Fatigue Crack Growth Analysis Under Mixed Mode Loading

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ABSTRACT. *In this paper mixed mode crack growth behaviour is studied. The simulation of fatigue crack path is considered. Furthermore, the experiment of crack path under mixed mode loading was performed with a plate with crack emanating from the edge of a hole. Therefore, the fatigue lives for an Arcan specimen subjected mixed mode loading are estimated. The stress intensity factors for Mode I and Mode II were calculated using analytical and numerical approaches. The effect of stress/load ratio and loading angle on fatigue life up to failure of the Arcan specimen is analysed. The numerical evaluations are in a good agreement with the experimental results.*

INTRODUCTION

Real structural components contain randomly oriented flaws either from metallurgical defects or from the damage induced during service. Presence of those flaws within structure could cause appearance of sort of combination of the three loading modes. Based on fracture mechanics solutions for crack tip fields, those modes are known as Mode I, Mode II and Mode III. In the case with two of three different type of loadings subjected structural component it is considered as dealing with mixed mode loading. Some of those flaws could even lead to catastrophic failure of structures subjected to fatigue loading. The most important aspect in engineering practice is to develop an accurate procedure for life estimation of structural components.

During fatigue crack growth propagation, mixed mode loading as external loading causes chances of crack path. Consequently, a few problems have to be solved in the case of mixed mode loading: the direction of the crack propagation, to find the full length of crack path and calculate a number of cycles up to failure.

A series of criteria for the case of mixed mode loading have been proposed in the past decades to simulate crack path. Namely, for analysis of crack growth direction, different criteria can be used such as MTS – criterion or a maximum tangential stress criterion [1], G – criterion based on a maximum strain energy release rate [2], S - criterion or the minimum elastic strain energy density criterion [3], J – criterion [4], T – criterion based on the maximum dilatational strain energy density [5], M – criterion or the maximum stress triaxiality criterion [6], etc.

On the other hand, for mixed mode fatigue crack problems, special attention should be given to mathematical models for the simulation of fatigue life to failure in order to enable safety design and maintenance of structural components. Estimation of fatigue life to failure requires determination of the fatigue crack growth rates. For mixed mode problems relations for the fatigue crack growth rate and with it number of cycles up to failure can be expressed as a function of equivalent stress/strain intensity factor [7,8] or the J-integral [9].

In engineering application external loadings are with different levels. Those levels of loading could have significant impact on fatigue life to failure of structural component and it is necessary to analyse their affect in the case of mixed mode loading.

The objectives of this paper are simulation of crack path, experimental investigation of crack path and estimation of a realistic life of two different plates subjected mixed mode loading. Additionally, the paper analyses the effect of stress ratio on the fatigue life up to failure, the final crack growth length and the equivalent stress intensity factor of an Arcan specimen.

CRACK GROWTH LIFE ESTIMATION

In structural components, cracks can exist that may weaken the structure and lead to a reduction in its service life. Consequently, the most important aspect is to predict how fast the cracks grow and how strong cracked structural component is. In order to answer to those questions it is necessary to provide adequate models/procedures for fatigue crack growth estimation. In this section is presented procedure for specimens or structural elements subjected mixed mode loading. Formulated procedure is then used for analysis of affects of stress ratio and loading angle on service life of considered specimen.

The type of loading such as mixed mode is complex special for cracked problems so it is necessary to briefly define a stress field around crack tip. Erdogan and Sih [1] analysed mixed mode problem based on the fact that the crack starts to grow from the tip in the direction along which the tangential stress ($\sigma_{\theta\theta}$) is maximum. They proposed that the elastic crack tip stress field in terms of the crack tip polar coordinate system (r,θ), by keeping only the singular terms, can be expressed as:

$$\sigma_{rr} = \frac{1}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[K_I\left(1+\sin^2\left(\frac{\theta}{2}\right)\right)+K_{II}\left(\frac{3}{2}\sin\theta-2\tan\left(\frac{\theta}{2}\right)\right)\right], \tag{1}$$

