Investigation on the Shrink-fitted Assemblies with a Circumferential Groove

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ABSTRACT. *In this study, the endurance limit of shrink-fitted assemblies with a circumferential groove subjected to rotating bending is investigated experimentally as well as numerically. So called staircase method is used for the realisation of the fatigue test series. These assemblies fail always in the circumferential groove, therefore an "infinite" life can be assumed at this load level. Statistical evaluation of the experimental results is applied using several methods. Moreover, influence of mechanical and heat surface treatment (shot peening and induction hardening) on the endurance limit is investigated experimentally.*
By means of the FE-Analyses the influence of static pre-stress in circumferential groove due to interference fit on fatigue limit is determinated. The impact is characterised in dependence on the shaft diameter, shoulder geometry and surface pressure in the shrink fit.
The verification of the numerical and experimental results shows a very good agreement with calculation according to German standard DIN 743 (based on a nominal stress approach).

INTRODUCTION

Shrink-fitted assemblies can be found in numerous applications, especially in drive and powertrain engineering. Besides the main function of torque transmission between shaft and hub rotating machine elements are often subjected to additional cyclic bend loading. This leads to a complex multiaxial stress state within the critical cross section of the shaft. Moreover tribological damage (fretting fatigue and fretting wear) occurs in the contact area resulting in a collateral reduction of the fatigue strength.
Fig. 1 shows a shaft failure of a passenger carriage caused by a fatigue crack close to a wheel set press-fit (left) and a fatigue failure of a shrink-fitted assembly subjected to rotating bending in laboratory conditions (right). In order to prevent unsafe shaft design under consideration of shrink-fitted hub an integral engineering approach is needed for the endurance limit evaluation.
This study combines experimental and numerical fatigue assessment with a special focus on shrink-fitted assemblies with a circumferential groove. Numerous material and test data are necessary for a precise endurance limit estimation. Numerical analyses of

the tested geometry allow to identify the local stress and suitable fatigue prediction parameters. Furthermore, the experimental results can be transferred within the simulation to other assembly designs.

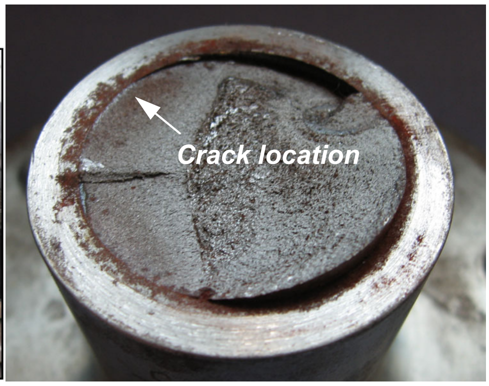


Figure 1. Shaft failure of ICE3 train caused by fatigue crack [1] (left); fatigue fracture surface of shrink-fitted assembly subjected to rotating bending [2] (right).

EXPERIMENTAL ANALYSES

Geometry and Test Stand

Fatigue tests of the assemblies were performed using test apparatus for rotating bending shown in Fig. 2. The loading frequency was 15 Hz. The geometry of the tested assembly is shown in Fig. 2. The Function of the test stand is described in detail in [2]. All tested shafts and hubs were made of 1.0503 (AISI 1045) normalised steel characterized by yield strength of 350 MPa and ultimate tensile strength of 690 MPa. Especially in gear boxes shaft shoulders with a circumferential groove are widely-used as axial adjustment and for axial load transmission. The circumferential groove according to DIN 509 [3] form E 0.6 x 0.3 was chosen as representative test geometry. Nominal shaft diameter was 40 mm, outer hub diameter was 80 mm and the assembly length was 44 mm, see also Fig. 4. The relative interference of the shrink-fit according to [2] was set to 1.7 ‰ leading to a mean contact pressure of 133 MPa. The shafts and hubs were thermally shrink-fitted.

Statistical Evaluation

The fatigue test procedure was estimated by so called "arcsin√P-transformation". In this method presented by Dengel in [4] fatigue failures and not failed specimens (no cracks) of one load level are plotted and afterwards referred to each other. Using this method the average value $P_{Ü}$ = 50% and the borders of survival probabilities (10% and 90%) can be estimated. Another benefit of this method is the functional correlation between survival probabilities and the experimental load levels. The fatigue strength evaluation algorithm uses an approximation of linear regression with coefficients a and b. In this way a correlation between load level and the so called transformation value z is given, which is defined as arcsin$\sqrt{P_{Ü}}$.

