Fatigue Assessment of Smooth Metallic Specimens Using a Simplified Multiaxial Criterion Based on the Critical Plane Approach

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ABSTRACT. *In the present paper, a high-cycle critical plane-based multiaxial fatigue criterion is employed to evaluate the multiaxial fatigue strength of smooth metallic specimens subjected to biaxial cyclic loading. According to such a criterion, the critical plane orientation is correlated to the weighted mean directions of the principal stresses, and the multiaxial fatigue strength is assessed through an equivalent stress expressed by a quadratic combination of the shear stress amplitude and the amplitude of an equivalent normal stress acting on the critical plane. The criterion is a modified version of the original Carpinteri-Spagnoli (C-S) criterion. The proposed modifications are related to: the weighting procedure of the principal stress axes; the definition of the equivalent normal stress by taking into account the mean normal stress effect; the expression of the quadratic combination of stresses. Several high-cyle fatigue strength results for smooth metallic specimens, subjected to in-phase or out-of-phase biaxial loading with different loading ratios and mean stress values, are analysed by employing the C-S criterion, its modified version, and other criteria available in the literature.*

INTRODUCTION

High-Cycle Fatigue (HCF) failures of structural metallic components are characterized by more than around 10^5 cycles of repeated loading. Several fatigue experimental campaigns have shown that loadings, which cause HCF failure, generally produce only elastic strains into the material. Moreover, most of load states arising during service in structural components are multiaxial. Therefore, many high-cycle multiaxial fatigue criteria proposed in the literature are stress-based, and aim at reducing a given multiaxial stress state to an equivalent uniaxial stress, which is then compared to the uniaxial fatigue strength at a specified number of loading cycles [1-15].

Some of the above criteria are based on the so-called critical plane approach according to which the fatigue failure assessment is performed on a plane where the amplitude or the value of some stress components or a combination of them attains its maximum [3-5, 8, 10]. Alternatively, the position of the critical plane may be correlated with the principal stress directions and, since such directions are generally time-varying under fatigue loading, averaged principal stress directions should be considered by deducing them, for example, through appropriate weight functions [12-15].

In the present paper, a multiaxial high-cycle fatigue criterion based on the critical plane approach [15] is applied to the case of in-phase and out-of-phase sinusoidal biaxial normal and shear stress states. Such a criterion is a modified version of the original Carpinteri-Spagnoli (C-S) criterion [13], and assumes a fatigue damage parameter given by a combination of the shear stress amplitude and an equivalent normal stress amplitude acting on the critical plane.

Relevant experimental tests on metals available in the literature [16-19] are analysed by means of the present criterion and other common stress-based criteria [1-13]. For such tests, the ratio $\tau_{af,-1}/\sigma_{af,-1}$ between the fatigue strength at a number of loading cycles $\overline{N}$ ($>10^5$ cycles) under fully reversed torsion and that under fully reversed bending (or tension) falls within the following range: $1/\sqrt{3} \le \tau_{af,-1}/\sigma_{af,-1} \le 1$.

THE MODIFIED C-S CRITERION

The main steps of the modified C-S criterion [15] are described hereafter.

Averaged directions of the principal stress axes

The averaged directions of the principal stress axes $\hat{1}, \hat{2}$ and $\hat{3}$ are obtained from the averaged values $\hat{\phi}, \hat{\theta}, \hat{\psi}$ of the principal Euler angles. Such values are computed by independently averaging the instantaneous values $\phi(t), \theta(t), \psi(t)$ as follows:

$$\hat{\phi} = \int_0^T \phi(t) W(t)\, dt \qquad \hat{\theta} = \int_0^T \theta(t) W(t)\, dt \qquad \hat{\psi} = \int_0^T \psi(t) W(t)\, dt \tag{1}$$

with T = period of the loading cycle. By assuming the following weight function $W(t)$:

$$W(t) = H[\sigma_1(t) - \sigma_{1,\max}] \quad \text{with} \quad \begin{array}{l} H[x] = 1 \text{ for } x \ge 0 \\ H[x] = 0 \text{ for } x < 0 \end{array} \tag{2}$$

where $\sigma_{1,\max}$ is the maximum value (in the loading cycle) of the maximum principal stress σ_1, no averaging procedure is required since the averaged principal stress axes coincide with the instantaneous principal directions corresponding to the time instant at which the maximum principal stress σ_1 achieves its maximum value during the loading cycle, and this makes the implementation of the criterion rather simple.