$$\sigma_{\theta\theta} = \frac{1}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[K_I\cos^2\left(\frac{\theta}{2}\right)+\frac{3}{2}K_{II}\sin\theta\right], \tag{2}$$

$$\tau_{r\theta} = \frac{1}{2\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[K_I\sin\theta+K_{II}\left(3\cos\theta-1\right)\right], \tag{3}$$

$$\sigma_{zz} = \begin{cases} 0 & (plane\,stress), \\ \nu(\sigma_{\theta\theta}+\sigma_{rr}) & (plane\,strain), \end{cases} \tag{4}$$

where K_I and K_{II} are stress intensity factors for Mode I and Mode II. Due to the fact that all stresses are functions of stress intensity factors (K_I, K_{II}) and the crack kinking angle θ it shows that stresses can be expressed as functions of the loading configurations, magnitude of the loads and geometry of the specimen or structural element.

In order to calculate the critical crack length and number of cycles up to failure accurately, it is necessary to determine accurate the crack growth rate for the considered specimen or structural element.

For the fatigue crack growth rate under mixed mode loading Tanaka [7] suggested relation using a Paris type equation as a function of an equivalent stress intensity factor. This relation can be modified if a coefficient C is substitute with a new coefficient C^*. By introducing the new coefficient in calculation, mixed mode loading is even more adequately included in analysis. Namely, from experimental investigations it was realized that the new coefficient C^* could be expressed in terms of stress intensity factors for Mode I and Mode II. Thus, the relation for crack growth rate can be written as:

$$\frac{da}{dN} = C^* \left(\Delta K_{eq}\right)^m , \tag{5}$$

where K_{eq} is the equivalent stress intensity factor i.e.

$$\Delta K_{eq} = \left(\Delta K_I^4 + \Delta K_{II}^4\right)^{0.25} . \tag{6}$$

Due to the fact that external loading is mixed mode, the new coefficient C^* has to be expressed as function of elastic mixity parameter M^e,

$$C^* = f\left(M^e\right) = C\left(1 + \beta\left(M^e - 1\right)^2\right), \tag{7}$$

where β is a constant obtained from experiments (β = 3, for all CTS specimens).

For mixed mode loading, Shih [10] found that the elastic mixity parameter can be expressed as:

$$M^e = \frac{2}{\pi} \arctan\left|\frac{\Delta K_I}{\Delta K_{II}}\right| . \tag{8}$$

Since that the relations for C^* and M^e are defined as functions of two stress intensity factors (Mode I and Mode II), with substitution Eqs.7 and 8 in Eq. 5 and after integration it is possible to obtain the number of cycles up to failure:

$$N = \int_{a_0}^{a_f} \frac{da}{C^* \left(\Delta K_{eq}\right)^m} , \tag{9}$$

where N is number of cycles up to failure required for the initial crack a_0 which grow to final length a_f.

As it can be seen, relations for C^* and K_{eq} are complicated functions, so it is necessary to use numerical integration for calculation of number of cycles up to failure.

STRESS INTENSITY FACTOR

In practical problems of the fracture mechanics analysis, geometry of the considered specimen or structural element and external loading are included in crack growth analysis by the stress intensity factor. Depending on complexity of geometry of any type of loading for calculation of the stress intensity factors both analytical and/or numerical approaches [11, 12] could be used.

Only correctly determined stress intensity factors enable the exact calculation of critical length and fatigue life up to failure of specimen or structural element. In this paper, an Arcan specimen and a plate with crack emanating from the edge of a hole subjected mixed mode loading were analysed.

