Is Miner's law valid for asphalt mixes?

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ABSTRACT. Within the analytical pavement design process, Miner's law is used for linear damage accumulation. The aim of the described research project was to investigate the validity of Miner's law especially for asphalt mixes. The investigations were carried out using the cyclic indirect tensile test according to AL Sp-Asphalt 09. The macro crack formation based on the energy ratio was defined as fatigue criterion. Tests with five different load configurations (with increasing, decreasing, alternating and cyclic loading) at five test temperatures have been carried out to investigate the validity of Miner's law. The results show a clear dependency of the damage sum on loading order and test temperature. Considering a real temperature distribution in combination with the damage sum obtained in the tests shows a good conformity with Miner's law.

1 PURPOSE OF THE STUDY

Various stresses occur in asphalt pavements as a result of traffic loading and climate impact. Currently, the analytical pavement design process of asphalt pavements is performed regarding the fatigue state at the lower surface of the asphalt base layer and under consideration of the material properties of the used mixes. Within the design process, the damages caused by the numerous load states are added linearly using Miner's law. Miner's law has been developed on the basis of uniaxial tensile tests on aluminium samples. Hence the validity of Miner's law must be verified for asphalt mixes.

2 INTRODUCTION

2.1 Miner's law
Miner's law allows the accumulation of different damages. Based on cyclic uniaxial tensile and tensile/compression tests on aluminium samples, Miner introduced the following hypothesis:

$$\sum_i \frac{n_i}{N_i} = 1 \qquad (1)$$

n_i = number of load cycles applied at stress i; N_i = load cycles to failure at stress i

Each load cycle consumes a small quantity of the material's lifetime. According to Miner's law, the chronological order of the damage does not influence the damage value.

It has already been proved for other materials, in particular steel, that the independence of the damage from the chronological order postulated in Miner's law does not exist. That means that load cycles with elastic strains below the endurance limit (which exist for steel) cause damage when they occur in combination with load cycles with large elastic strains (Radaj [1]).

The tests Miner conducted on aluminium samples already showed a large variance of the summation of the different damage portions. The damage sum for the test results published varies between 0.60 and 1.49 (Miner [2]). Miner used the fracture of the sample as damage criterion. Nevertheless, Miner's law is used within the analytical pavement design process for asphalt pavements.

2.2 Evaluation of fatigue tests

Different fatigue criteria can be used for the evaluation of fatigue tests. For the described tests (cyclic indirect tensile tests, see section 3.1), the method of energy ratio ER based on the concept of dissipated energy (Hopman [3]) has been used to determine the moment of macro-crack formation.

The energy ratio ER relates the dissipated energy of load cycle i to the dissipated energy of the initial state 0. Assuming that during the test the induced stress and also the phase angle are constant ($\sigma_0 = \sigma_i$; $\varphi_i = $ const.) and $\varepsilon = \sigma/E$, ER for harmonic periodical stresses can be calculated using equation 2.

$$ER(N_i) = \frac{N_i \cdot W_0}{W_i} = \frac{N_i \cdot (\pi \cdot \sigma_0 \cdot \varepsilon_0 \cdot \sin \varphi_0)}{\pi \cdot \sigma_i \cdot \varepsilon_i \cdot \sin \varphi_i} = N_i \cdot E_i \qquad (2)$$

N_i=number of load cycles; W=dissipated energy; σ=maximum stress; ε=elastic strain; φ= phase angle between stress and strain, E_i=stiffness modulus

Fatigue functions can be arranged on the basis of stress or strain. This research project studies strain-based fatigue functions.

3 LABORATORY TESTS

Cyclic indirect tensile tests (CITT) are conducted to examine the validity of Miner's law for asphalt mixes: on the one hand, these were tests with constant stress amplitudes and, on the other, tests with five different loading configurations. The tests and the test conditions are specified in the following sections.

3.1 Cyclic indirect tensile test (CITT)

In CITTs, a circular disk specimen is loaded diametrically between two loading strips (see Figure 1). Because of the specimen shape and the linear transmission of force to the lat-

eral area, an inhomogeneous state of stress occurs in vertical and horizontal directions (see Figure 1). FEM calculations verify that the horizontal tensile stress in the middle of the specimen is nearly constant over approximately 70 % of the specimen's diameter. In the area of force transfer, horizontal compressive stresses appear. The mean value of the vertical compressive stress is located in the middle of the specimen and the maximum value in the area of load transfer. In the centre of the specimen, the ratio between vertical compressive stress and horizontal tensile stress is 3 to 1.

