Life Estimation in Specimens with Stress Gradient: Notches and Fretting Fatigue

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ABSTRACT. *Fatigue in notched specimens and fretting fatigue are two different phenomena but they have in common the existence of a stress gradient. In these cases life assesment is usually done with models that consider the process as a superposition of an initiation and propagation phase. The model employed in this paper combines both phases without defining a priori the separation between them. In the propagation phase a crack length dependent growth threshold is introduced in the Paris law. On the other hand, the S-N curve used in the initiation phase is modified by subtracting the propagation cycles. The proposed model is applied to a group of fretting fatigue tests in spherical and cylindrical contact as well as in tests with notched specimens to check the validity of the model in predicting life in different conditions.*

INTRODUCTION

Fatigue in notched specimens and fretting fatigue are two phenomena very common in real practice. They appear in any machine or structure, although not always are the cause of the failure [1]. Fretting may appear when two elements come into contact and there is friction between them due to the variable forces applied. Its effect is to create a stress concentration in the contact zone which induces an early initiation of cracks. In fretting fatigue there is the additional complication of having a more complex multiaxial stress state and a nonproportional variation of stresses. Nevertheless, the two phenomena have in common the stress gradient. This fact makes it reasonable to use the same model for life estimation in both situations.

In the fatigue process two phases are usually distinguished: initiation of the crack and its subsequent propagation. The principal difficulty in combining both phases is to decide where one finishes and the other begins, i.e. what is the crack length a_i where it is considered that the propagation phase starts. The models that combine both phases usually define the initiation length, a_i, a priori depending on what is the smallest detectable crack length, on the definition of the *S-N* curve, or choosing the value that better fits the experimental results, etc.

The relative weight between these phases depends on many factors. In high cycle fatigue with plain specimens or notched specimens with a low stress concentration the propagation phase is negligible compared to initiation. In the case of a high stress gradient the initiation of the crack takes place rapidly. In some cases, if the stress concentration is high enough the initiation phase can be neglected. This is the reason why in fatigue with notched specimens and in fretting fatigue it is convenient to use a model capable to combine the initiation and propagation phase.

There are several models that estimate life combining both phases [2-5]. This paper proposes a life estimation model with the particularity that combines the initiation and propagation phase without defining where one finishes and the other begins. The model is an improvement over others proposed previously by the authors and applied only to fretting fatigue [6-8].

The following sections will present the experimental data used to analyze the model: fretting fatigue with spherical and cylindrical contact and notched specimens. Afterwards, the proposed model will be described and finally the results compared to the experimental data.

EXPERIMENTAL DATA

The schematic of the fretting tests setup is shown in Fig.1. In this type of tests, a normal constant force P is applied. Then, a cyclic variable force, F, is applied generating the bulk stress, σ. The stiffness, K, of the support of the contact elements produces a tangential load, Q, in the contact. The force Q is also variable and in phase with F.

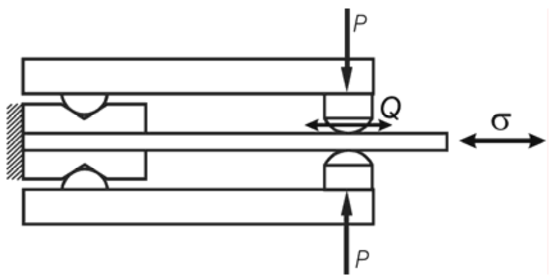


Figure 1. Schematic of the fretting fatigue setup.

The fretting fatigue tests with spherical contact were performed by the authors with the aluminium alloy 7075 T651 [8]. The material properties are shown in Table 1. The fretting fatigue tests with cylindrical contact were performed with the aluminium alloy 2024 T351, where different radii of the contact pads were used [2]. The fatigue tests with the notched specimens were performed by the authors using the aluminium alloy 2024 T3. The geometry of the specimen was a 4 mm thick plate, 50 mm wide with a centred hole 4 mm in diameter. The tests were done with a load ratio of R = 0. The maximum loads, nominal stresses based on the net section and the number of cycles until failure, are shown in Table 2. The fatigue properties of this material were obtained from [9] and the crack growth properties from [10].