The expressions for stress intensity factors under mixed mode loadings (Mode I and Mode II) for geometry shown in Fig.1 are:

$$K_I = \frac{P}{bt}\cos\varphi \, f_I\left(\frac{a}{b}\right)\sqrt{\pi a}\,, \tag{10}$$

$$K_{II} = \frac{P}{bt}\sin\varphi \, f_{II}\left(\frac{a}{b}\right)\sqrt{\pi a}\,, \tag{11}$$

where: a is the crack length, t is the thickness of the specimen, P is external load, φ is the loading angle and corrective functions for the Arcan specimen [13] are:

$$f_I\left(\frac{a}{b}\right) = 1.12 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.27\left(\frac{a}{b}\right)^3 + 30.39\left(\frac{a}{b}\right)^4, \tag{12}$$

$$f_{II}\left(\frac{a}{b}\right) = \frac{1.122 - 0.561\left(\frac{a}{b}\right) + 0.085\left(\frac{a}{b}\right)^2 + 0.180\left(\frac{a}{b}\right)^3}{\left(1 - \left(\frac{a}{b}\right)\right)^{1/2}}. \tag{13}$$

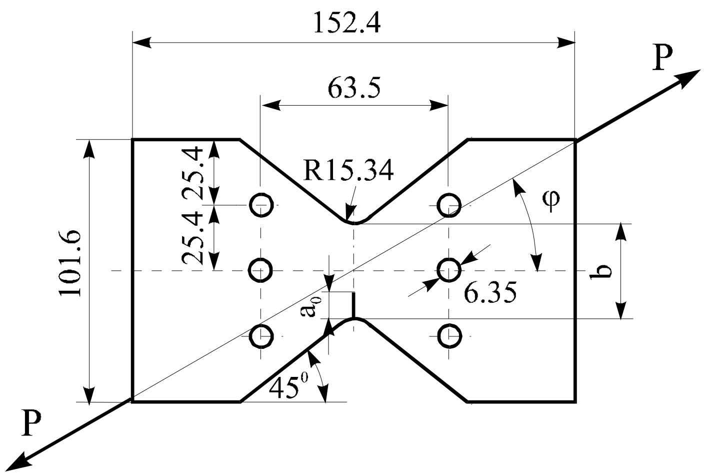

Figure 1. Geometry of the Arcan specimen [14].

Additionally, when analysing complex problems (it is often complex geometry of structural element and/or complex type of loading) it is necessary to use numerical method (FEM) for crack growth analysis. One of these complex problems is the plate with crack emanating from the edge of the hole subjected mixed mode loading and it is analysed in the following section.

NUMERICAL RESULTS

In this section a few numerical examples are considered. In those examples are presented numerical simulation of crack path and fatigue life calculation of two specimens. Moreover, the effect of different parameters on mixed mode fatigue crack growth was analysed.

Example 1. Crack Path under Mixed Mode Loading using Finite Element Method

In this example, crack path simulation of a plate was carried out. Additionally, for the same plate crack path was obtained experimentally. FEM was used for crack growth simulation. The considered plate is with crack emanating from the edge of a hole (Fig.2). The angle between the initial crack and direction of external loading is 45^0. The plate is made of 2024 T3 Al Alloy. External loading is with constant amplitude (P=5200N and stress ratio R = 0.1). Material characteristics are: E = 73174 MPa, ν=0.33, K_{IC} = 120 $MPam^{1/2}$ and geometry characteristics: a_0=0.0125 m, r = 0.01 m, 2w=0.050 m, L = 0.2 m, t = 0.005 m.