The dimensions of the samples are chosen as a function of the maximum grain size. For a maximum grain size of 11 mm, the specimen diameter should be 100 mm and the height 40 mm. The loading strips had a width of 12.7 mm (AL Sp-Asphalt 09 [4]).
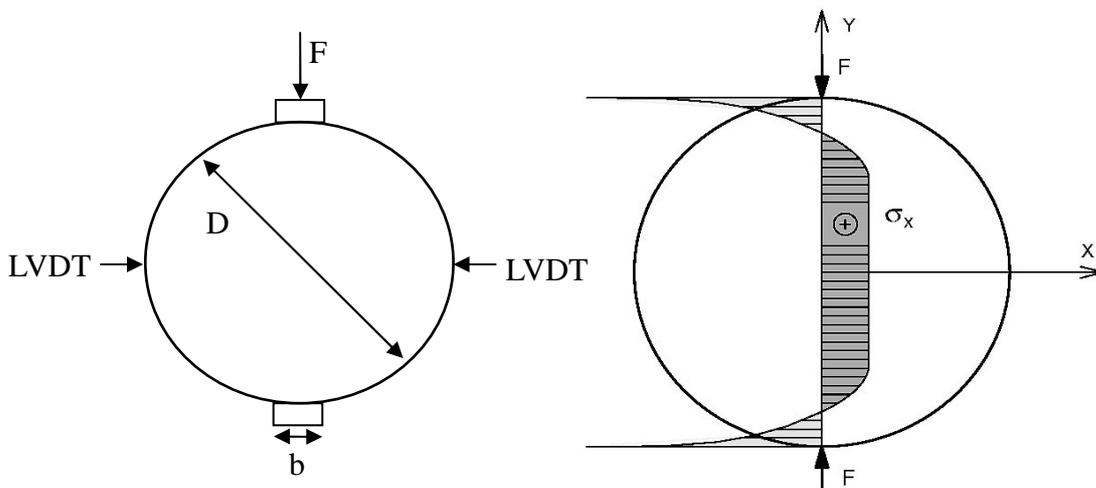
Figure 1. Principle of the CITT and horizontal tensile stress distribution in the middle of the specimen

3.2 Asphalt mixes investigated and sample preparation

A stone mastic asphalt with a maximum grain size of 11 mm and 6.5% polymer modified bitumen (SMA 11 S with bitumen 25/55-55A) was used for the tests. The void content of the samples was between 2.69 and 3.77 Vol.-%. This corresponds to a bulk density variance of 0.03 g/cm³. The asphalt was produced in a mixing plant. The asphalt slabs were fabricated at laboratory using a segmented roller compactor and a deformation-controlled compaction regime. Afterwards, specimens were drilled out of the slabs and the void content as well as the dimensions were determined for each specimen.

3.3 Test conditions and load configuration

The CITTs have been carried out at 5 Hz. The chosen test temperatures are 10, 2.5, 5, 12.5 and 20 °C. They are evenly spread over the possible temperature range of the CITT. The load, which was applied to the specimen, had the form of a harmonic sinusoidal wave without any rest periods. The wave is defined by lower and upper stress. Depending on the test temperature, the cryogenic stress or the necessary contact stress have been chosen as lower stress value. For the determination of fatigue functions, the upper stress has been

varied three times.

Fatigue tests with constant stress amplitudes according to AL Sp-Asphalt 09 [4] have been carried out and evaluated and formed the basis for the following tests to verify Miner's law. Tests with five different load configurations were conducted to prove the validity of Miner's law. Figure 2 illustrates the chronological order of the different load levels. The duration of each load level depends on the test temperature. The load configurations can be described as:

- configuration A: increasing loading
- configuration B: decreasing loading
- configuration C: alternating loading
- configuration D: cyclic loading.

Load configuration D was tested in two versions. In version D50, each loading level has a length of 50 load cycles. In version D500 however, each load level has a duration of 500 load cycles. The lengths of the load levels 1 (low), 2 (medium) and 3 (high) for load configurations A, B and C are chosen in dependency of the testing temperature.
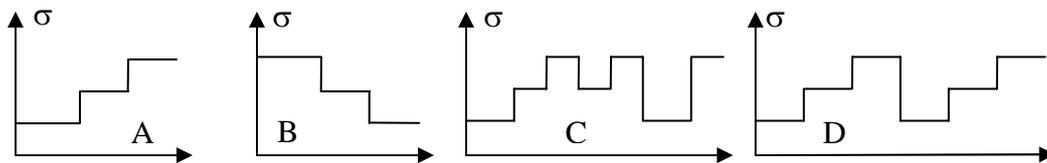

Figure 2. Load configurations

3.4 Evaluation of tests

Evaluating the fatigue tests with constant stress amplitudes produces strain-based fatigue functions. The fatigue functions are generated for each test temperature to ensure maximum accuracy. Figure 3 shows the five fatigue functions. These results are used to determine the lengths of the different load levels (1, 2, 3) and the number of load cycles N_i until failure at stress i.