Table 1. Properties of the materials employed in the tests.

		7075 T651	2024 T351	2024 T3
Young's Modulus	***E* (GPa)**	71	74.1	71
Poisson's ratio	**ν**	0.33	0.33	0.33
Tensile stress	σ_u **(MPa)**	572	470	505
Yield stress	σ_y **(MPa)**	503	310	375
Paris law coefficient	C	8.83 10^{-11}	6.53 10^{-11}	2.22 10^{-11}
Paris law exponent	n	3.322	3.387	3.545
Threshold	ΔK_{th} **(MPa m$^{0.5}$)**	2.1	2.1	2.5
Fatigue strength	$\Delta\sigma_{FL}$ **(MPa) (10^6 cyc.)**	169	230	207
Fatigue strength coeff.	σ'_f **(MPa)**	1610	714	835
Fatigue strength exponent	b	-0.1553	-0.078	-0.096
Friction coefficient	μ	1.27	0.65	-
Grain size	$2l_0$ **(µm)**	50	50	50
***R* (mm)**		100	127 - 229	-
Geometry		Spher. frett.	Cyl. frett.	notch

An important difference between these three groups of tests is the gradient and the level of stresses which appear on the surface. Figure 2 shows the amplitude of the normal stress perpendicular to the crack path in two tests for each of the three groups. The two tests chosen as representative of each geometry were the ones producing the highest and the lowest stress on the surface. It can be observed that in the fretting fatigue tests the stress on the surface and the gradient are higher, mainly in spherical contact. The importance of the initiation phase depends on the chosen model, but if the same model is applied, this implies that the initiation in fretting is faster than in the notched specimen.

Table 2. Forces, stresses and no. of cycles until failure with notched specimens.

Test	Force max. (N)	Stress (MPa)	Cycles
1	22080	120	5 10^6 *
2	25760	140	541410
3	25760	140	2.98 10^6
4	25760	140	5 10^6 *
5	31280	170	99341
6	31280	170	111070
7	34960	190	64366
8	34960	190	85604
9	38640	210	38950
10	38640	210	38276

* Interrupted test

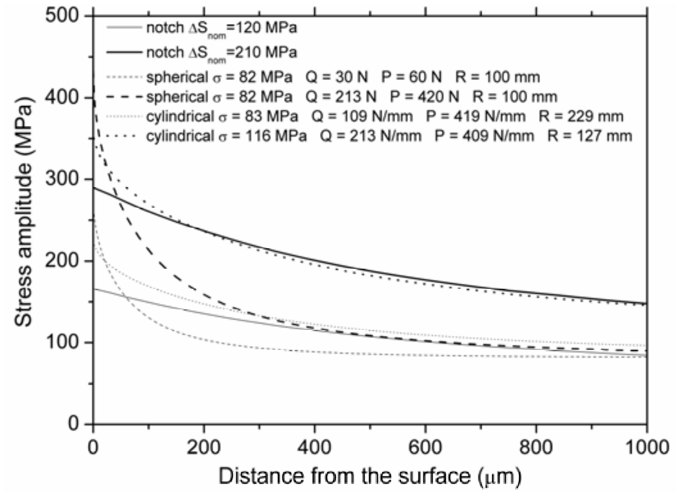


Figure 2. Stress amplitude as a function of the distance from the surface.

THEORETICAL MODEL

As mentioned earlier, the life estimation model is based on another one proposed by the authors [6]. Each phase is analyzed separately producing two curves. One of them gives the number of cycles needed to propagate a crack from each initiation length, a_i, until failure using fracture mechanics, ($a_i - N_p$). The other curve gives the number of cycles necessary to generate a crack of length a_i, ($a_i - N_i$). This curve is calculated from the stresses along the assumed crack path and an *S-N* curve that will be detailed further on. The sum of these two curves would be the estimated total life as a function of the value of the crack length chosen to separate the initiation and propagation phases. As shown in previous publications [6,11], these curves show that close to the surface the initiation process is the dominant one and away from the surface the opposite. The nexus between the two zones can be found at the minimum of the total life curve mentioned earlier. As this life value is the most conservative, it will be taken as the estimated total life.