Orientation of the critical plane

The orientation of the critical plane is assumed to be correlated with the averaged directions of the principal stress axes. The empirical expression of the off angle δ between the normal versor $\mathbf{w}$ to the critical plane (where $\mathbf{w}$ belongs to the averaged principal plane $\hat{1}\hat{3}$) and the averaged direction $\hat{1}$ of the maximum principal stress σ_1 is given by:

$$\delta = (3\pi/8)\left[1 - \left(\tau_{af,-1}/\sigma_{af,-1}\right)^2\right] \tag{3}$$

Mean value and amplitude of the normal stress and shear stress

The stress vector $\mathbf{S_w}$, the normal stress vector $\mathbf{N}$ and the shear stress vector $\mathbf{C}$ acting on the critical plane are given by:

$$\mathbf{S_w} = \boldsymbol{\sigma} \cdot \mathbf{w} \qquad \mathbf{N} = (\mathbf{w} \cdot \mathbf{S_w})\ \mathbf{w} \qquad \mathbf{C} = \mathbf{S_w} - \mathbf{N} \tag{4}$$

with $\boldsymbol{\sigma}$ = stress tensor.

For multiaxial constant amplitude cyclic loading, the direction of the normal stress vector $\mathbf{N}(t)$ is fixed with respect to time and, consequently, the mean value N_m and the amplitude N_a of the vector modulus $N(t)$ can readily be calculated. As far as the shear stress vector $\mathbf{C}(t)$ is concerned, the definitions of the mean value C_m and amplitude C_a of the vector modulus $C(t)$ are not unique due to the time-varying direction of $\mathbf{C}(t)$. The procedure proposed by Papadopoulos [20] to determine C_m and C_a is here adopted.

Fatigue strength estimation

As is well-known, the effect of a tensile mean normal stress superimposed upon an alternating normal stress strongly reduces the fatigue resistance of metals, while a mean shear stress superimposed upon an alternating shear stress does not affect the fatigue life (e.g. see Ref. [20]). Therefore, the following multiaxial fatigue strength condition is here adopted:

$$\left(N_{a,eq}/\sigma_{af,-1}\right)^2 + \left(C_a/\tau_{af,-1}\right)^2 = 1 \tag{5}$$

where:

$$N_{a,eq} = N_a + \sigma_{af,-1}\ \left(N_m/\sigma_u\right) \tag{6}$$

with σ_u = ultimate tensile strength. Equation (6) is based on the well-known linear interaction between normal stress amplitude and normal stress mean value (diagram of Goodman).

In order to transform the actual periodic multiaxial stress state into an equivalent uniaxial normal stress state (with amplitude $\sigma_{a,eq}$), Equation (5) can be rewritten as follows:

$$\sigma_{a,eq} = \sqrt{N_{a,eq}^2 + \left(\sigma_{af,-1}/\tau_{af,-1}\right)^2 C_a^2} = \sigma_{af,-1} \tag{7}$$

EXPERIMENTAL VALIDATION

In the present section, the above fatigue criterion is applied to some experimental HCF results, related to smooth cylindrical specimens subjected to in-phase or out-of-phase loading: bending and torsion [16, 18, 19], axial loading with non-zero mean stress and internal pressure [17], bending with non-zero mean stress and torsion [18, 19].

The mechanical and fatigue properties of the materials examined are summarised in Tables 1 and 2, respectively, where m indicates the slope of the S-N curve for fully reversed bending (the stars indicate the values assumed by the present authors).

Firstly, the critical plane criterion previously described is applied to the experimental tests reported in Refs [16-19]. Different values of both the amplitude ratios ($r = \sigma_{x,a} / \tau_{xy,a}$; $r^* = \sigma_{x,a} / \sigma_{y,a}$) and the phase angles ($\alpha =$ phase angle between the longitudinal normal stress σ_x and the shear stress τ_{xy}; $\beta =$ phase angle between the longitudinal normal stress σ_x and the hoop normal stress σ_y) are analysed.

By plotting the shear stress amplitude C_a against the equivalent normal stress amplitude $N_{a,eq}$ acting on the critical plane (Fig.1), fatigue failure occurs according to the modified C-S criterion if the points with coordinates ($N_{a,eq}, C_a$) lie out of the ellipse with semi-axes equal to $\sigma_{af,-1}$ and $\tau_{af,-1}$ (see Eq. (5)). For the different materials and loading conditions analysed, Figure 1 shows the correlation between this theoretical ellipse and the test results related to the failure state.

