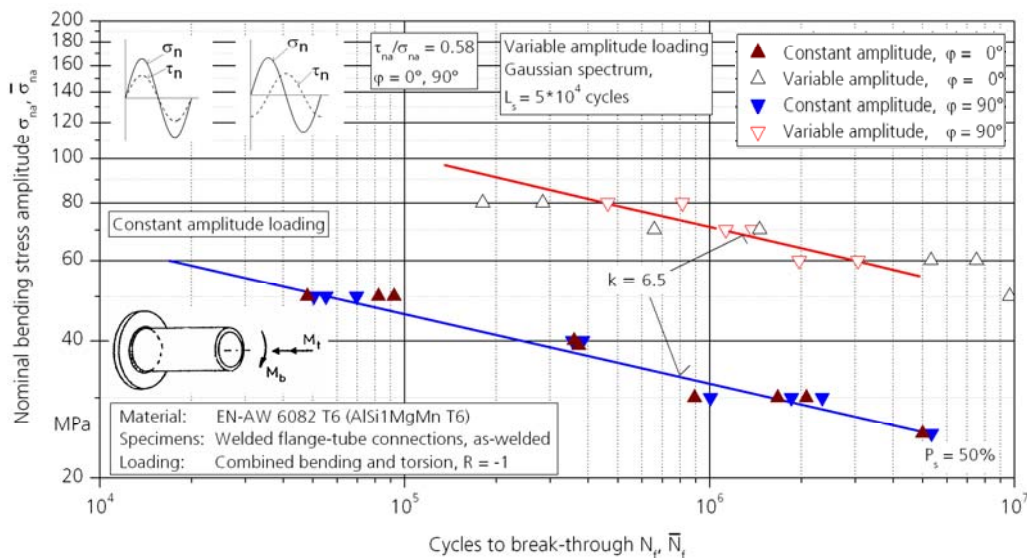
Also the laser beam welded overlapped thin tubes ($t = 1.0\text{ mm}$) of the ferritic steel S235 G2T (St 35) ($e = 26\%$) behave in the same way under non-proportional loading [22, 23], as the seam welded thicker flange-tube connections [20], without any intersection of the SN-curves after $N < 10^6$ cycles.



DIA 8075e

Figure 16. Fatigue testing results obtained with as-welded thick ($t = 10 \text{ mm}$) steel specimens under combined bending and torsion ($\varphi = 0^\circ$ and 90°)

In contrast to the seam welded steel flange-tube connections, similar thickness joints ($t = 10 \text{ mm}$) of the age hardened aluminium alloy AISi1MgMn T6 (AW-6082 T6) ($e = 14 \%$) render a neutral behaviour under both out-of-phase constant and variable amplitude loading in comparison to in-phase loading [21, 24], Fig. 17. This means that this alloy possesses a semi-ductile material behaviour with regard to multiaxial fatigue.