Fracture mechanics is used during the propagation phase. The crack growth law that has been used includes the behaviour of short cracks since the initiation length can be in the order of microns. This is done by including a modified crack growth threshold as a function of crack length [8]:

$$\frac{da}{dN} = C\left(\Delta K^{n} - \left(\Delta K_{th\infty}\cdot\left(\frac{a^{f}}{a^{f}+a_{0}^{f}-l_{0}^{f}}\right)^{1/2f}\right)^{n}\right) \tag{1}$$

In the Eq. 1 $\Delta K_{th\infty}$ is the growth threshold for long cracks, f is a parameter that in general is taken as 2.5 [12], l_0 is the distance to the first microstructural barrier and a_0 is the so called El Haddad constant, defined as $a_0 = 1/\pi\ (\Delta K_{th\infty}/\Delta\sigma_{FL})^2$, where $\Delta\sigma_{LF}$ is the fatigue strength, usually taken at 10^6 cycles. The factor multiplying the growth threshold for long cracks in Eq. 1 derives from the theoretical approximation to the Kitagawa-Takahashi diagram: stress threshold versus crack length. Other crack growth laws have been tested to estimate the short cracks growth [8,13]. Eq. 1 is the one that better fitted the experimental observations in fretting fatigue.

The model presented in this paper analyzes the initiation phase based on the work by McClung et al. [14] in notches. The first step is to obtain a fatigue curve, $S-N|_{a_i}$, in plain fatigue specimens which gives the number of cycles necessary to generate a crack of length a_i as a function of the stress applied, S. For each stress level, S, the number of cycles in this curve is obtained by subtracting the number of cycles to propagate a crack from a_i to failure from the number of cycles until failure obtained in a plain fatigue test with stress S. These curves, $S-N|_{a_i}$, which will be referenced as initiation curves, are shown in Fig. 3 for four different cases: total fracture (the original S-N curve) and generation of a 100, 50 and 10 microns crack.

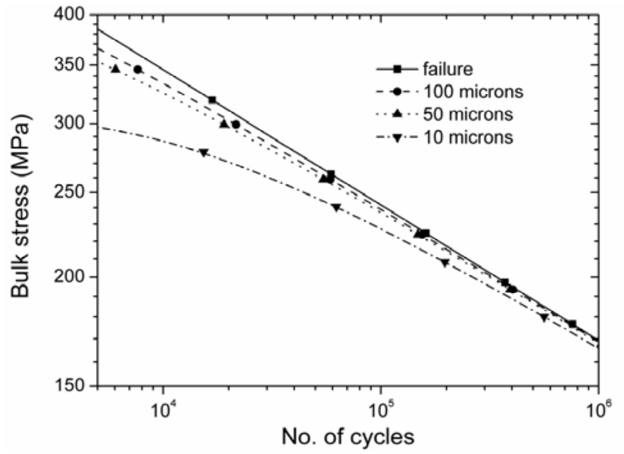


Figure 3. Initiation curves for different crack lengths in Al 7075 T651 in plain fatigue.

This procedure makes it possible to estimate the number of cycles needed to initiate and grow a crack until a certain length for any stress applied in plain fatigue using the appropriate curve $S-N|_{a_i}$. In the case of a specimen with a stress gradient as in fretting fatigue or in a notched specimen, the same process can be applied although with some modifications. In this situation the stresses rapidly diminish with depth from a maximum on the surface, Fig. 2. Therefore, the initiation life depends on where and how the stresses are evaluated. Based on the results obtained previously [7], the average stress between the surface and a depth equal to the initial crack length, a_i, seems as a reasonable option to obtain the number of cycles to initiate and propagate a crack to the length a_i from the curve $S-N|_{a_i}$. This option assumes that, in any situation, the same average stress from the surface to a_i produces the same life to initiate a crack a_i.

For the initiation phase, due to the stress state, a multiaxial fatigue criterion should be considered, mainly in fretting fatigue. It has been shown that in fretting fatigue the stress state is highly multiaxial and its evolution nonproportional [15]. Several multiaxial fatigue criteria are available [11], being the Fatemi-Socie criterion one of the most accepted [16].