Table 1. Mechanical properties of the examined materials

Authors	Ref.	Material	ν	E (MPa)	σ_u (MPa)
Nishihara et al	[16]	982 FA	0.30*	210000*	681.30
		5695	0.30*	210000*	374.25
		IC_2	0.24*	105000*	180.99
		D-30	0.33*	70000*	433.11
Rotvel	[17]	C=0.35%	0.29	209934	559.17
Froustey et al.	[18,19]	30NCD16	0.30*	195000	1160.00
		XC18	0.30*	210000	520.00

Table 2. Fatigue properties of the examined materials

Authors	Ref.	Material	$-1/m$	$\sigma_{af,-1}$ (MPa)	$\tau_{af,-1}$ (MPa)	$\overline{N}$
Nishihara et al	[16]	982 FA	18.33	313.92	196.20	10^7
		5695	15.17	235.44	137.34	10^7
		IC_2	9.07	96.14	91.23	10^7
		D-30	9.57	155.98	100.06	10^7
Rotvel	[17]	C=0.35%	12.19	215.82	138.37	$2.5x10^6$
Froustey et al.	[18,19]	30NCD16	10.00*	710.00	450.00	10^6
		XC18	10.00*	310.00	179.00	10^6

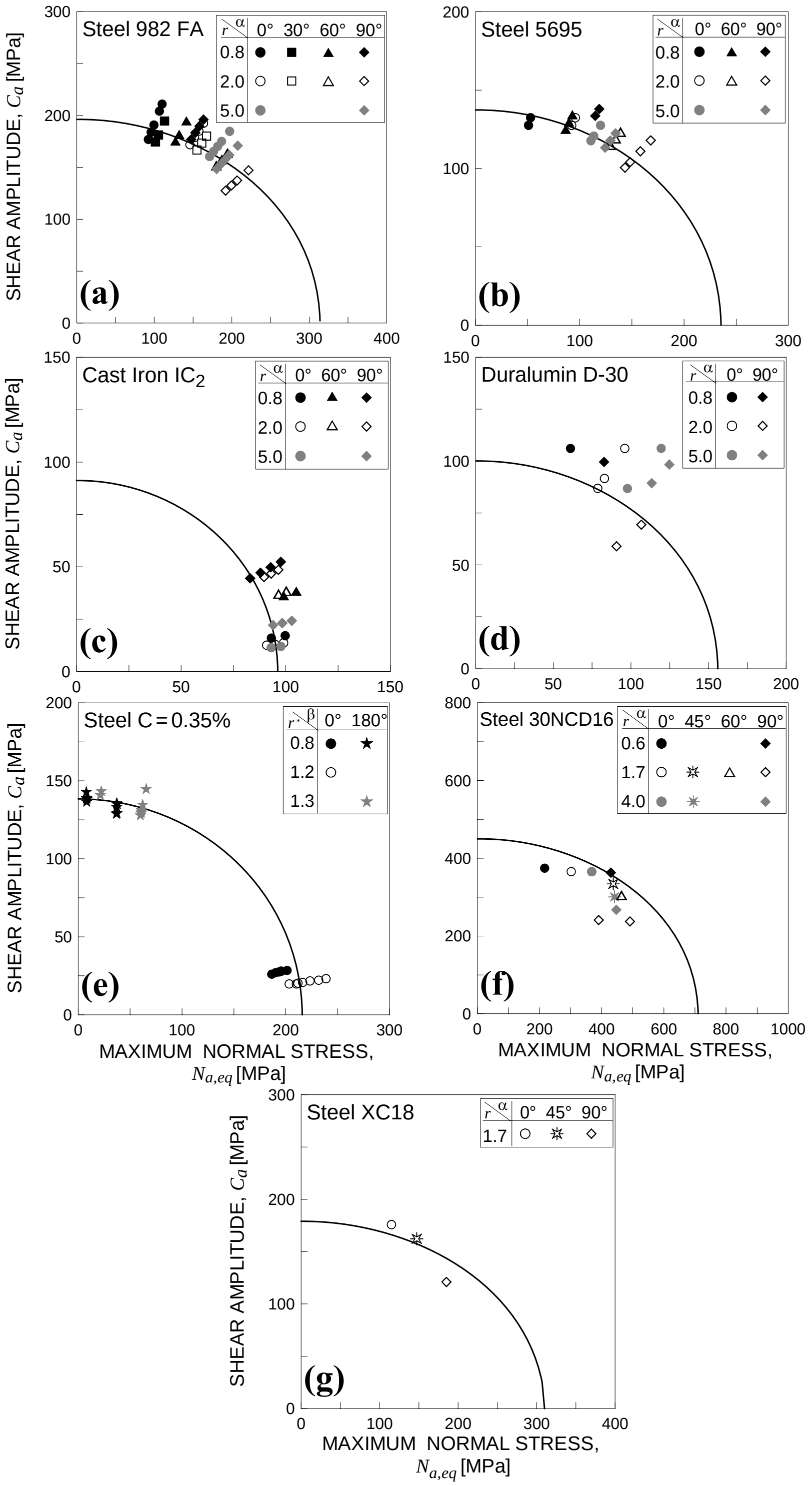

Figure 1. Shear stress amplitude against equivalent normal stress amplitude acting on the critical plane: theoretical evaluations and experimental results for the examined materials