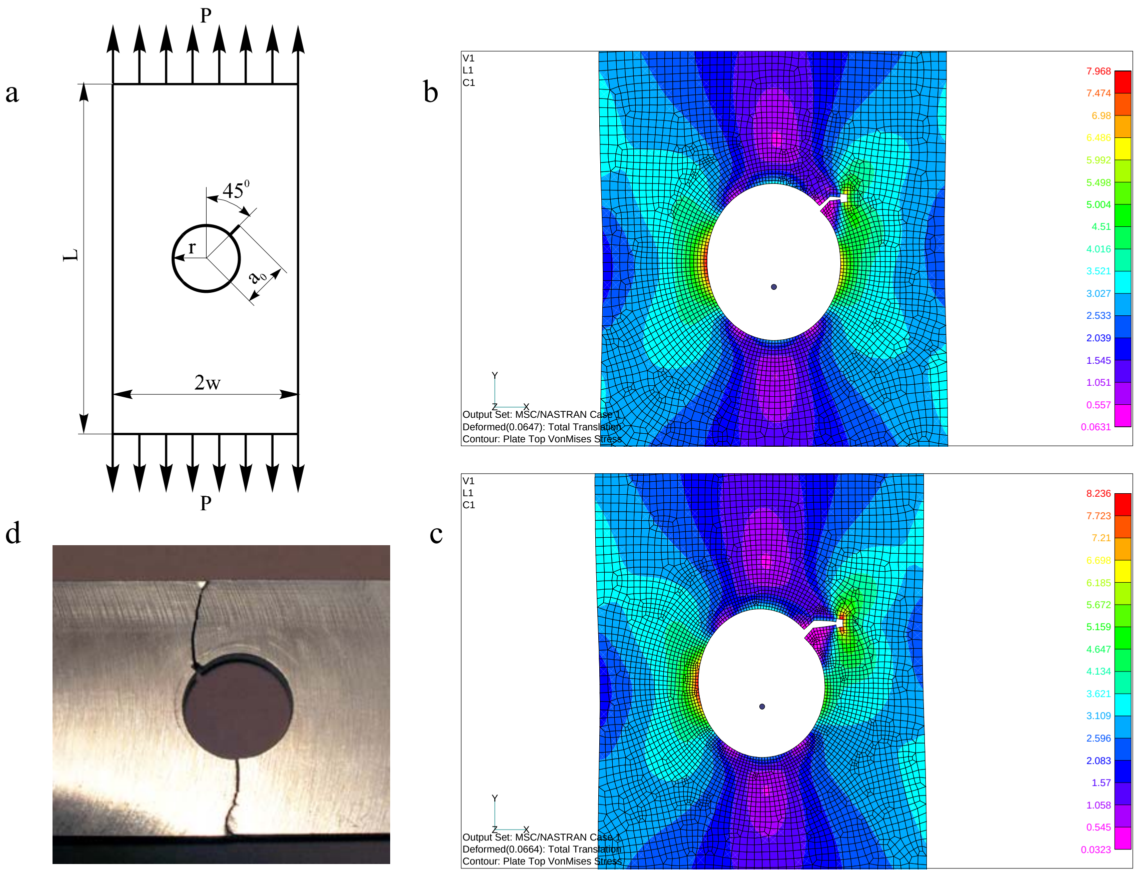

Figure 2. The plate with crack emanating from the edge of a hole
(a – geometry, b - numerical simulation of crack path – the second step,
c - numerical simulation of crack path – the third step,
d – experiment (failure of the plate)).

Geometry of considered plate presents complex problem due to crack position comparing to the direction of external loading, so numerical method was used for crack growth simulation. Based on known characteristics of material, geometry and loading the stress intensity factors (Mode I and Mode II) and the crack kinking angles θ have to be calculated for each increment of the crack length, step by step. A representation of the finite element analysis of considered plate for two different steps is shown in Fig.2 (b-c). As observed from Fig.2 (b-c), the calculated crack path using numerical method (FEM) is in a good agreement with the experimental observation (Fig.2.d).

Example 2.a Equivalent Stress Intensity Factor Calculation

In this example, the equivalent stress intensity factor calculation for the Arcan specimen (Fig.1) was analysed. External loading was mixed mode loading with different loading angle φ. Geometry characteristics are: a_0 = 0.00635 m, b = 0.0381 m, t = 0.0023 m. The considered specimen is made of 2024 T3 Al Alloy and K_{IC} = 120 MPa $m^{1/2}$.

Based on known geometry characteristics it is possible to define the stress intensity factors K_I and K_{II} (Eqs.10 and 11 with Eqs.12 and 13). With determinated K_I and K_{II}, the equivalent stress intensity factor K_{eq} can be calculated using Eq.6.