Before each test with loading configurations (A, B, C, D) a pre-test of three times 50 load cycles was performed to determine the initial elastic strain of each load level. Thanks to this procedure, the number of load cycles until failure can be investigated. Directly after the pre-test, the intended loading configuration was applied to the specimen.
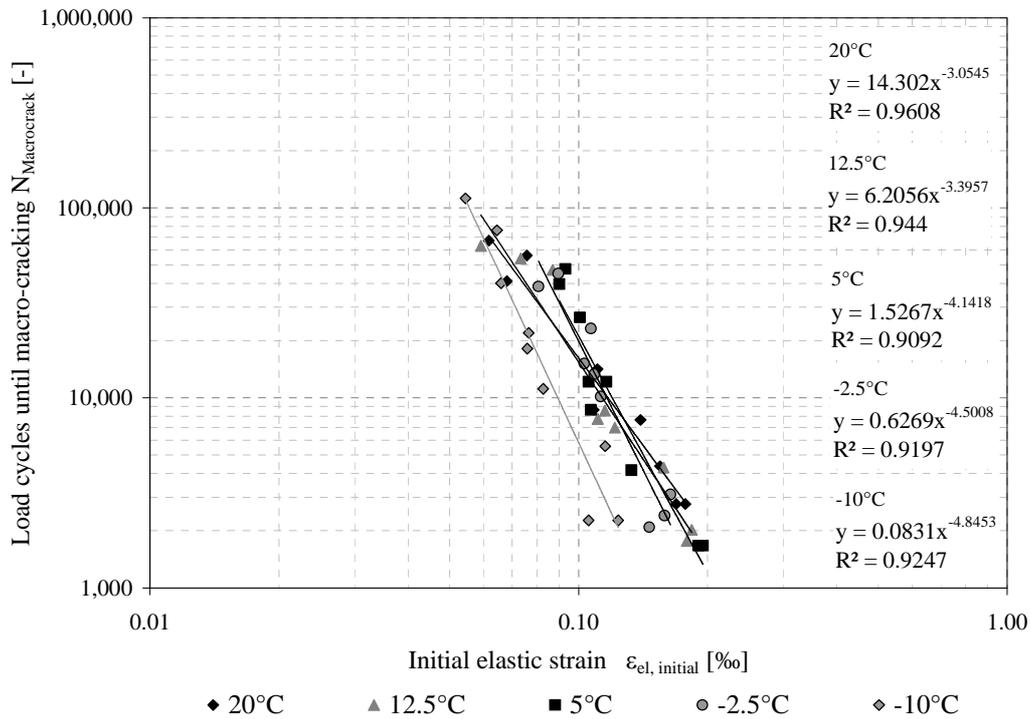
Figure 3. Fatigue functions for the different test temperatures (constant stress amplitudes)

The chart shows the following fitted equations:

20°C
$y = 14.302x^{-3.0545}$
$R^2 = 0.9608$

12.5°C
$y = 6.2056x^{-3.3957}$
$R^2 = 0.944$

5°C
$y = 1.5267x^{-4.1418}$
$R^2 = 0.9092$

-2.5°C
$y = 0.6269x^{-4.5008}$
$R^2 = 0.9197$

-10°C
$y = 0.0831x^{-4.8453}$
$R^2 = 0.9247$

Legend: ♦ 20°C ▲ 12.5°C ■ 5°C ● -2.5°C ◇ -10°C

Axes: x-axis "Initial elastic strain $\varepsilon_{el,\,initial}$ [‰]", y-axis "Load cycles until macro-cracking $N_{Macrocrack}$ [-]"

4 TEST RESULTS

4.1 Impact of the loading order

The damage sum of the different loading configurations in Figure 4 shows a clear dependence on the loading configuration and the load order, respectively. The following statements can be derived from the test results:

(1) The damage sum is lower for increasing loading compared with decreasing loading (difference between configuration A and B). That means that a high initial loading does not lead to early failure.

(2) The length of each load level has an important impact on the damage sum. Fast changes in the load level, as it occurs in situ, lead to a lower damage sum (difference between configuration D500 an D50).

