Investigation of multiaxial fatigue in the prospect of turbine disc applications: Part II – Fatigue criteria analysis and formulation of a new combined one

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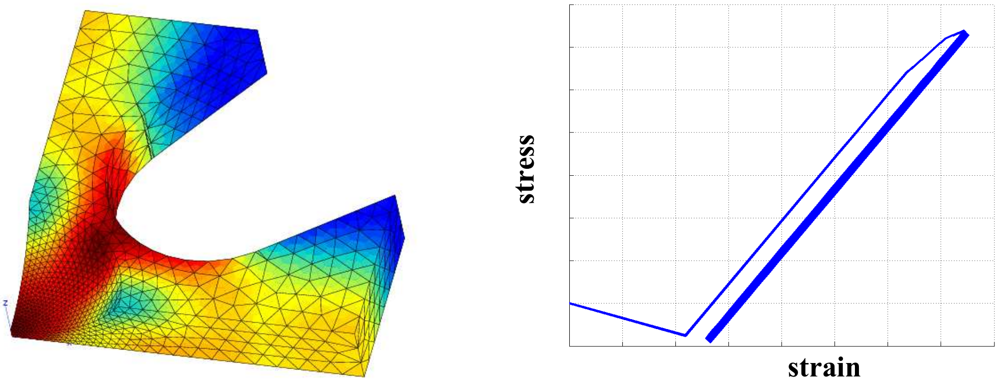
ABSTRACT. *Different multiaxial fatigue criteria, including those based on octahedral shear stress amplitude combined with hydrostatic pressure, and more recent models based on critical plane approaches are evaluated using the multiaxial fatigue experimental data generated in the present study. This includes various loading conditions (tension/torsion, tension/internal pressure, biaxial). The analyses show that none of them is able to correctly correlate all the considered uniaxial and multiaxial conditions. A new combined criterion is then proposed that offers a possible compromise and significantly improves the predictions.*

INTRODUCTION

Many engineering components that undergo fatigue loading experience multiaxial loadings. Rotating parts in turboengines, like turbine or compressor discs are typical examples. Nowadays, many multiaxial fatigue models and data are available in the literature. In spite of the number of proposed criteria, no universally accepted approach yet exists. Several good reviews of multiaxial fatigue criteria are available in the literature (Garud [1], Brown and Miller [2] or Macha and Sonsino [3]). These approaches can be divided into three categories. One popular approach has traditionally been to extend the static yield criteria to fatigue by combining octahedral shear stress amplitude with hydrostatic pressure. By similarity, several strain based criteria have been proposed. More recently, special attention has been paid to energy criteria. The Energy approach has been proposed and tested by several researchers (Garud [4], Ellyin [5] and Radakrishan [6]) and but presents some difficulties for quasi elastic cycles. A third method for multiaxial fatigue life evaluation has been to use critical plane approaches. In all case, as it will be underlined in the present paper, the largest difficulty with any such fatigue criterion is to take simultaneously into account both the mean-stress effects (in uniaxial conditions) and the multiaxiality effects. To assess current methodologies for multiaxial fatigue, Snecma, Turbomeca, Onera and CEAT has conducted a significant experimental program on two classes of classical disc materials. The experimental devices and results are presented and discussed in the part I joint paper. In the present paper, several existing multiaxial fatigue criteria (namely Sines [7], Crossland [8], Brown-Miller [9], Fatemi-Socie [10], Smith Watson Topper [11] and Gonçalvès et al. [12]) are evaluated to determine their suitability at correlating the multiaxial fatigue data generated in the program. Regarding the observed limitations, a new criterion is proposed with the objective to estimate correctly shear and equibiaxial fatigue conditions together with a correct description of mean stress effects.

FINITE ELEMENT ANALYSIS

The experimental program presented in the part I joint paper includes three types of multiaxial tests: tension/torsion in-phase and out-of-phase conditions, tension/internal pressure in-phase conditions, biaxial fatigue loadings. All the multiaxial conditions have been calculated by various methods, strength of materials rules, (thermo-)elastic Finite Element analysis and full cyclic inelastic Finite Element analysis. The nonlinear constitutive equations, used in the present work, are consistent with the unified viscoplastic formalism developed by Chaboche [13], which allows a unified description of yielding, creep, stress relaxation, and a variety of other mechanical effects, such as Bauschinger effects and time recovery. When considering strain control tests, the cyclic mean stress relaxation is an important phenomenon that has to be taken into account, due to the great impact influence of a non completely relaxed mean-stress on fatigue life, as presented in Military Handbook [14]. During the cycles, there is effectively a continuous relaxation of the mean stress until a non zero stabilised value which depends on the applied strain amplitude. Here is then involved a multi-kinematic hardening rule including thresholds [15] in order to correctly reproduce the cyclic curve, the stabilized stress-strain loop and the corresponding mean-stresses.



