Estimating the Notch and Mean Stress Effect in Fatigue through the Degree of Multiaxiality of the Local Elasto-Plastic Stress/Strain Fields

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ABSTRACT. *The present paper is concerned with the estimation of fatigue lifetime of engineering materials by directly taking into account the degree of multiaxiality of the local elasto-plastic stress/strain fields acting on the fatigue process zone. The proposed approach takes as a starting point the assumption that Stage I is the most important stage to be modelled to accurately predict fatigue damage. This is done through the so-called Modified Manson-Coffin Curve Method (MMCCM), which postulates that the critical plane is that material plane experiencing the maximum shear strain amplitude. Subsequently, the MMCCM is used to show that the mean stress effect in fatigue can directly be addressed as a problem of inherent multiaxiality. Finally, the above critical plane approach is reformulated in terms of the Theory of Critical Distances (TCD) in order to correctly account for the detrimental effect of stress/strain gradients arising from stress concentration phenomena.*

INTRODUCTION

Multiaxial Fatigue is classically treated as a problem involving external systems of complex forces and moments resulting in multiaxial stress/strain states that damage engineering materials' critical sites: a typical example is a shaft subjected to combined bending and torsion. It is the writers' opinion that the above situation is just a sub-case of the more complex multiaxial fatigue problem: in fact, multiaxial fatigue involves not only external but also inherent multiaxiality [1]. This firm belief is supported by the well-known fact that also in a sample subjected to uniaxial cyclic loading the presence of a notch results, close to the stress raiser's apex, in a local cyclic stress/strain field which is multiaxial [1, 2, 3]. The important detail which makes such a problem somehow easier to be addressed in practise is not the fact that the applied force is uniaxial, but the fact that the local stress/strain fields always vary proportionally (i.e., in-phase). This should explain the reason why when notched materials are subjected to uniaxial loading, fatigue lifetime can accurately be estimated by considering only the maximum principal stress or strain: since the relevant stresses and strains in the fatigue process zone are in phase, either σ_1 or ε_1 are in any case representative of the entire stress/strain field distribution.

Owing to the scenario as described above, it is logical to believe that, given both the material and the degree of multiaxiality of the local stress/strain fields, the corresponding fatigue damage has to be the same independently of the source from which the multiaxiality itself arises. Accordingly, the present paper addresses the problem of estimating fatigue damage due to inherent multiaxiality through an approach which was devised and validated by considering situations involving external multiaxiality [4].

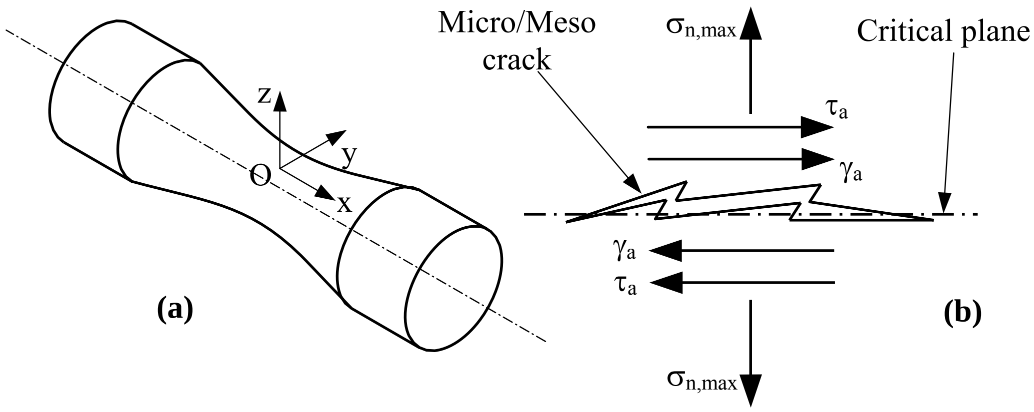

Figure 1. The fatigue damage model the MMCCM is based on.