The analysis of the initiation and propagation phases leads to obtaining the two curves mentioned at the beginning of this section and represented in Fig. 4 for a specific test in fretting fatigue with spherical contact where the bulk stress is. The total number of cycles is obtained adding these two curves. This would be the number of cycles to failure (initiation plus propagation) depending on which initiation length is chosen. Finally, the minimum of this curve is taken as the fatigue life and the point where it happens is the initiation length, i.e. the length where it is considered that initiation finishes and propagation begins. Figure 4 shows that the estimated life does not vary greatly for a certain range of the initiation length. The extension of this approximately flat region is different from one test to another and it is usually smaller when the amplitude of stresses near the surface is lower, as in the analyzed fretting fatigue tests with cylindrical contact and even more with the fatigue tests with notched specimens.

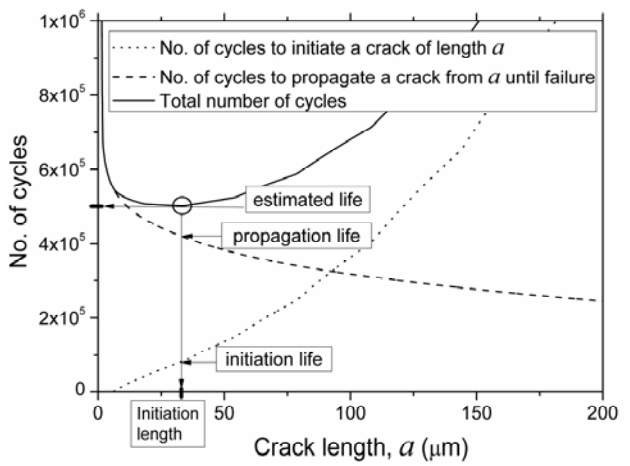
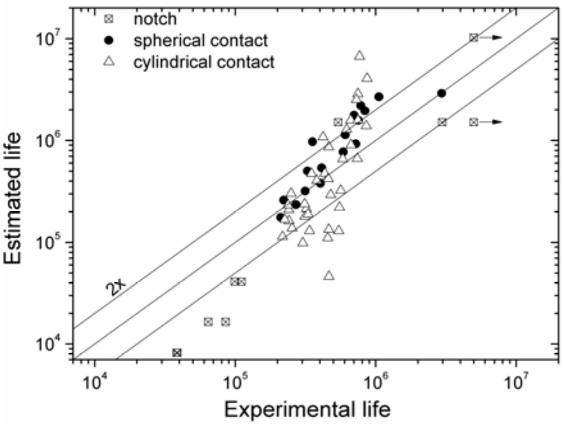


Figure 4. Application of the proposed model to a fretting fatigue test with spherical contact: σ = 110 MPa, Q = 29 N and P = 70 N.

RESULTS

The results of applying the proposed model to the three groups of tests are shown in Fig. 5, where the estimated life is represented vs. the experimental life. The majority of the estimated lives lie in a 2x band around the experimental values. This shows that this model is valid for very different situations.

Other parameters can be analyzed besides the estimated life: initiation length, Fig. 6, and percentage of initiation life, Fig. 7. Figure 6 shows that in the tests analyzed the separation between initiation and propagation occurs almost for every case, between 10 and 40 microns. This is relevant if the wide range of values of stress level and gradient is taken into account. The values of the initiation length are slightly lower in fretting fatigue with cylindrical contact than in fretting with spherical contact and no clear trend is observed related to fatigue life. In the case of fatigue with notched specimens there is a clear trend: the higher the fatigue life the lower the initiation length.