Von Mises equivalent stress (equibiaxial loading)

σ_{11}-ε_{11} loops in the critical region

Figure 1 : F.E. calculation of equibiaxial loading on cruciform specimen (TA6V).

The full cyclic inelastic analysis was mandatory only for the compact cruciform specimens, in order to obtain in the critical regions the local multiaxial cyclic stress-strain responses and their stabilised mean-stress level, Fig.1. The Finite Element simulations have shown that the minimum number of successive calculated cycles necessary to achieve the stabilised state is around 100.

MULTIAXIAL FATIGUE MODELS

The objective of the present work was to identify reliable multiaxial fatigue models in the context of turbine disc applications. The existing models evaluated here can be classified in two categories, equivalent effective stress models and critical plane models.

Critical Plane models

The critical plane concept has been used by different authors and still receives great attention. Critical plane models were mainly developed on the basis of phenomenological observations of fatigue crack development. It is generally recognised that fatigue cracks often nucleate and propagate on critical planes. The models assume then that crack nucleation on a particular plane is a function of the normal and /or shear stresses and strains on that plane. The various proposed formulae are different, but the process to follow is merely the same. One must firstly, find the critical plane and secondly check if the criterion is satisfied on this plane. One of the first such approach is the Dang-Van criterion, quite popular in France [16]. Three other popular critical plane models are evaluated in this paper, the critical plane approach of Brown and Miller [9], Fatemi-Socie [10] and Smith-Watson-Topper [11].

Models based on an effective stress amplitude

Effective stress models are essentially extensions of static yield criteria, such as the Von Mises criterion. The multiaxial stress amplitude is then reduced to an effective uniaxial cyclic value and the hydrostatic stress is often introduced to take into account the mean stress effects. This is the case for the popular Sines and Crossland criteria. The three effective stress models, Sines [7], Crossland [8] and Goncalvès [12] considered in this study can be written in the same format:

$$\sigma_{a_{eff}} = \sigma_{a_{eq}} \left(1 + b\,\sigma_{D_0} t_{eq}\right) \tag{1}$$

The triaxiality factor t_{eq} takes one of the three following forms associated respectively with Sines, Crossland and Gonçalvès criteria :

$$t_{eq} = t_F = \frac{(Tr\sigma)_{mean}}{\sigma_{a_{eq}}}, \quad t_{eq} = \frac{(Tr\sigma)_{\max} - \sigma_{a_{eq}}}{\sigma_{a_{eq}}}, \quad t_{eq} = s_F - 1 = \frac{\sigma_{p_{\max}} - \sigma_{a_{eq}}}{\sigma_{a_{eq}}} \tag{2}$$

where $\sigma_{a_{eq}}$ is the octahedral shear stress amplitude, $(Tr\sigma)_{mean}$ the mean value of the first stress invariant during the cycle. The term σ_{D0} represents the fatigue limit for reversed cycle at 10^7 cycles and b is a material parameter that will be identified from uniaxial tests with mean stress effects.

A new combined criterion

As it will be described later on, the confrontation of these existing models with the multiaxial fatigue data have shown that they do not correlate well all the variety of mean stress levels and loading conditions generated in the experimental program. A specific combined model has then been formulated with the objective to estimate correctly shear and equibiaxial fatigue conditions. The proposed criterion is written as defined in Eqn. 1 but with the following expression for the triaxiality factor:

$$t_{eq} = \xi \frac{s_F\ t_F}{1+\left|t_F\right|} + (1-\xi)\left(s_F - 1\right) \tag{3}$$

Here ξ is a material parameter. This new model gives results similar to the Sines one when t_F tends to zero and similar to a model with $t_{eq} = s_F$ when t_F takes high values. For uniaxial loadings Sines, Crossland, Gonçalvès and the new combined criteria are identical and $t_{eq} = (1+R)/(1-R)$ where R is the stress ratio. In the present work, all the different criteria have been correlated to the lifetime by using a simple Basquin type function $\sigma_{a_{eff}} = BN_f^{-\beta} + \sigma_{D_0}$ involving the fatigue limit for reversed cycle at 10^7 cycles. In this expression, B and β are here material parameters, that are identified on LCF (strain or stress control) and HCF tests (stress control).