THE ADOPTED FATIGUE DAMAGE MODEL

In Ref. [5], Kanazawa, Miller and Brown affirmed: *"Stage I cracks form on crystallographic planes, being slip planes within individual grains of metal. These are not necessarily the planes of maximum shear in the macroscopic sense, but rather the slip system most closely aligned to these planes. Clearly, the slip systems which experience the greatest amount of deformation are those which align precisely with the maximum shear direction, and therefore most fatigue cracks initiate in these grains. But slip systems with lesser degrees of shear also initiate cracks at a slower rate"*.

According to the above experimental evidence, the MMCCM postulates that, given the critical point, fatigue damage is maximised on that material plane experiencing the maximum shear strain amplitude, γ_a [1, 4] (see Figure 1). Further, it is hypothesised that, in order to correctly take into account the mean stress effect, according to Socie [6] also the maximum stress, $\sigma_{n,max}=\sigma_{n,m}+\sigma_{n,a}$, normal to the critical plane, weighed through the shear stress, τ_a, relative to the critical plane itself, has to be incorporated into the fatigue damage model, the combined effect of $\sigma_{n,max}$ and τ_a being evaluated through the following stress ratio [4]:

$$\rho = \frac{\sigma_{n,max}}{\tau_a} \tag{1}$$

To conclude, it is worth observing that ratio ρ is seen to be sensitive not only to the presence of superimposed static stresses, but also to the degree of multiaxiality and non-proportionality of the stress/strain state damaging the assumed critical point [1].

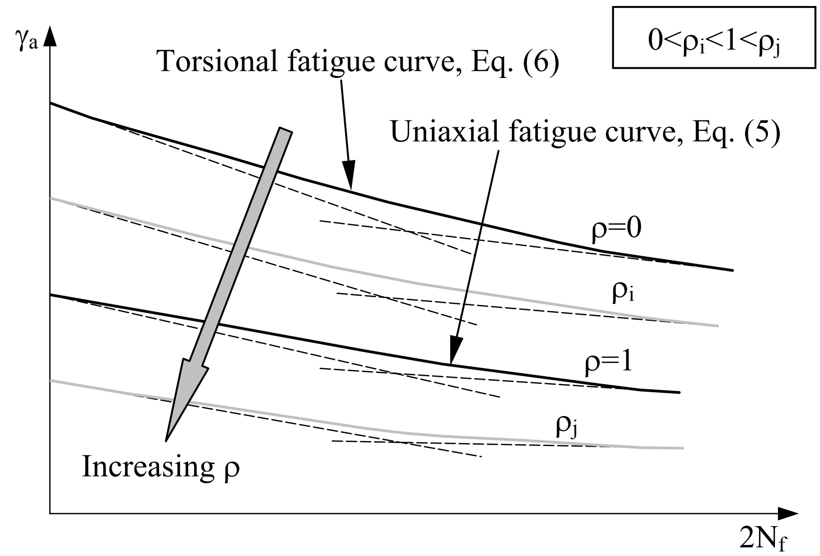

Figure 2. Modified Manson-Coffin Diagram.

THE MODIFIED MANSON-COFFIN CURVE METHOD

The MMCCM postulates that fatigue lifetime can accurately be predicted by using non-conventional bi-parametrical Manson-Coffin curves [4]. In more detail, such an approach takes as its starting point the idea that fatigue damage can be summarised in a log-log diagram plotting the shear strain amplitude, γ_a, relative to that plane experiencing the maximum shear strain amplitude (i.e., the so-called critical plane) against the number of reversals to failure, $2N_f$ (Fig. 2). By using a large number of experimental results generated under proportional and non-proportional multiaxial loading paths [4], it was shown that, as the value of ratio ρ changes, different fatigue curves are generated in the modified Manson-Coffin diagram (Fig. 2). According to the schematic chart reported in Figure 2, the equation describing any Modified Manson-Coffin curve can directly be expressed as:

$$\gamma_a = \frac{\tau'_f(\rho)}{G}\left(2N_f\right)^{b(\rho)} + \gamma'_f(\rho)\cdot\left(2N_f\right)^{c(\rho)} \tag{2}$$

where relationships $\tau'_f(\rho)$, $\gamma'_f(\rho)$, $b(\rho)$ and $c(\rho)$ are defined as follows [4]:

$$\frac{\tau'_f}{G} = \rho \cdot (1+\nu_e)\frac{\sigma'_f}{E} + (1-\rho)\frac{\tau'_f}{G}\ ;\ b(\rho) = \frac{b \cdot b_0}{(b_0 - b)\rho + b} \tag{3}$$

$$\gamma'_f(\rho) = \rho \cdot (1+\nu_p)\varepsilon'_f + (1-\rho)\gamma'_f\ ;\ c(\rho) = \frac{c \cdot c_0}{(c_0 - c)\rho + c} \tag{4}$$

By following a fairly articulated reasoning, such calibration functions were derived by directly using the conventional fully-reversed uniaxial and torsional Manson-Coffin fatigue curves rewritten in terms of maximum shear strain amplitude, that is [1, 4]:

$$\gamma_a = (1+\nu_e)\frac{\sigma'_f}{E}(2N_f)^b + (1+\nu_p)\varepsilon'_f(2N_f)^c \quad \text{(Uniaxial case, } \underline{\rho=1}\text{)} \tag{5}$$

$$\gamma_a = \frac{\tau'_f}{G}(2N_f)^{b_0} + \gamma'_f(2N_f)^{c_0} \quad \text{(Torsional case, } \underline{\rho=0}\text{)} \tag{6}$$

where ν_e and ν_p are Poisson's ratio for elastic and plastic strain, respectively.

Table 1. Static and fatigue properties of the investigated materials.

Material	**Inconel 718**	**SAE 1045**	**En3B**[a]	**Al 6082**[a]	**AISI 1141**[a]
Ref.	[7]	[8]	[10]	[10]	[11]
E *[MPa]*	208500	204000	208500	69090	200000
G *[MPa]*	77800	80300	80000	26000	77000
ε'_f	2.67	0.298	0.2113	30.8	1.0266
σ'_f *[MPa]*	1640	930	691	513.3	1296
b	-0.06	-0.106	-0.0795	-0.0574	-0.08855
c	-0.82	-0.49	-0.4859	-1.4864	-0.6868
γ'_f	18	0.413	0.366	53.4	1.7781
τ'_f *[MPa]*	2146	505	399	296.3	748
b_0	-0.148	-0.097	-0.0795	-0.0574	-0.08855
c_0	-0.922	-0.445	-0.4859	-1.4864	-0.6868
K' *[MPa]*	1530	1258	890.7	401.5	1205
n'	0.07	0.208	0.1635	0.024	0.122

[a]Torsional fatigue contants estimated according to Von Mises.

In Ref. [4] the accuracy and reliability of the MMCCM was checked by using several experimental results taken from the literature and generated under in-phase and out-of-phase biaxial loading, investigating also the effect of superimposed static stresses: the MMCCM was seen to be highly accurate, giving predictions falling mainly within a fatigue life error interval of a factor equal to about ±3 [3].