Then, the above experimental data are analysed through the original C-S criterion [13] and several criteria proposed in the literature [1-11] (for such criteria, a freeware software [21] is employed). A general form for each of such criteria is given by the following expression:

$$\sigma_{a,eq} \;=\; A\,f(C) + B\,g(N) \;=\; \sigma_{af,-1} \tag{7}$$

where A, B are parameters depending on the mechanical and fatigue material properties, and f, g are functions of the shear stress C and normal stress N, respectively.

The quality of the estimations can be evaluated through an error index, $I(\%)$, defined as the relative difference between the two sides of the equality in Eq. (7):

$$I\% = \frac{\sigma_{a,eq} - \sigma_{af,-1}}{\sigma_{af,-1}}\,100\% \tag{8}$$

Positive values of I for a given criterion indicate that such a criterion makes a conservative estimation with respect to the experimental results.

Figure 2 shows the relative frequency of the error index for the modified C-S criterion applied to the 142 experimental tests. Such a relative frequency represents, for each interval equal to 5%, the number of experimental tests whose error index falls in the interval considered, where such a number is normalised with respect to the total number of tests. In particular, the error index falls in a range $\pm 5\%$ and $\pm 10\%$ for 62% and 80% of the cases examined, respectively.

The average value ($\mu_{I\%}$) and the standard deviation value ($s_{I\%}$) of the error index related to the estimations deduced by applying the present approach and other criteria are shown in Table 3. Due to numerical problems, a few experimental data could not be assessed through the Fogue criterion [6] and the Liu-Zenner criterion [9].

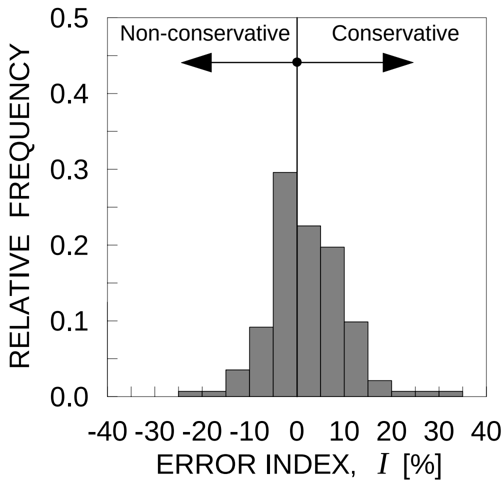

Figure 2. Relative frequency of the error index for the experimental tests [16-19], according to the modified C-S criterion

Table 3. Comparison between the results derived through different multiaxial fatigue criteria

Criterion	*Crossland*	*C-S*	*Modified C-S*	*Dang Van*	*Findley*	*Fogue*	*GAM*	*Liu - Zenner*	*Matake*	*McDiarmid*	*Papadopoulos*	*Robert*	*Sines*
							Steel 982 FA						
$\mu_{I\%}$	0.0	-1.3	2.3	2.6	6.2	3.7	5.5	3.0	7.2	-0.5	6.5	6.2	-5.7
$s_{I\%}$	9.1	7.2	5.9	9.3	7.0	6.8	6.8	6.9	7.1	8.0	6.8	7.0	9.0
							Steel 5695						
$\mu_{I\%}$	-2.2	-1.7	2.6	0.4	4.4	4.1	7.6	4.0	4.8	7.2	7.8	4.4	-3.0
$s_{I\%}$	4.6	4.2	4.8	5.3	4.8	6.2	8.3	6.2	5.4	4.9	8.5	4.8	4.6
							Cast iron IC_2						
$\mu_{I\%}$	6.0	4.0	6.0	8.3	9.7	-	3.0	6.3	25.8	14.8	12.6	9.7	27.4
$s_{I\%}$	9.3	5.3	6.1	11.2	7.3	-	5.1	9.8	23.7	16.0	11.9	7.3	12.9
							Duralumin D30						
$\mu_{I\%}$	10.3	7.0	10.4	12.2	16.1	14.2	15.7	13.5	17.0	2.6	16.9	16.1	1.1
$s_{I\%}$	16.5	16.0	13.9	16.8	12.6	12.2	12.0	12.5	12.1	13.6	11.5	12.6	15.2
							Steel (C=0.35%)						
$\mu_{I\%}$	5.6	3.7	-1.0	16.5	9.9	12.9	5.1	-	9.6	0.9	5.8	11.5	7.0
$s_{I\%}$	5.7	4.0	5.1	11.5	5.4	8.4	6.3	-	5.1	6.2	5.7	6.3	14.0
							Steel 30NCD16						
$\mu_{I\%}$	-13.9	-10.2	-8.7	-6.4	3.4	7.8	-4.2	2.0	3.2	-3.7	-6.5	16.6	1.5
$s_{I\%}$	9.3	9.5	6.5	8.4	8.3	8.8	3.5	8.6	8.9	8.1	3.2	13.6	15.7
							Steel XC18						
$\mu_{I\%}$	-6.6	-4.3	-0.8	-0.8	3.4	2.8	6.8	2.8	3.9	6.3	6.8	3.4	-6.6
$s_{I\%}$	11.2	10.9	7.9	8.0	4.1	1.6	2.6	1.6	3.7	3.8	2.6	4.1	11.2