APPLICATIONS AND DISCUSSION

The presented multiaxial models were evaluated based on their ability to correlate both uniaxial data and multiaxial data. All the models have been identified on the same uniaxial data base, namely strain and stress fatigue tests at several loading ratio.

Identification of mean stress effects on unixial tests

Our experimental program includes also LCF (strain or stress control) and HCF tests (stress control). A particular attention has been addressed on the identification of the mean stress effects. Figure 2 represents Haigh's diagram obtained on the titanium based alloy (TA6V) at two different temperatures for a fixed fatigue life of 10^7 cycles. Room temperature results well correlate literature data, showing for this material an extremely high dependency on the mean stress, but its saturation for large mean stresses. The material parameter b that appears in Eqn.1 corresponds then to the slope of the straight line that fits to the best the experimental data in the region $-1 < R < 0.45$. However, in order to reproduce correctly the experimental tendency, a bilinear fitting is introduced.

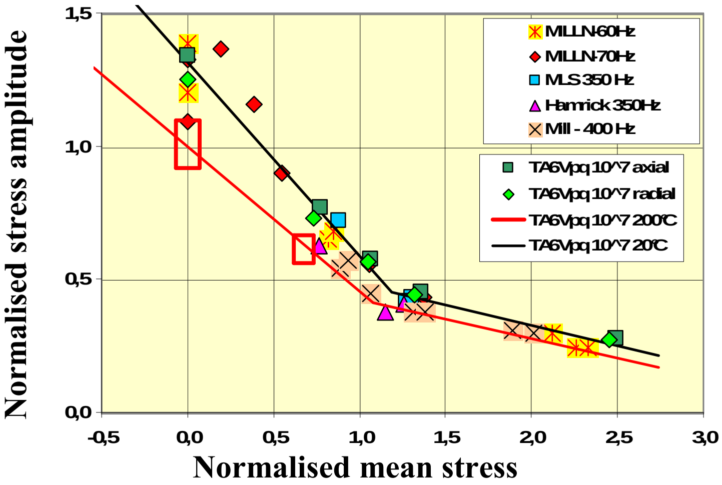


Figure 2 : uniaxial Haigh's diagram for TA6V at room temperature and 200°C (Rectangular red boxes indicate the overall uncertainty of results at 200°C)

The limit value t^* is defined as $t^* = (1 + R^*)/(1 - R^*)$ where R^* is the limit loading ratio, in uniaxial, for which a change of the slope of the straight line occurs in Haigh's diagram. In what follow, at 200°C, the red bilinear line in figure 2 has been used. These mean stress effects have also been validated for LCF stress fatigue tests. Using uniaxial strain and stress data, the mean stress effects are then identified and consequently the multiaxial mean stress effects considered in most criteria are also completely characterised.

Multiaxial Evaluation

The first comparisons to multiaxial experimental data are presented for Sines, Crossland, Gonçalvès and the new combined criterion in Figs 3 and 4 which can be seen as a multiaxial extension of Haigh's diagram. Such kind of representation has been first proposed by Dang-Van [16]. The octahedral shear stress amplitude $\sigma_{a_{eq}}$ is therefore plotted versus respectively $(Tr\sigma)_{mean}$, $(Tr\sigma)_{\max}$, $\sigma_{p_{\max}}$ and $t_{eq}.\sigma_{a_{eq}}$. The line represents the criterion for a specific fatigue live of 40000 cycles. As described earlier, all the criteria have been identified using the uniaxial fatigue data at various loading ratio, as it can be seen in Fig 2. The normalised experimental results here are obtained by roughly interpolating the true experimental data on TA6V at 200°C (measured life as a function of loading levels) for the same fatigue life $N_f^* = 40000$ (at crack initiation). The tension/torsion (T.T), tension/ internal pressure (T.IP) and the biaxial fatigue results of the conducted experimental program have been considered. As they are plotted in a normalised way, existing data at 250°C from Gomez [17] were also added. From these comparisons, it appears that Crossland criterion has to be rejected. Moreover, the results clearly demonstrate that only Sines and the modified combination proposed in this work are able to correlate results in the shear regime, though only Gonçalvès and the proposed combination may be acceptable for repeated equibiaxial conditions.