THE MEAN STRESS EFFECT AS A PROBLEM OF INHERENT MULTIAXIALITY

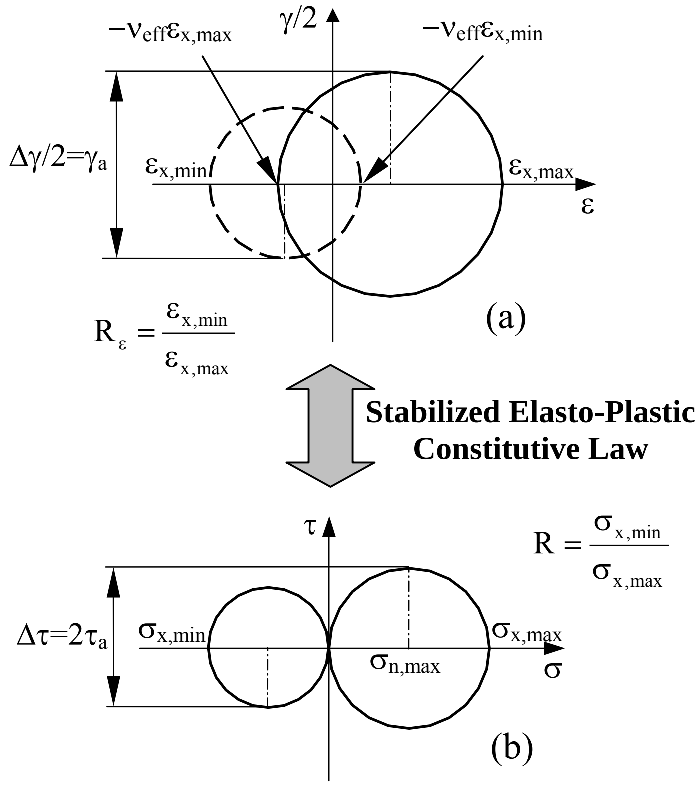

Figure 3. Stress/strain quantities relative to the critical plane under uniaxial fatigue loading.

As said in the previous section, the MMCCM proved to be highly accurate in estimating the detrimental effect of non-zero mean stresses and strains. This remarkable accuracy can be explained by forming the hypothesis that the mean stress effect is nothing but a problem of inherent multiaxiality. In more detail, consider the sample sketched in Figure 1a and assume that it is subjected to a cyclic axial strain characterised by a strain ratio, $R_\varepsilon=\varepsilon_{x,min}/\varepsilon_{x,max}$, larger than -1, such a situation being described in terms of Mohr's circles in Figure 3a. According to the specific stabilised elasto-plastic behaviour of the investigated material, the corresponding Mohr's circles describing the stress state are those sketched in Figure 3b.

The most important implication of the situation depicted in the above figure is that, in general, R_ε is different to the corresponding load ratio, $R=\sigma_{x,min}/\sigma_{x,max}$, due to well-known phenomena like stress/strain hardening and softening, mean stress relaxation and cyclic creep. Further, given the material, even though the strain ratio, R_ε, is kept constant, the corresponding load ratio R is seen to vary as the amplitude of the induced deformation increases. In a similar way, under a constant value of R, the resulting ratio R_ε changes as the amplitude of the applied stress increases. As to the latter scenario, it is straightforward to see that, under uniaxial loading, $\rho=2/(1-R)$ [1]: since ρ is an index suitable for measuring, in an engineering way, the degree of multiaxiality of the stress state at critical locations [1, 4], this seems to strongly support the idea that the mean stress effect can efficiently be treated as a problem of inherent multiaxiality.

To conclude, the experimental, N_f, vs. estimated, $N_{f,e}$, fatigue lifetime diagram reported in Figure 4 shows the accuracy of the MMCCM in taking into account the mean stress effect in plain samples subjected to strain controlled axial loading, the static and fatigue properties of the considered materials being summarised in Table 1.

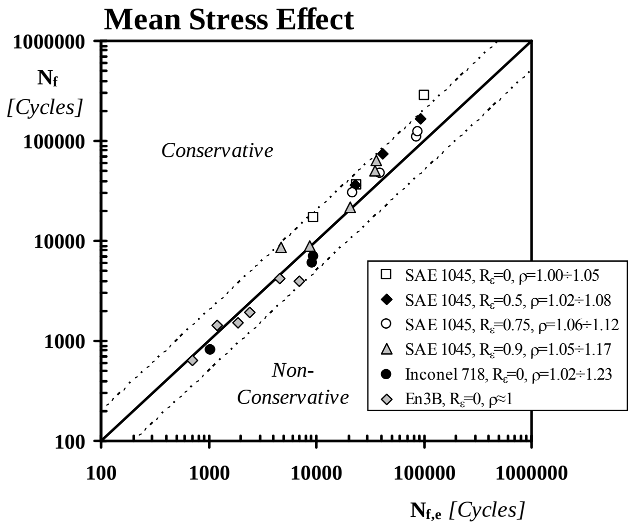

Figure 4. MMCCM's accuracy in estimating the mean stress effect under uniaxial fatigue loading