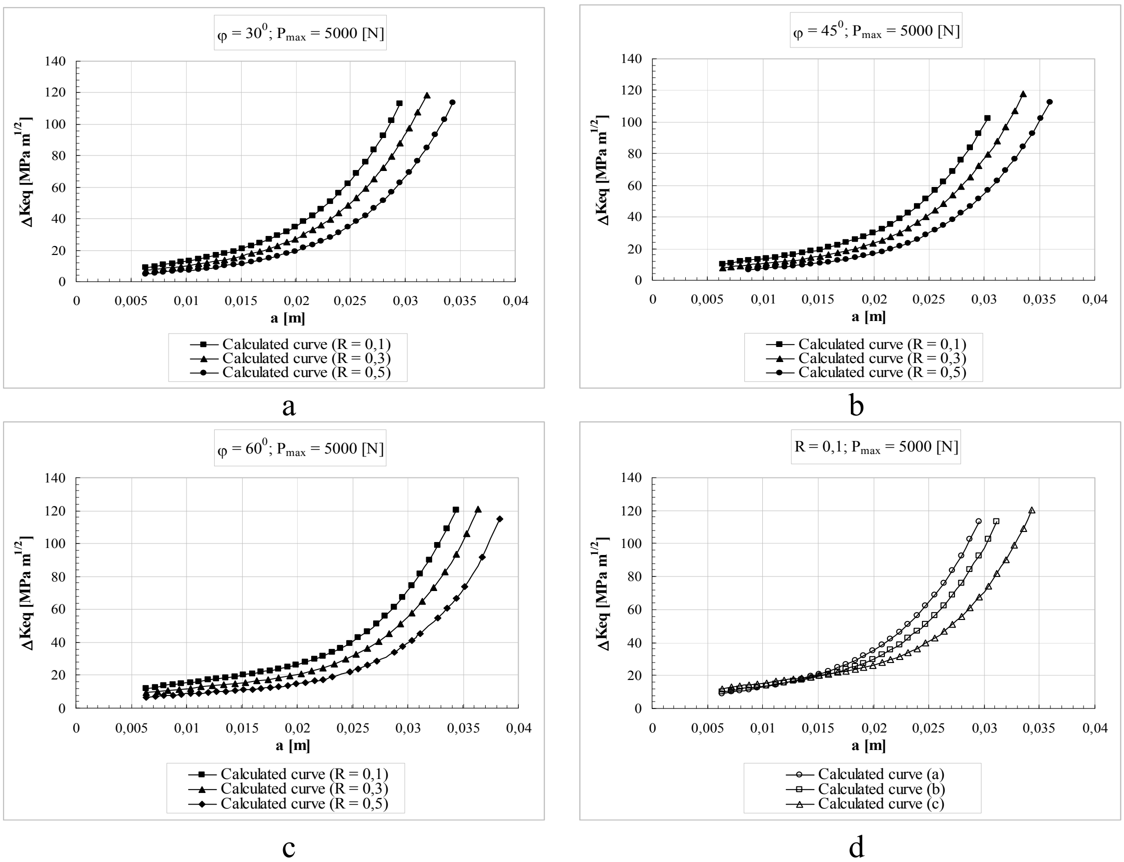

Figure 3. Equivalent stress intensity factor versus crack length
for different stress ratio and loading angles
(where are calculated curves: (a) $\varphi = 30^0$; (b) $\varphi = 45^0$; (c) $\varphi = 60^0$).

All calculated curves of the equivalent stress intensity factor for different stress ratios R and loading angles φ are shown in Fig.3 (a-d).

Example 2.b Crack Growth Rate Calculation

This example considered crack growth rate estimation under mixed mode loading. As in previous example, the Arcan specimen (Fig.1), made of 2024 T3 Al Alloy, was used. Material characteristics are: $C = 5\ 10^{-11}$, $m = 4.07$, $K_{IC} = 120$ MPam$^{1/2}$ and geometry characteristics: $a_0 = 0.00635$ m, $b = 0.0381$ m, $t = 0.0023$ m.