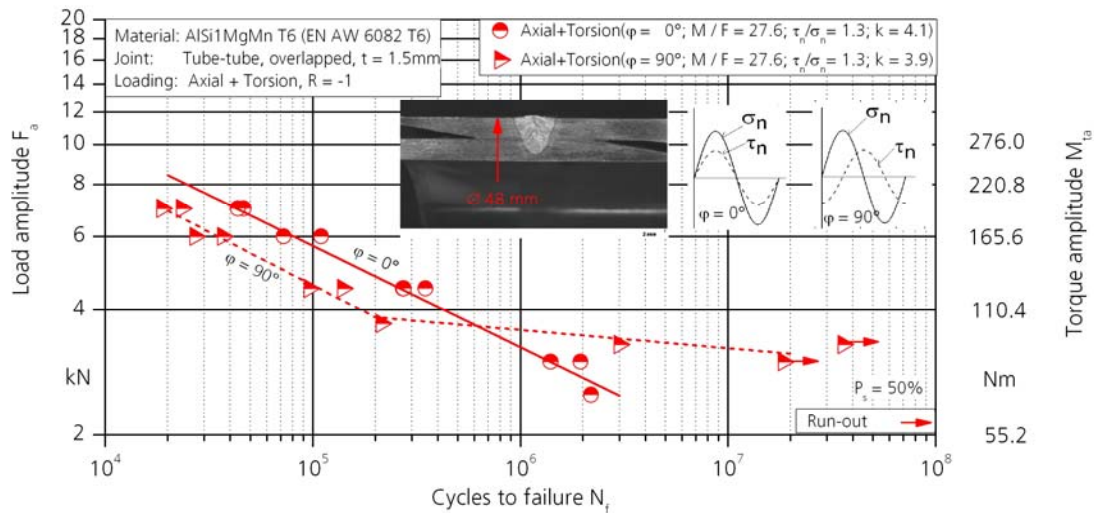
DIA 8076e

Figure 17. Fatigue testing results obtained with as-welded thick ($t = 10 \text{ mm}$) aluminium specimens under combined bending and torsion ($\varphi = 0^\circ$ and 90°)

However, new results show that this observed behaviour cannot be generalized for the alloy AlSi1MgMn T6. Laserbeam welded thin ($t = 1.5$ mm) overlapped tubes from nominally the same alloy but in a more ductile condition ($e = 17$ %) display a decrease of fatigue life under non-proportional out-of-phase loading [25], Fig. 18. This behaviour is not due to the difference between seam welding of a thick joint and laser beam welding of a thin joint, as displayed for the ductile steel joints in Fig. 16, but due to the differences in material ductility. Obviously, the manufacturing process and the heat treatment of thick and thin tubes (10 mm versus 3 mm thickness before machining) lead to differences in ductility despite the use of the same nominal material.

Also the laser beam welded aluminium alloy AlMg 3.5 Mn (AW-5042) ($e = 20$ %) results in a decrease of fatigue life under non-proportional loading compared to proportional loading.

However, the reduction of fatigue life is restricted for both aluminium alloys in the fatigue life region $N < 10^6$ cycles. At higher fatigue lives, an intersection or a crossing of the SN-lines occurs. The non-proportional loading may even become more advantageous in this region. This may be a result of contact of the overlapped tubes at low load levels. However, more tests in this region are necessary to clarify the course of the SN-curves.



DIA 8297e

Figure 18. Laserbeam welded overlapped thin ($t = 1.5$ mm) aluminium tube-tube specimens under constant amplitude multiaxial loading

Damage causes

Obviously, the ductile materials react in a different way to non-proportional loading with changing principal stress or strain directions than low-ductility (or brittle) and semi-ductile materials. What is generally accepted is that, for low-ductility (or brittle) materials, damage is activated by normal stresses or strains and, for ductile materials, damage is activated by shear stresses or strains. It is logical to assume that, in the case of semi-ductile materials, an interaction between shear and normal stresses or strains

will be responsible for the damage. What is also known is that, in the case of changing principal stress or strain directions in ductile materials, dislocations are activated in different directions blocking each other. This leads to the so called multiaxial stress hardening in contrast to in-phase loading with constant principal stress or strain directions even if the material reveals a cyclic softening under uniaxial strain controlled cyclic loading [7]. The multiaxial stress hardening under out-of-phase deformation control causes the fatigue life decrease compared with in-phase multiaxial deformation controlled loading. However, it has to be mentioned that there are also materials like Ti-6.5Al-3.4Mo which reveals a fatigue life decrease under deformation controlled out-of-phase loading without a cyclic hardening compared to in-phase loading [26].

Several critical plane and integral hypotheses have been developed for considering the described damage causes in past years e.g. [1-4, 8-10, 20-34], but none of them were introduced in advance without having a knowledge of the multiaxial behaviour and the estimation of the ductility related damaging parameters. The selection of the appropriate hypothesis requires an assessment of the ductility of the material to be evaluated, but the example with the seam welded and laserbeam welded aluminium alloy, Figs. 17 and 18, shows the difficulty in deciding which level of ductility may cause a change of the multiaxial fatigue response for nominally the same material.