THE STRUCTURAL VOLUME

The strain based approach postulates that fatigue damage in components containing geometrical features can directly be estimated through the notch root strains. Unfortunately, this *modus operandi* results in estimates whose degree of conservatism is seen to increase as the sharpness of the assessed notch increases [9, 11]. In order to overcome the above problem, recently Susmel and Taylor [10] have proven that fatigue lifetime of notched components can efficiently be estimated by addressing the problem in terms of the Theory of Critical Distances, TCD (applied in the form of the Point Method, PM). In more detail, in such a preliminary investigation fatigue damage was estimated by post-processing the stabilized elasto-plastic stress/strain fields according to the maximum principal stress/ strain criterion, i.e. by neglecting the actual degree of multiaxiality of the stress/ strain state at the critical points. In order to reformulate the above idea according to the fatigue damage model adopted in the present study, consider a notched component subjected to an external system of cyclic forces resulting in a multiaxial stress/strain field acting on that material portion close to the notch tip (Fig. 5). According to Kanazawa,

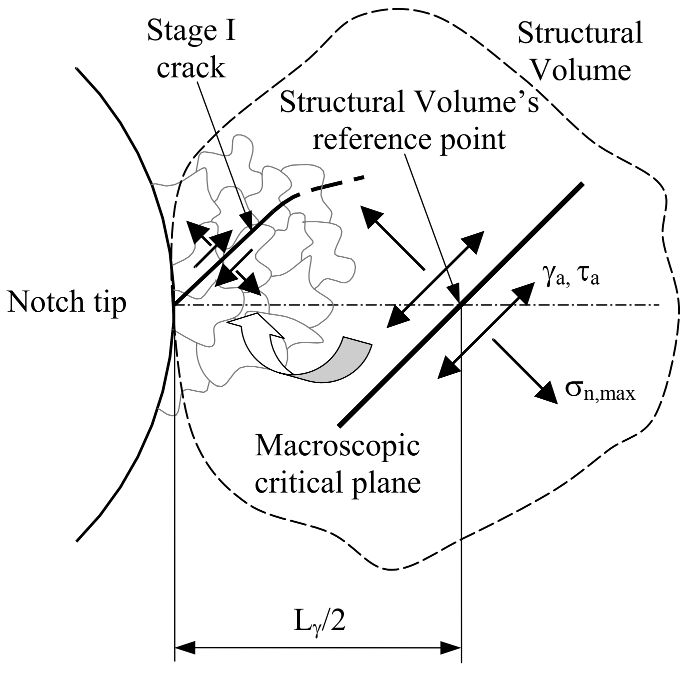

Figure 5. Structural Volume, TCD and MMCCM.

