Cyclic calculations and life estimation in thermomechanical fatigue

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ABSTRACT *The present paper describes a new numerical procedure devoted to the calculation of aeronautic engines and to the fatigue life assessment evaluation. The original aspects consists (1) in a new method used to determine the mechanical steady state of a structure under thermomechanical loading, (2) in a short review of an advanced fatigue model which can reproduce the Woehler curve, including the influence of the mean stress.*

INTRODUCTION

To be able to develop new aeronautical engines, it is necessary to have efficient numerical tools to evaluate the stress and strain fields and associated life time. These tools must not only take into account all relevant physical phenomena (cyclic plasticity, creep, low cycle fatigue, ...) but also be easy to use in an industrial context from material model to the final exploitation results.
The hot section of aeronautical engines have to support creep and fatigue. Due to the complex shapes and the applied loadings, simplified methods generally lead to an underestimation of life. It is proposed an improved method that permits to accurately evaluate the stress-strain fields.
Rotating disks for instance present local stress fields of biaxial or equibiaxial character, associated with high mean stresses. One of the difficulties of the inelastic analysis of the component is then to correctly evaluate the local triaxiality and its redistribution during cycling. These quantities play a very significant role on the fatigue life calculated.

THE ACCELERATED CALCULATION PROCEDURE

General description

The method proposed and tested in the present paper consists to add explicit local cycles in an incremental cyclic calculation in order to accelerate the search of a stabilized material response. The main steps are:

- Storage in a cycle of primary variables (strain and strain increment) obtained at integration points after convergence of Newton-Raphson iterations of a classical incremental calculation;
- At the end of a cycle calculation and for each integration point, local integration of the constitutive equations on the previous local cyclic strain history until stabilization of the material response at this point.

More precisely, the implemented algorithm is detailed in figure 1. Note the following points:

- The only modification to a standard incremental calculation are located in lines 17 to 34 of figure 1;
- Explicit cycles are carried out separately for each integration point (lines 22 to 33). Note that the number of cycles needed to obtain the stabilized response may be different from an integration point to another;
- The stabilization criterion focuses on the variation of the maximum value of von Mises equivalent stress from a local explicit cycle to another.

VALIDATION TEST

An axisymmetric structure which is representative of a turboengine rotating disk is calculated. The disk is made of titanium alloy.

Material model behavior

Because of the unsymetric loadings applied on a such structures, taking into account in cyclic computations the continuous relaxation of the mean stress until a non zero stabilised value, which depends on the applied strain amplitude, is important and requires an accurate representation of the material behavior.
The classical viscoplastic Chaboche model with isotropic and nonlinear kinematic hardening leads to a complete relaxation of the mean stress to zero.
For stage with this problem, a multi-kinematic hardening rule including thresholds [1] must be used. This model is an extension of a phenomenological model [2] based on thermodynamic principles and the state variable approach.

1 Initialization, $t0 = 0$

2 • Opening of the backup file of the local quantities in writing

3 Loop on the cycles of period T

4 • $t = t0$

5 Loop on the increments Δt of cycle

6 • Application of load increment

7 • Initialisation of displacement increment: $\{\Delta U\} = \{0\}$

8 Global equilibrium iterations

9 • (i) External forces evaluation $\{F^{ext}(\Delta U)\}$

10 • Evaluation of internal forces $\left\{F^{int}(\Delta U)\right\}$

11 and global tangent matrix $\left[K^T(\Delta U)\right]$

12 • Evaluation of the residue of total balance: $\{R\} = \left\{F^{int}\right\} - \{F^{ext}\}$

13 • if $||\underline{\mathbf{R}}|| < \varepsilon$, then convergence, goto (ii)

14 • Update displacement increment: $\{\Delta U\} = \left[K^T\right]^{-1}\{R\}$

15 • goto (i)

16 (ii) End increment: $t = t + \Delta t$

17 • Writing of increment results in the local quantities file:

18 for each integration point ip, writing of $\underset{\sim}{\varepsilon}^{ip}(t)$

19 **Explicit local cycles**

20 • Closing of backup file and opening in reading

21 Loop on integration points ip

22 (iii) Reading in backup file of the strain history

23 of cycle $\left\{ \underset{\sim}{\varepsilon}^{ip}(\tau)\ ,\ t0 < \tau < t0 + T \right\}$ for the integration point ip

24 • $J0^{ip}_{max} = mises\left(\underset{\sim}{\sigma}^{ip}(t0+T)\right)$

25 Loop on local cycles ic

26 (iv) $J^{ip}_{max} = 0\ ,\ \tau = t0$

27 Loop on the increments Δt of local cycle ic

28 • Integration of the constitutive equations for $\Delta\underset{\sim}{\varepsilon}^{ip} = \underset{\sim}{\varepsilon}^{ip}(\tau+\Delta t) - \underset{\sim}{\varepsilon}^{ip}(\tau)$

29 • if $mises\left(\underset{\sim}{\sigma}^{ip}(\tau+\Delta t)\right) > J^{ip}_{max}\ ,\ J^{ip}_{max} = mises\left(\underset{\sim}{\sigma}^{ip}(\tau+\Delta t)\right)$

30 • Stabilization test at integration point ip:

31 if $\frac{J^{ip}_{max} - J0^{ip}_{max}}{J0^{ip}_{max}} < \varepsilon$ then stabilization, goto (v)

32 • $J0^{ip}_{max} = J^{ip}_{max}\ ,\ ic = ic+1\ ,\ \tau = \tau + \Delta t\ ,$ goto (iv)

33 (v) end local cycles point d'integration ip: $ip = ip+1$, goto (iii)

34 • Closing of backup file and opening in writing

35 • $t0 = t0 + T$

36 • goto 5

Figure 1: Newton-Raphson algorithm with explicit local cycles to accelerate the search of the stabilized solution.

Complete constitutive equations

- **Partition of the strain tensor**

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^{e} + \underset{\sim}{\varepsilon}^{th} + \underset{\sim}{\varepsilon}^{v} \tag{1}$$

$\underset{\sim}{\varepsilon}^{e}$, $\underset{\sim}{\varepsilon}^{th}$, $\underset{\sim}{\varepsilon}^{v}$ are elastic strain, thermal strain, classical viscoplastic strain tensors respectively.

- **Criterion**: We suppose a von Mises material and use the following expression for the yield surface.

$$f(\underset{\sim}{\sigma}) = J(\underset{\sim}{\sigma} - \underset{\sim}{\mathbf{X}}) - R \tag{2}$$

where $\underset{\sim}{\sigma}$ the stress tensor, $\underset{\sim}{\mathbf{X}} = \sum_i \underset{\sim}{\mathbf{X}}_i$ the back-stress tensors associated with different kinematic hardening mechanisms, R the yield radius, and J the second invariant of $\underset{\sim}{\sigma}$.

- **Normality rule**

$$\dot{\underset{\sim}{\varepsilon}}^{v} = \dot{p}\,\underset{\sim}{\mathbf{n}} \qquad \text{with} \qquad \dot{p} = \left(\frac{\langle f(\underset{\sim}{\sigma})\rangle}{K}\right)^{n} \qquad \text{and} \qquad \underset{\sim}{\mathbf{n}} = \frac{3}{2}\,\frac{\underset{\sim}{\mathbf{s}}}{J(\underset{\sim}{\sigma})} \tag{3}$$

where:

- $\underset{\sim}{\mathbf{n}}$ is the normal to the flow surface in the stress space and $\underset{\sim}{\mathbf{s}}$ the stress deviator tensor.