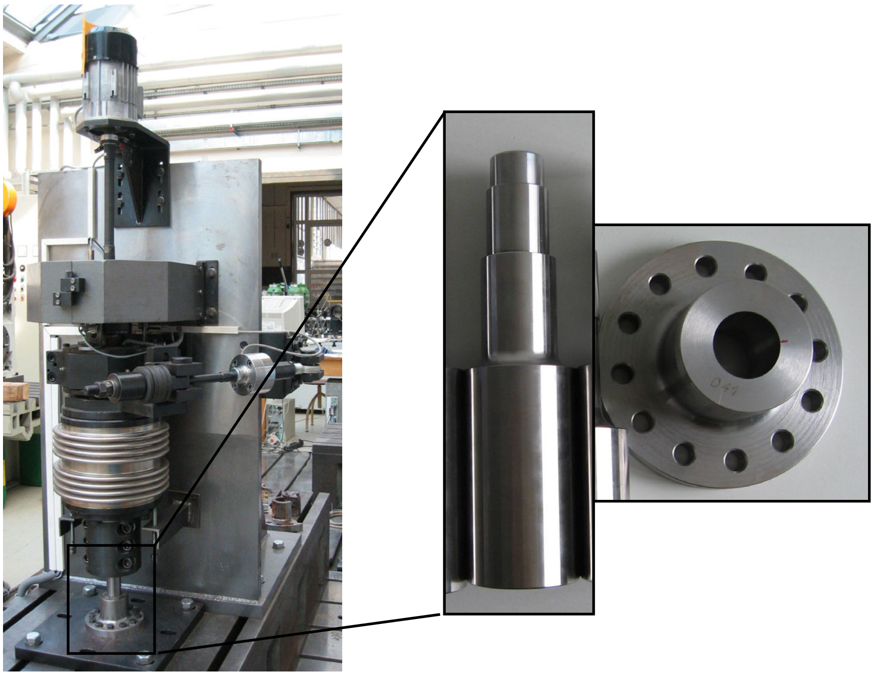


Figure 2. Experimental set-up for fatigue testing of shaft-hub-connections under rotating bending conditions with corresponding specimen geometry (shaft and hub).

First the experimental value of the survival probabilities $P_{Ü}$ can be found by the number of specimen n per load level and the number of cracks r.

$$P_{Ü} = \frac{r}{n} \tag{1}$$

For the approximation of the single survival probabilities the known equations for linear regression with coefficients a and b are used.

$$F(P_{Ü})_i = a + b \cdot \arcsin\sqrt{P_{Ü}} \tag{2}$$

Fatigue Tests

So called improved staircase method introduced by Hück in [5] was used for the realisation of the fatigue test series. The experimentally estimated stair case sequence for rotating bending of the tested shaft geometry is shown in Tab. 1. Shaft failures are signed with "x", no cracks with "o". The number of load cycles for reaching the "infinite life" was set to $5 \cdot 10^6$. The stair case method was started by a bending amplitude value of 118 MPa.

Table 1. Stair case fatigue evaluation method of a shrink fit subjected to rotating bending; ellipse marks one specimen.

Bending stress amplitude	σ_{ba} [N/mm^2]	stair case method: shrink-fit bending charge														
	135															
	126															
	118	X		X				X						X		
	110		0		X		0		X		X		0		X	
	103					0				0		0				
	96															
	90															
	Nr.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

X – failure 0 – no crack

The test sequence in Tab. 1 was evaluated by the arcsin√P-Transformation. This method provides a structural durability of σ_{bADK} = 108 MPa for 50% survival probability under rotating bending. More detailed work about the statistical evaluation was published by the authors earlier, [2]. The crack appeared always in circumferential groove (Fig. 3). Therefore an "infinite" life can be assumed for shaft witout failure. Resulting from crack location the circumferential groove leeds to a higher fatigue notch factor than the shrink-fit aside. Thus the fatigue limit of the shaft can be increased by more favourable groove geometry.