In addition to the classical ductility values of elongation or area reduction, the ratio between the local fatigue strength amplitudes τ_a/σ_a to be determined by pure axial loading or bending and by pure torsion is also often used as an indicator for multiaxial fatigue behaviour [1, 4, 8-10, 31-34]:

$\tau_a/\sigma_a = 1/2 = 0.5$, according to Tresca, or $1/\sqrt{3} = 0.58$, according to von Mises, indicate that the shear stress (strain) will be the dominating parameter, and $\tau_a/\sigma_a = 1.0$ (principal stress /strain hypothesis) points to the normal stress as being the damage steering parameter. Intermediate values suggest an interaction of shear and normal stresses/strains. However, this ratio also depends on several parameters, which restrict its applicability as an indicator for the selection of an appropriate hypothesis.

Neither the elongation nor the ratio τ_a/σ_a are values, which may generally help as criteria for selecting the suitable hypotheses. Their relation with regard to the mentioned damage scenarios (normal, shear or their combination) can only be determined by multiaxial fatigue tests contrasting proportional and non-proportional loading.

Also, other damaging causes with regard to multiaxial fatigue are suggested, e.g. the hydrostatic stress state [35, 36]. However, this suggestion also has its limitation and is not of general use [27].

Furthermore, the introduction of particular factors into the Gough-Pollard equation [37] considering the influence of ductility on multiaxial fatigue, e.g. in design codes [38, 39], is only an experience based consideration.

SUMMARY AND CONCLUSIONS

For the multiaxial fatigue response of materials, one important parameter is the mode of local deformation under multiaxial loading. Load control of unnotched specimens, if

macroscopic plasticity occurs, results in a quite different fatigue life response than strain control of unnotched specimens. However, load control of notched specimens or of components results in a similar behaviour to unnotched specimens under strain control, because in the critical areas, i.e. notches, the local deformations are strain controlled as long as ratchetting or cyclic creep does not occur. Only under locally multiaxial elastic deformations does the loading mode play no role: the deformations are also locally elastic under load control. In this case, the right component related material behaviour is also determined with unnotched specimens.

While ductile wrought steels, non-welded as well as welded, under changing principal stress directions, e.g. caused by an out-of-phase non-proportional loading between normal and shear stresses, show a significant fatigue life reduction compared to an in-phase multiaxial loading with constant principal stress directions, less ductile (semi-ductile) cast steels reveal an insensitive (neutral) behaviour and the even less ductile (low-ductility) cast nodular iron even show an increase of fatigue life under out-of-phase non-proportional loading. In this context, cyclic softening (quenched and tempered bainitic) steels or cyclic hardening (austenitic) steels reveal no influence with regard to the fatigue life response under non-proportional loading. Also, for porous sintered steels, a behaviour like cast nodular iron is determined.

Similar tendencies are also observed for aluminium and magnesium alloys. While cast aluminium alloys with a very low elongation result in an increase of fatigue life under non-proportional loading, as does cast magnesium, welded semi-ductile aluminium extrusions show a neutral behaviour and more ductile welded extrusions even show a decrease of fatigue life.

Even short-fibre reinforced polyamides, which show an increase of fatigue life under non-proportional loading at room temperature, behave in a neutral (semi-ductile) way at elevated temperatures due to the increase of ductility.

A further difficulty with regard to fatigue life calculations is that the slopes of the SN-curves for proportional and non-proportional loading are not always parallel. For wrought non-welded ductile steels, they are significantly apart in the finite fatigue life region ($N < 10^6$), but they intersect in the high-cycle region ($N > 10^6$). This means that a single hypothesis would not be able to describe the multiaxial fatigue behaviour along the whole SN-curve. A similar behaviour is also observed for laser beam welded thin aluminium joints, but not for seam or laser beam welded steel connections.

It is obvious that ductility decides the failure mechanisms. However, a hypothesis suggesting ductility, e.g. expressed by area reduction or elongation, as a steering parameter has not yet been proposed. Despite this fact, there are suggestions for considering the influence of ductility on failure mechanisms, e.g. critical plane approaches or modifications with regard to the evaluation of shear and normal stresses or strains. However, as a general multiaxial fatigue hypothesis does not yet exist, the selection of the appropriate hypothesis still requires much testing effort. Therefore, especially in the case of safety components, e.g. axle systems, or complete body and chassis, experimental verifications are still indispensable.

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