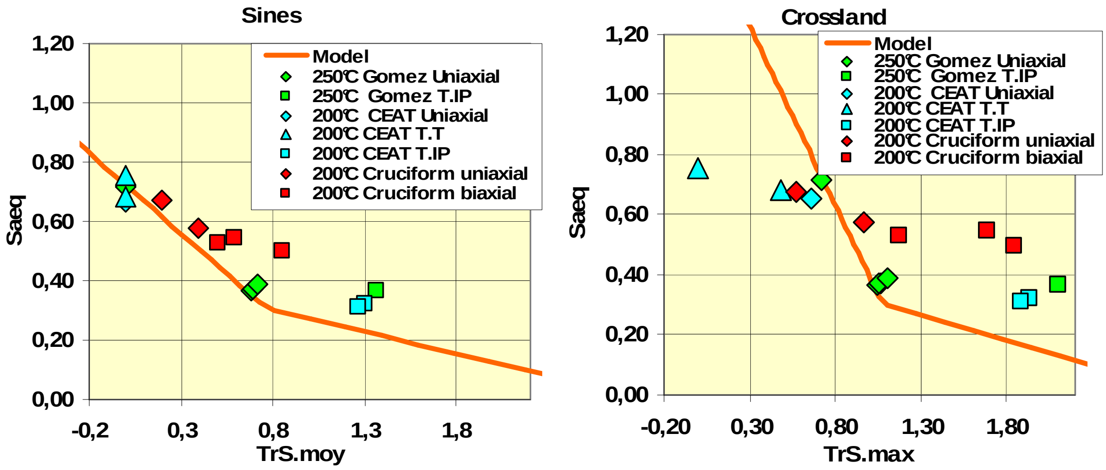


Figure 3 : Comparisons of the Sines sand Crossland criteria with interpolated multiaxial results on TA6V. Left :Sines; right: Crossland.

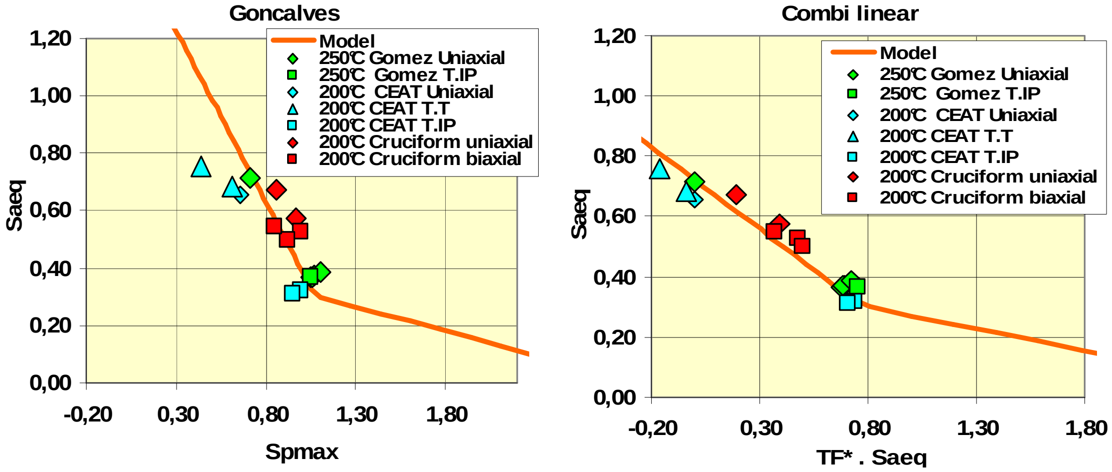


Figure 4: Comparisons of the Goncalvès criterion and the new combined one with interpolated multiaxial results on TA6V. Left: Goncalvès; right: New combined

Figure 5 confirms previous results for the various configurations tested in the present work. The three criteria Sines, Gonçalvès and the new combined criterion are compared here. This is made on a biaxial stress diagrams ($\sigma_3 = 0$), for proportional loading conditions, very useful to compare various criteria on the same basis, all of them giving identical results for fully reversed and repeated uniaxial conditions. The envelops are plotted for equals fatigue lives of 40000 cycles, consistently with experimental data on TA6V. On the left figure we observe, in a different way the great conservatism of Sines criterion for equibiaxial conditions, and the slight non conservatism of Gonçalvès one, both for shear and for equibiaxial conditions. On the right, we observe the quite good compromise obtained with the combined criterion proposed in the present paper, both for shear conditions and equibiaxial ones (where it is only with a limited conservatism). Other criteria based on critical plane approaches are presented on Fig. 6, both for the reversed conditions (on the left) and for $R = 0.3$ (on the right).