It seems to be impossible to derive the dependence of the damage sum from the loading configuration using the test results because random loading configurations occur in situ. Furthermore, recovery, which has a positive effect on the damage sum, cannot be taken into account. Recovery is caused, for example, by rest periods of different lengths. The study of this effect would require real time tests. Furthermore, the scatter of the test results (on the one hand, the number of load cycles until failure and the damage sum, on the other) is really high. It could be observed that the scatter is especially high for low test temperatures and for loading configurations with alternating and cyclic loading. In Figure 4, the range of the damage sum for 5 °C is also given.
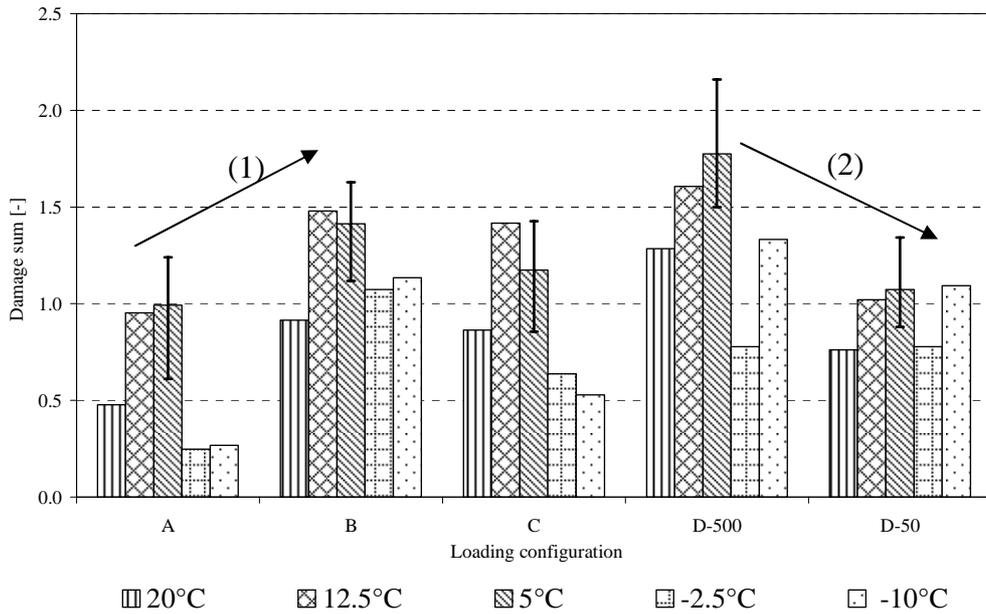
Figure 4. Impact of loading configuration on the damage sum and the scatter for 5 °C
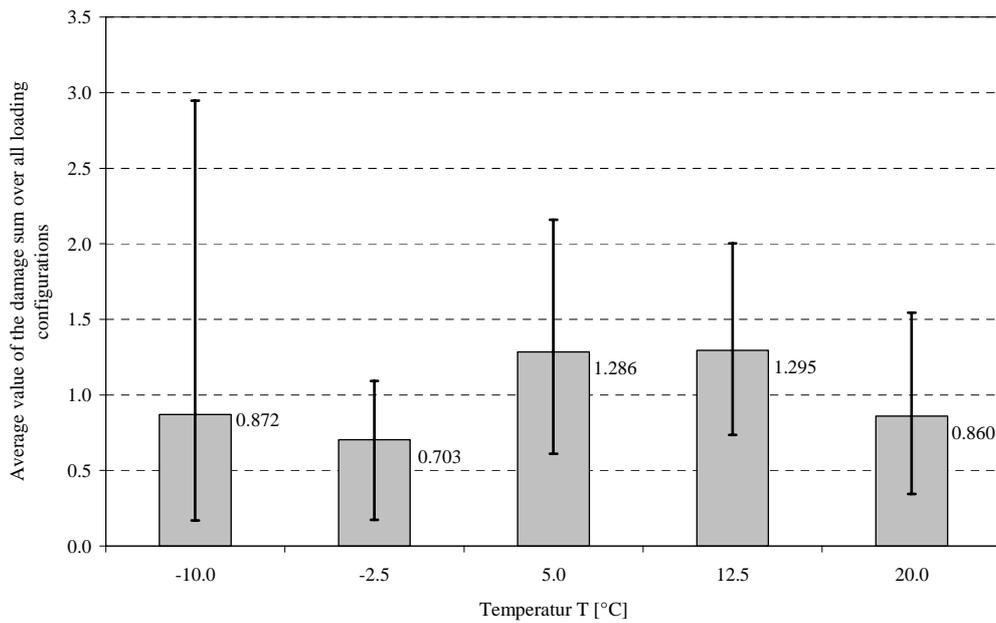