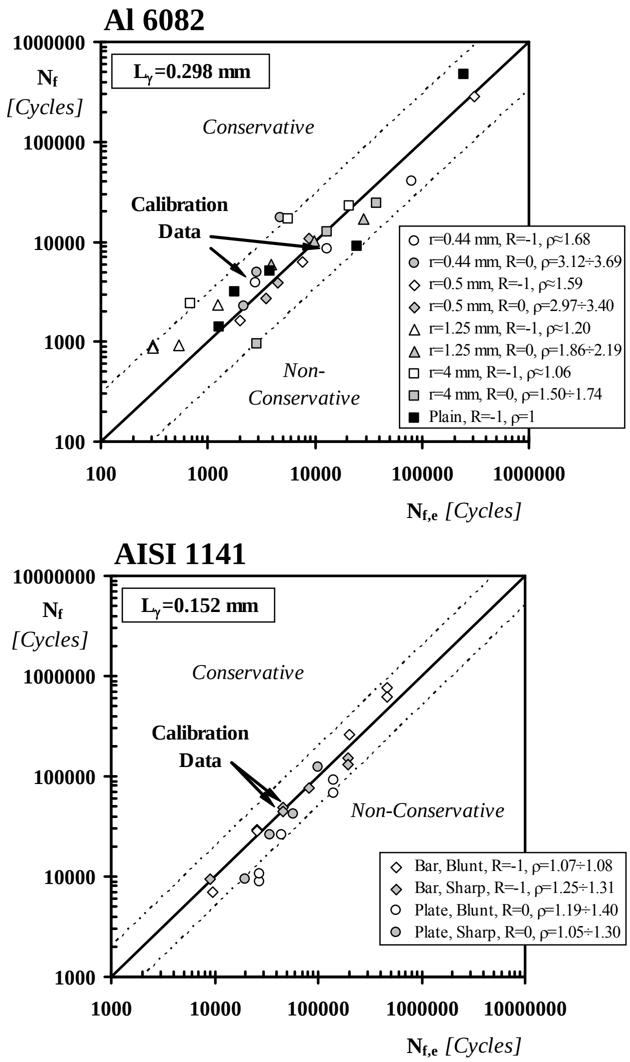

Figure 5. MMCCM's accuracy in estimating the notch effect under uniaxial fatigue loading

Miller and Brown [5], the formation of Stage I cracks depends on the micro-stress/strain components relative to those slip planes most closely aligned to the macroscopic material planes experiencing the maximum shear strain amplitude. Since, in situations of practical interest, the actual orientation of such crystallographic planes is never known, by using the Volume Method argument [2] the hypothesis can be formed that a macroscopic stress/strain state representative of the fatigue damage in those grains situated in the vicinity of the notch apex can be estimated by simply averaging the elasto-platic stress over the fatigue process zone (i.e., over the so-called structural volume). Further, according to the TCD (see Ref. [2] and references reported therein), the reference stress/strain state determined in terms of the Volume Method is the same as that determined at a given distance from the stress concentrator apex (Fig. 5). If the stress/strain state determined at the centre of the structural volume is assumed to give an engineering information representative of the average microscopic stress/strain states damaging those grains located in the vicinity of the crack initiation site, it can be hypothesised that the shear and normal macroscopic stresses and strains relative to the critical plane are somehow related to the corresponding microscopic quantities acting on the most damaged glide planes (Fig. 5). According to the above idea, fatigue damage not only in notched, but also in plain engineering materials can then be estimated by considering the stabilised stress/strain state at a distance from the assumed crack initiation site equal to $L_\gamma/2$, L_γ being treated as a material constant. The validity of this hypothesis will be tested in the next section.

To conclude, it is possible to highlight that, since the idea summarised above does not allow any reasonable hypothesis on the actual shape of the structural volume to be formed, the irregular area sketched in Figure 5 is nothing but as schematic representation of the structural volume idea.

ESTIMATING FATIGUE LIFETIME OF NOTCHED COMPONENTS

The diagrams reported in Figure 5 show the accuracy of the proposed approach when employed to estimate fatigue lifetime of notched samples tested under uniaxial loading (see also Table 1). In particular, the cylindrical V-notched specimens of Al6082 [9] had gross diameter equal to 10 mm, net diameter equal to 6.1–6.2 mm, notch opening angle equal to 60°, and notch root radii equal to 0.44 mm, 0.5 mm, 1.25 mm, and 4.0 mm, respectively. The samples were tested in force control at R ratios of -1 and 0. Two different geometrical configurations were investigated for the specimens of Vanadium-based AISI 1141 MA forging steel [10]: cylindrical bars with circumferential V-notches and flat samples with lateral U-notches (where "sharp" corresponds to a net K_t value of 2.8, whereas "blunt" to a net K_t value of 1.8). The elasto-plastic stress/strain fields were determined from elasto-plastic Finite Element analyses done using commercial software ANSYS. A multilinear isotropic hardening rule was adopted, by running 6 virtual cycles to allow the material in the vicinity of the notch tip to reach a stabilized configuration.

To conclude, it can be observed that the charts of Figure 5 seem to fully support the validity of all the hypothesis formalised in the present paper.

CONCLUSIONS

The high accuracy level shown by the multiaxial elasto-plastic approach proposed in the present paper seems to strongly support the idea that both the mean stress and notch effect in fatigue can efficiently be treated as problems of inherent multiaxiality. More work needs to be done in this area to check the validity of such a *modus operandi* also in the presence of sharp notches subjected to complex systems of external cyclic forces.

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