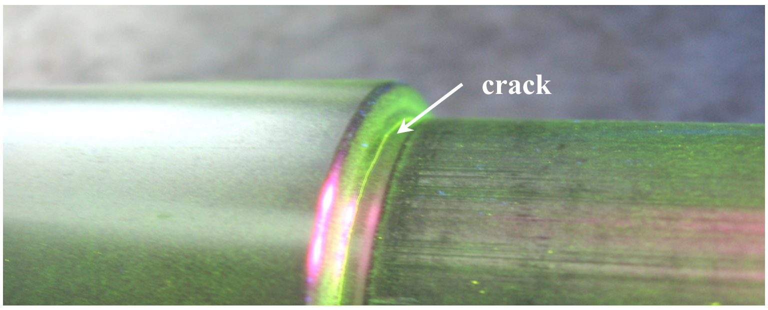
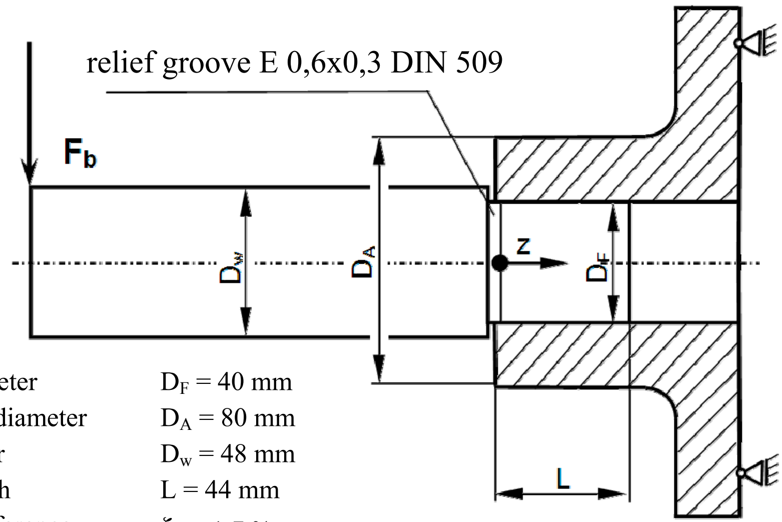


Figure 3. Crack location on the shaft with circumferential groove.

NUMERICAL ANALYSES

Finite-Element Model

The numerical simulations described below were performed using the commercial finite-element (FE) software, ABAQUS. The shaft-hub connection with shrink fit was reproduced as a fully three-dimensional FE model. The individual components, i.e. shaft and hub were meshed exclusively with eight-node linear hexahedron elements. C3D8I elements (incompatible-modes formulation) were used within contact and high-stress-gradient areas of the shaft to obtain more accurate results. C3D8R (reduced-integration) elements were used to fill the major part of the hub as well as the free end of the shaft to reduce the number of model variables. The meshing strategy used for creating matching contact meshes was chosen in order to obtain an accurate reproduction of stresses in critical areas and of contact displacements (splippage). Geometry of the simulated shaft-hub connection is shown in Fig. 4. Between the hub front surface and the shaft shoulder a gap was modelled so that no contact occurs during the bend loading. In general, penalty normal contact formulation with high contact stiffness (10^7 MPa/mm) was used to keep the penetration negligible (less than 0.1 μm).



Press-fit diameter	D_F = 40 mm
Hub external diameter	D_A = 80 mm
Shaft diameter	D_w = 48 mm
Press-fit length	L = 44 mm
Relative interference	ξ = 1.7 ‰
Coefficient of friction	μ = 0.21

Figure 4. Geometry of the basic type of shaft-hub connection with circumferential relief groove according to DIN 509.

Stress Distribution

The distribution of maximal principal stress over the relief groove is shown in Fig. 5. Stress maximum occurs in radius of the relief groove as expected. In accordance with Fig. 3 the crack location is predicted correctly in the radius.