Based on known characteristics of material, geometry and loading, calculated curve of the crack growth rate (Eq.5 with Eqs.6 and 7) for stress ratio R = 0.1 is shown in Fig.4.

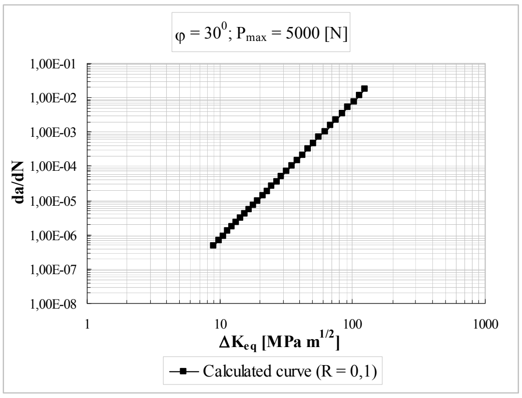

Figure 4. Crack growth rate versus equivalent stress intensity factor.

Example 2.c Crack Growth Life Estimation

In this example fatigue life calculation up to failure was considered. Geometry of the Arcan specimen, material and the type of loading used here are the same as in example 2.b. Using needed material and geometry parameters enabled determination of the fatigue life up to failure. Actually, for the crack paths (experiment [14]) shown in Fig.5, using Eq.9 with Eqs.6, 7 and 8, it is possible to determine number of cycles up to failure.

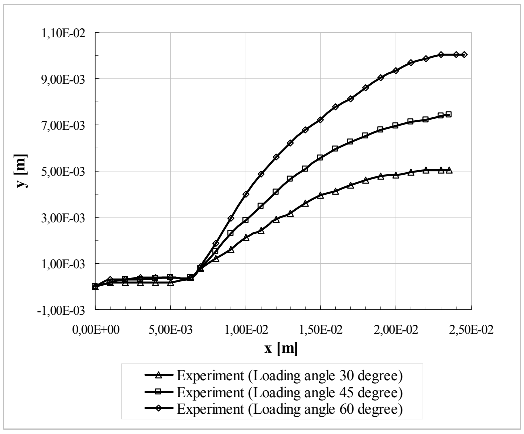

Figure 5. Crack paths for different loading angles [14].

For calculation of number of cycles up to failure it is important to determine final crack growth length a_f for different stress ratios and loading angles. All final crack growth lengths, for different loading angles φ, are obtained so that they meets condition: K_{IC}=120 MPa $m^{1/2}$. Calculated number of cycles, as a function of crack length for different stress ratios R and loading angles φ are presented in Fig. 6 (a-d).