CONCLUSIONS

In the present paper, a high-cycle critical plane-based multiaxial fatigue criterion is employed to evaluate the multiaxial fatigue strength of smooth metallic specimens subjected to biaxial cyclic loading. Such a criterion is a modified version of the original C-S criterion. The modifications introduced take into account the effect of the mean value of the normal stress acting on the critical plane, and make the implementation of the criterion rather simple.

Several conventional fatigue strength results for smooth metallic specimens, subjected to in-phase or out-of-phase biaxial loading with different values of the loading ratio and mean stress, are analysed by employing the modified C-S criterion (a good correlation between experimental and theoretical results is observed) and other multiaxial fatigue criteria available in the literature.

REFERENCES

1. Sines, G. (1955). In: *NACA-TN-3495*, Washington.
2. Crossland, B. (1956). In: *Proc. Int. Conf. on Fatigue of Metals*, pp. 138-149, Institution of Mechanical Engineers, London.
3. Findley, W. N. (1959) *Trans. ASME Ser B* **81**, 301-306.
4. Dang Van, K. (1973). In: *Sur la résistance a la fatigue des métaux*, pp. 647-722, These de Doctorat es Sciences, Sci. Techniq. l´Armement, 47.
5. Matake, T. (1977) *Bull. JSME* **20**, 257-263.
6. Fogue M. (1987). In: *Critère de fatigue à longue durée de vie des états multiaxiaux de containtes sinusoidales en phase et hors phase.* Thèse de l'Institut National des Sciences Appliquées (INSA) de Lyon.
7. Papadopoulos, I.V. (1987). In: *Fatigue polycyclique des métaux: Une nouvelle approche.* PhD thesis, Paris, École Nationale des Ponts et Chaussées.
8. Robert, J.-L (1992). In: *Contribution à l'étude de la fatigue multiaxiale sous sollicitations périodiques ou aléatoires*. Thèse Laboratoire de Mécanique des solides (INSA) de Lyon.
9. Liu, J. and Zenner, H. (1993) *Mat.-wiss. u. Werstofftech* **24**, 240.
10. McDiarmid, D.I. (1994) *Fatigue Fract. Engng Mater. Struct.* **17**, 1475-1484.
11. Goncalves, C.A., Araùjo, J.A. and Mamiya, E.N. (2005) *Int. J. Fatigue* **27**, 177-187.
12. Carpinteri, A., Macha, E., Brighenti, R. and Spagnoli, A. (1999) *Int. J. Fatigue* **21**, 83-88 and 89-96.
13. Carpinteri, A. and Spagnoli, A. (2001) *Int. J. Fatigue* **23**, 135-145.
14. Carpinteri, A., Spagnoli, A. and Vantadori, S. (2003) *Fatigue Fract. Engng Mater. Struct.* **26**, 515-522.
15. Carpinteri, A., Spagnoli, A. and Vantadori, S. (2009) *Int. J. Fatigue* **31**, 188-196.
16. Nishihara T. and Kawamoto M. (1945). In: *Memories of the College of Engineering*, pp. 85–112, Kyoto Imperial University.
17. Rotvel F. (1970) *Int. J. Mech. Sci.*, **12**, 597–613.
18. Froustey C. and Lasserre S. (1989) *Int. J. Fatigue* **11**, 169-175.
19. Froustey C. and Lasserre S. (1992) *Int. J. Fatigue* **14**, 113-120.
20. Papadopoulos I.V. (1994) *Int. J. Fatigue* **16**, 377–384.
21. Papuga J. *PragTic freeware software*, http://www.pragtic.com/.