The maximal tensile stress due to the statical pre-stress of the shrink-fit is $\sigma_{1,Vor}$ = 200 N/mm^2 (s. Fig. 5; right). The maximal tensile amplitude (dynamical stress component) resulting from rotating bending is $\sigma_{1,dyn}$ = 308 N/mm^2. The stress concentration factor can be calculated as follows:

$$\alpha_{\sigma,FEM} = \frac{\sigma_{1,dyn}}{\sigma_{b,nom}} = 2,73 \tag{3}$$

There is a good agreement with the value of the stress concentration factor according to DIN 743 [6] $\alpha_{\sigma F}$ = 2.91. The difference is only about 6 %.

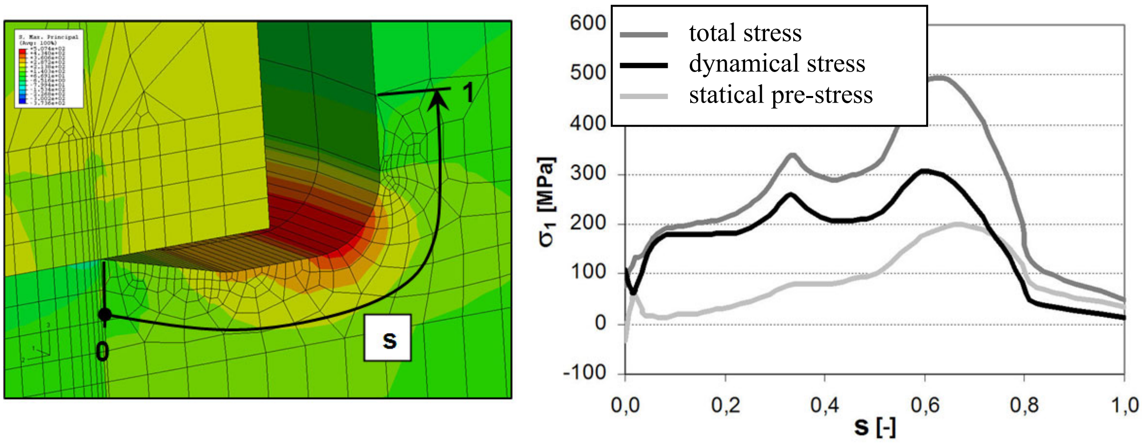


Figure 5. Distribution (left) and plot (right) of the maximal principal stress in relief groove of the shaft subjected to rotating bending, $\sigma_{b,nom} = 113\ \mathrm{N/mm^2}$.

Influence of Pre-Stress

Furthermore the fatigue notch factor for the relief groove was determinated. Therefore the tensile stress resulting from the shrink-fit has to be implemented as the statical pre-stress. This value was determinated in dependence on diameter ratios D_F/D_A and D_F/D_W and on relative interference ξ. The contact pressure p_F was chosen as a reference value, because it is linear depending on relative interference and easy to calculate.

By means of comprehensive FE-analyses the following empirical equation was formulated for the normalised pre-stress in the relief groove according to DIN 509 [3] depending on shaft shoulder diameter ratio D_F/D_W (cp. [2]):

$$\sigma_{1,\mathrm{Vor}} / p_F = \left(-7{,}88 \cdot \left(\frac{D_F}{D_W} \right)^2 + 8{,}95 \cdot \left(\frac{D_F}{D_W} \right) + 0{,}007 \right) \cdot \frac{-0{,}0003 \cdot D_F^2 + 0{,}058 \cdot D_F \cdot \mathrm{mm} - 0{,}418 \cdot \mathrm{mm}^2}{1{,}9 \cdot \mathrm{mm}^2} \tag{4}$$

Using Eq. 4 it is possible to calculate the maxial tensile pre-stress in the relief groove for any shrink-fit defined by D_F and diameter ratio $D_F/D_W \in < 0.70 \ldots 0.95 >$.

Fatigue Notch Factor

As already shown the tensile pre-stress for the considered shaft geometry is $\sigma_{1,\mathrm{Vor}} = 200\ \mathrm{N/mm^2}$. According to this the assessed fatigue limit of the relief groove using FE-analyses as well as DIN 743 [6] was $\sigma_{bADK,FEM} = 110\ \mathrm{N/mm^2}$ and $\sigma_{bADK,DIN} = 105\ \mathrm{N/mm^2}$. The comparison with the experimental estimated fatigue limit $\sigma_{bWK} = 108\ \mathrm{N/mm^2}$ shows a very good agreement with the theory. The calculated fatigue notch factors are shown in Fig. 6.