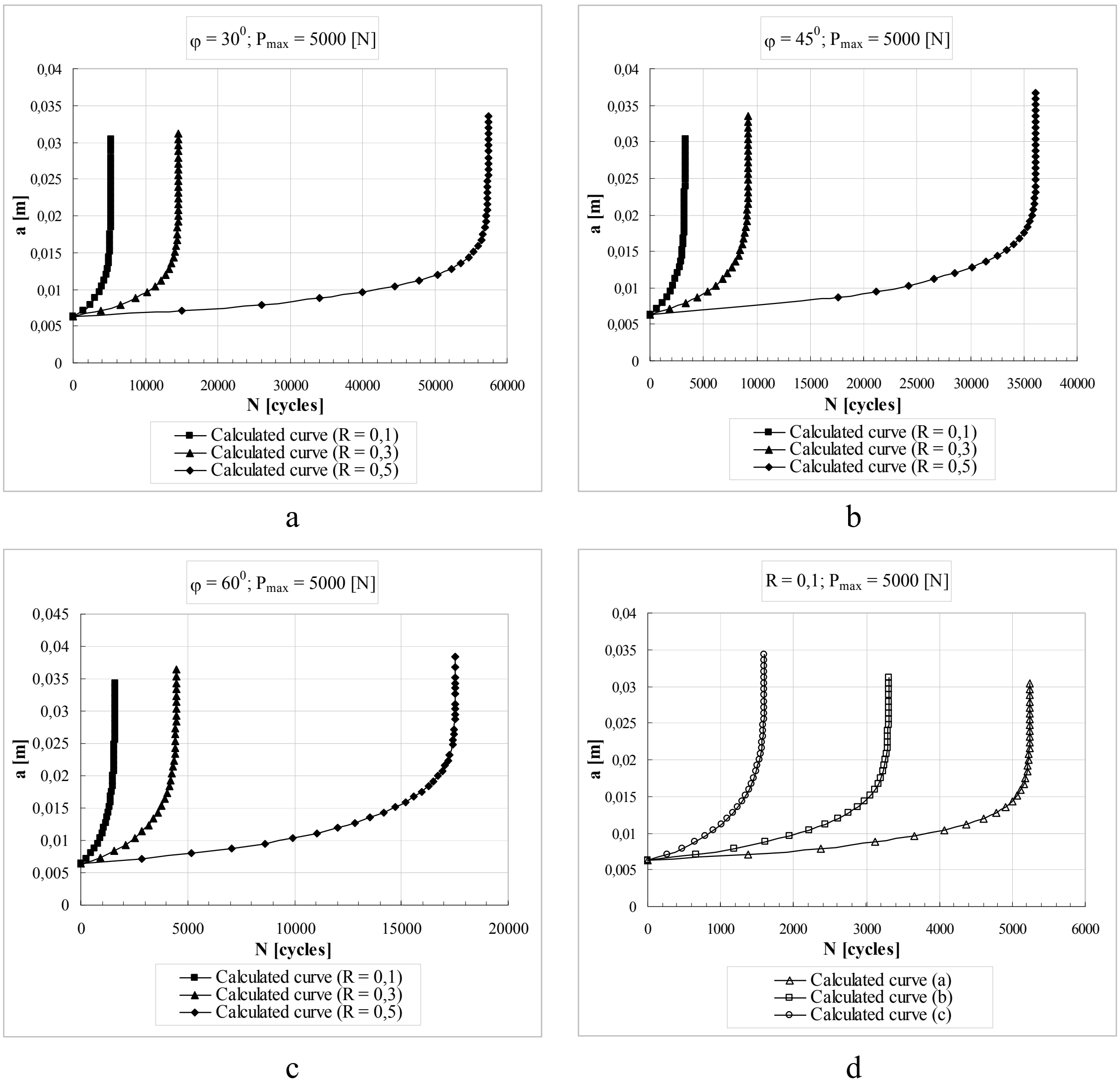

Figure 6. Number of cycles up to failure versus crack length for different stress ratio and loading angles (where are calculated curves: (a) $\varphi = 30^0$; (b) $\varphi = 45^0$; (c) $\varphi = 60^0$).

It is indicated in Fig.6 that the calculated values of final crack length are in a good agreement with the experimentally obtained crack paths (Fig.5) and estimated number of cycles up to failure are listed in Table 1.

Table 1. Calculated number of cycles up to failure
for different stress ratio R and loading angles φ (P_{max} = 5000 [N])

Loading angle φ [^{0}]	N [cycles]		
	R = 0.1	R = 0.3	R = 0.5
30	5245	14587	57376
45	3305	9193	36159
60	1601	4453	17517

CONCLUSIONS

The fatigue crack growth of two different plates under mixed mode loading conditions are studied using analytical and numerical approaches as well as experimental investigation. Namely, in this paper are considered the crack path simulation and fatigue life calculation. As experimental investigation the crack path of the plate with crack emanating from the edge of a hole is performed. Furthermore, a numerical model for fatigue life estimation is presented in order to analyse the influence of stress/load ratio and loading angle on the equivalent stress intensity factor and number of cycles up to failure. All comparisons between numerical and experimental rasults are in a good agreement. Finally, presented procedures can be used for the crack path simulation and life estimation of different structural components in engineering practice.

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