Figure 5. Impact of test temperature on damage sum

4.2 Impact of the test temperature

Figure 5 shows the temperature-dependent average values of the damage sum for the five loading configurations. Without considering the loading configuration, the average value of the damage sum is nonlinear considering the test temperature. The minimum and maximum values caused by different loading configurations are also given in Figure 5. The average value of the damage sum is larger than 1.0 for test temperatures 5 and

12.5 °C. For the other temperatures, the values are below 1.0. If you calculate the average over the given values again, you obtain 1.002 which is the failure criterion postulated by Miner. This approach assumes that Miner's law is valid for asphalt mixes under the given conditions.
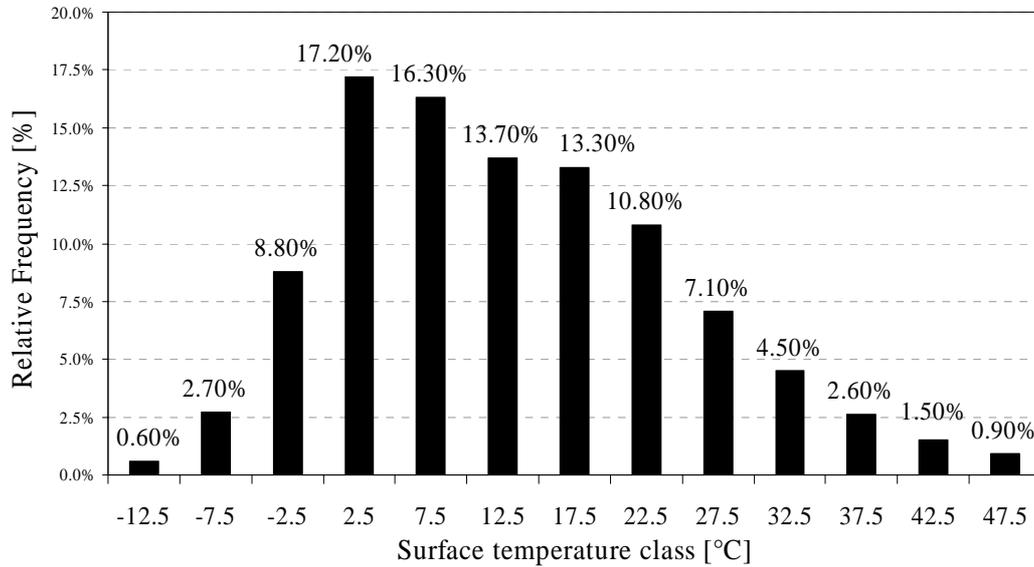

Figure 6. Frequency distribution of the surface temperature for Zone 5 in Germany [5]

But it must be taken into account that temperatures within an asphalt pavement did not appear at the same frequency. Figure 6 gives the temperature distribution of the surface temperature for Zone 5 [5] in Germany including e.g. Dresden. The temperature levels are divided into 5 K intervals and characterised by the mean value. The first step was to calculate the frequencies of the temperature levels -10, 5 and 20 °C using the characteristic values of the two neighbouring values. Because the sum of the frequencies of the five test temperatures is not 100% the frequencies have to be reweighted. By linking the determined temperature frequencies h(T) with the damage sum $n_i/N_i(T)$ and adding all single terms we obtain different damage sums for the six temperature zones defined for Germany (see Table 1). These results indicate that the used temperature distributions obviously lead to a compensation of the high and low parts of the damage sum.

Table 1. Damage sums for the temperature zones defined for Germany

Temperature Zone	Temperature weighted damage sum
1	1.092
2	1.070
3	1.075
4	1.086
5	1.082
Mountains	1.036

5 CONCLUSION

The investigation of five different loading configurations at five test temperatures shows a clear impact of the load order and the test temperature on the damage sum at failure. The impact of the test temperature is of particular importance because the loading order cannot be taken into account especially with regard to the impact of rest periods. It is not possible to determine this effect using accelerated tests. The damage sum obtained on the basis of the test results in combination with a real temperature distribution is 1.07. The CITT and also other fatigue tests can be conducted at a maximum test temperature of 20 °C. Hence, the impact of higher temperatures on the damage sum can only be presumed. It seems to be realistic that the obtainable damage sum at these temperatures does not increase compared with the value of 20 °C.

The test results published by Miner already showed a large scatter which can be confirmed by the current test results. From this follows that the use of statistic methods is required. In the future, it will be possible to consider different security levels by using cumulative frequency curves.

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