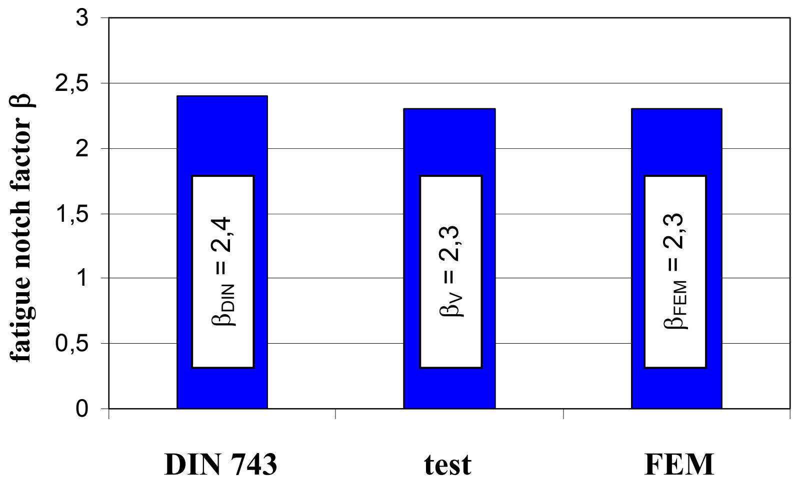


Figure 5. Comparison of the fatigue notch factors evaluated according to DIN 743 as well as FEM with the experimental determinated value of 2.3.

Surface Treatment

Moreover the fatigue limit of the shaft was increased using following technological procedures. Laser beam hardening and shot peening was chosen as appropriate surface treatments of the relief groove. The shrink-fit was covered during the surface treatment and was not treated. Fig. 6 shows the experimental evaluated fatigue limits in comparison to the basic not treated variant.

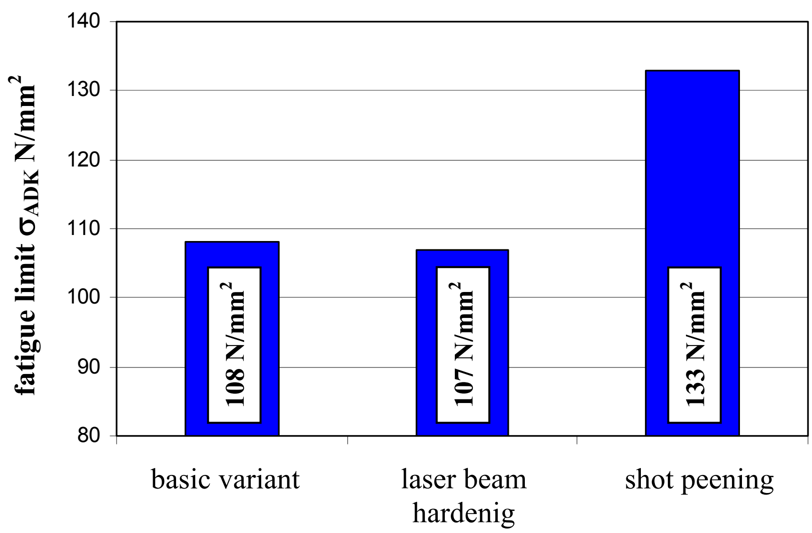


Figure 6. Experimentally evaluated fatigue limits of a shrink-fitted assemblies with relief groove; basic variant without surface treatment, laser beam hardened groove and shot peened groove.

The results show that the laser hardening no benefit delivers in the present case. Against it the shot peening increase the fatigue limit of the undercut at 20 %.

CONCLUSION

The endurance limit evaluation for prevalent machine elements is indispensable regarding to safe construction design. The presented integral approach combines statistical fatigue testing with numerical stress evaluation. First, the endurance limit of the tested shrink-fitted assembly with a circumferential groove subjected to rotating bend loading was estimated experimentally using Hück's modified stair case method. Numerical methods as well as nominal stresses were analysed regarding to endurance limit prediction. Good correlation was achieved with DIN 743. The shot peening of the undercut increase the fatigue limit of the tested geometry by 20%.

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