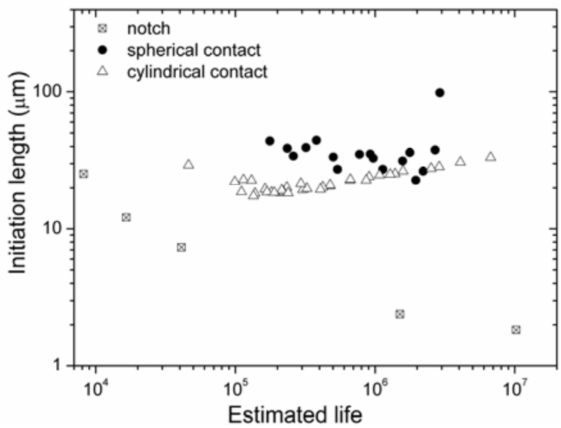


Figure 5. Estimated vs. experimental life. Figura 6. Initiation length vs. estimated life.

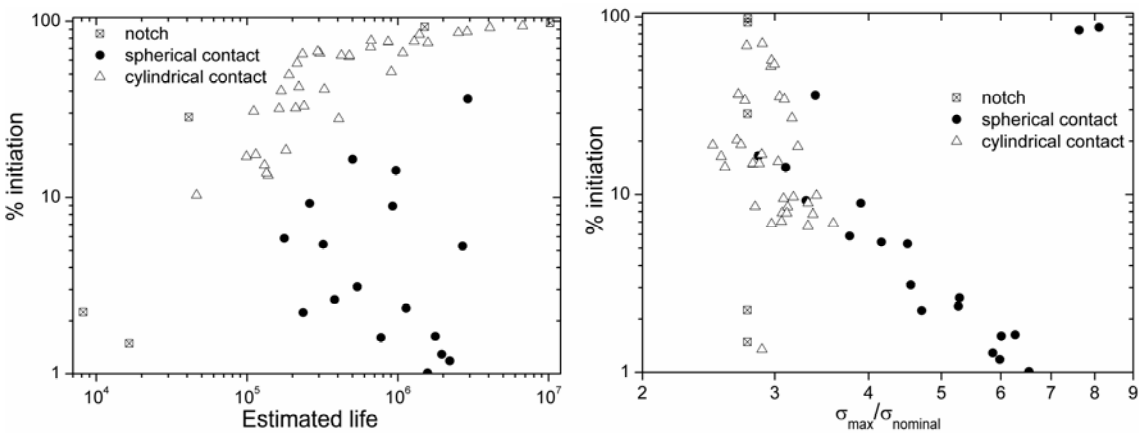


Figure 7. a) Percentage of estimated initiation life vs. estimated total life.b) Percentage of initiation life vs. the stress concentration, $\sigma_{max}/\sigma_{nominal}$.

Figure 7a shows the proportion of life dedicated to initiate a crack as it is obtained through the estimation model proposed as a function of the estimated life. It can be observed that while in fatigue with a notched specimen and fretting fatigue with cylindrical contact the initiation life increases with the number of cycles, reaching values as high as 99%, in spherical contact there is no clear trend. Nevertheless, a clear relation is found between the initiation life and the ratio $\sigma_{max}/\sigma_{nominal}$ [11], i.e., the stress concentration, Fig. 7b. In the fretting fatigue tests $\sigma_{nominal}$ is the amplitude of the bulk stress applied to the specimen and in the notched specimens is the nominal stress applied based on the net section. In both cases σ_{max} is the stress amplitude on the surface. Figure 7b shows that nearly all the results lies in a band where there is a potential relationship between the initiation phase and the stress concentration, the higher the stress concentration the lower the initiation life. This stress concentration is not the only parameter determining the initiation phase. In the case of the notched specimens the stress concentration is always the same but there is a considerable variation in the initiation life. As can be seen in Fig. 7a, in this case it is the fatigue life, i.e. the stress level, the factor influencing on the initiation life.

CONCLUSIONS

This paper has presented a fatigue life estimation model that combines the initiation and propagation phases without defining a priori where the first finishes and the second begins. The model is thought to be applied to situations where there is a stress gradient. It has been applied to three groups of very different tests, fretting fatigue with spherical and cylindrical contact and notches, obtaining good results.

The results show that the initiation life can be very different from one test to another and that neither of the two phases, initiation and propagation, can be neglected a priori. This makes the model useful and robust. It has been shown that the higher the stress concentration the lower the initiation life. The same influence has the level of stresses. Finally, the calculated crack length that separates the initiation and propagation phases is very similar in the whole set of tests analyzed.

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