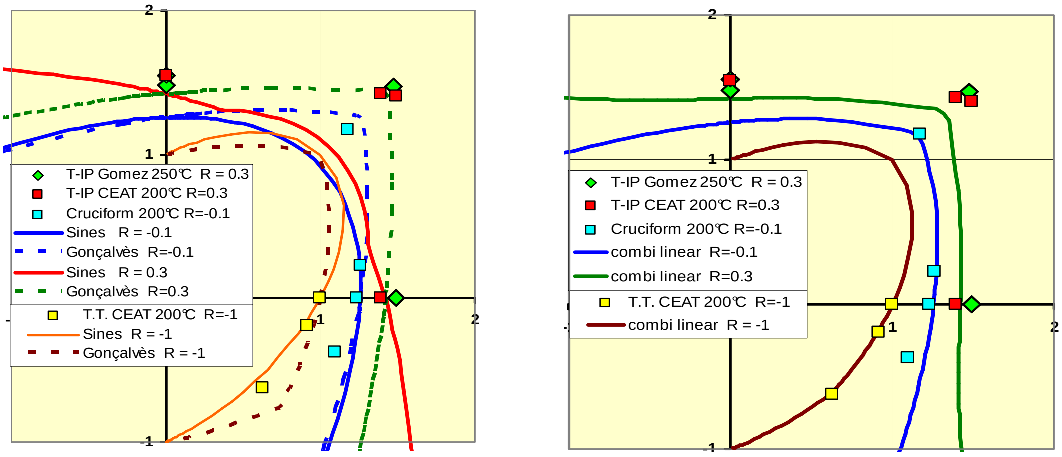


Figure 5: predicted multiaxial fatigue envelops for TA6V at 200°C, and comparisons with 3 kinds of loading conditions; left : Sines and Gonçalvès; right: new combined

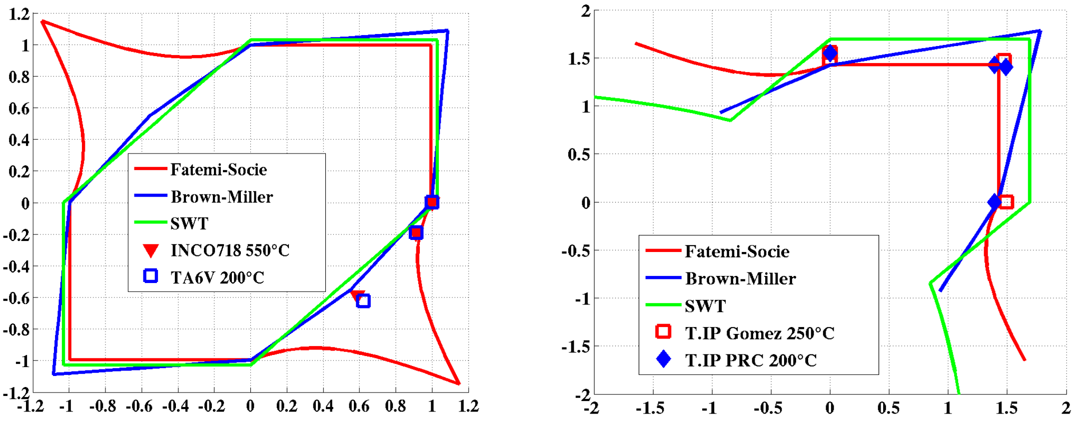


Figure 6: multiaxial fatigue envelops for 3 critical approaches with interpolated results on INCO 718 and TA6V. Left : $R = -1$; right: $R = 0.3$

It can be noticed that these models do not show improvements in what concerns the simultaneous description of the shear regime and the repeated equibiaxial regime. Fatemi-Socie is acceptable for the equibiaxial conditions (slightly non conservative) but totally inefficient for shear. Brown & Miller is acceptable for reversed shear but very non conservative for the repeated equibiaxial conditions. SWT model is conservative for reversed shear, but, as there is no material parameter for the mean stress effect, both uniaxial and equibiaxial conditions are non conservatively predicted for $R = 0.3$.

CONCLUSION

The evaluation of existing multiaxial fatigue criteria in the context of the life assessment of turbine discs has shown that, among all criteria evaluated, no one is able to correctly predict the observed multiaxiality effects for both pure shear and equibiaxial conditions. This is true for both the stress invariant based models and for the critical plane approaches. This is mainly due to the imposed choice to simultaneously reproduce, with the same criteria, the mean stress effects as observed under uniaxial conditions. A new special combined form of a multiaxial criterion has been proposed in the present work that simultaneously respects the pure shear conditions (both reversed and repeated ones), the equibiaxial regimes and the mean stress effects for uniaxial and multiaxial conditions. The experimental conditions of equibiaxial loadings have been obtained with two different procedures, tension-compression with internal pressure on tubular specimens and biaxial loads on cruciform specimens. The observations made against classical criteria are consistent with the two configurations, for different loading ratio.

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