- **Nonlinear isotropic hardening**

$$R = R_0 + Q\left(1 - e^{-bp}\right) \tag{4}$$

- **Nonlinear kinematic hardening with thresholds**

$$\underset{\sim}{\mathbf{X}}_i = \frac{2}{3}\,C_i\,\underset{\sim}{\alpha}_i \qquad \text{with} \qquad \dot{\underset{\sim}{\alpha}}_i = \left[\underset{\sim}{\mathbf{n}} - \frac{3\,D_i}{2\,C_i}\,\underset{\approx}{\phi} : \underset{\sim}{\mathbf{X}}_i\right]\dot{p}$$

$$\text{and} \qquad \underset{\approx}{\phi} = \left\langle \frac{D_i\,J(\underset{\sim}{\mathbf{X}}_i) - \omega_i\,C_i}{1 - \omega_i} \right\rangle^{m1} \frac{1}{\left(D\,J(\underset{\sim}{\mathbf{X}}_i)\right)^{m2}}\,\underset{\approx}{\mathbf{I}} \tag{5}$$

This rule was proposed by Chaboche in order to refine the modeling of mean-stress relaxation, since the introduction of the threshold allows to delay the effect of nonlinear kinematic hardening which cause a total mean-stress relaxation until zero.
$K, n, R_0, Q, b, C_i, D_i, \omega_i$ are the material parameters and $0 \leq \omega < 1$.
The material parameters of this model have been identified in the particular case of titanium at different temperatures. Figure 2 shows the typical results observed on titanium

alloy for strain controlled fatigue test at different strain ratio. During the cycles, there is a continuous relaxation of the mean stress until a non zero stabilized value which depends on the applied strain amplitude.

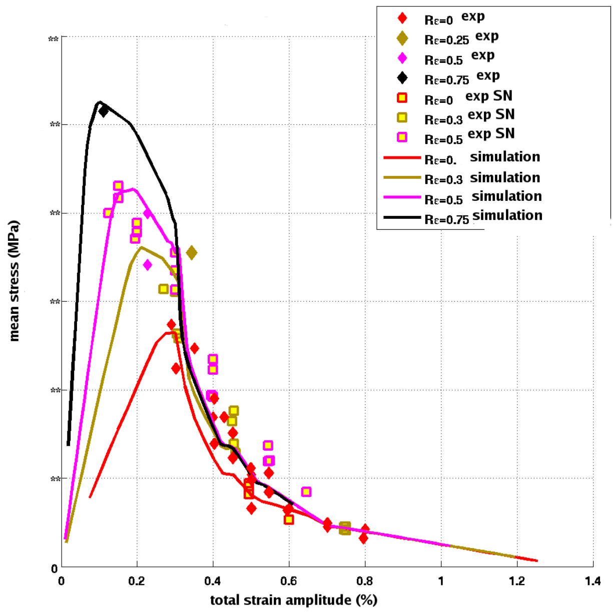

Figure 2: Mean stress relaxation curve for various strain ratio on TA6V 200°C.

Numerical simulation of a turboengine rotating disk

A turboengine rotating disk made of titanium alloy is analysed. The behavior of the material (titanium) has been modelled using the elastoviscoplactic constitutive law previously introduced. The mesh, as shown in figure 3 is a 2D axisymmetric quadratic model, composed of 14967 nodes and 4756 elements with reduced integration. The disk is submitted to a cyclic centrifugal load whose variation is summarized in figure 4.

The validation tests consist to perform two finite element analysis. The first one is a reference FE analysis of 500 cycles with the standard incremental method. The second one consists to perform 100 cycles with the accelerated calculation procedure which is activated once every two cycles.

With the classical incremental approach, the CPU time was 18 hours. The analysis of the results show that the stabilization of stress amplitude and mean stress quantities are very slow and may require the calculation of several hundreds of cycles. With the accelerated method, the CPU time was 2.5 hours. The maximal number of local explicit cycles needed to obtain the stabilized response is summarized in figure 5. It my be noted that, the average number of local explicit cycles required during the incremental cyclic calculation are between 50 and 20 during the first 10 cycles, to progressively decrease to 1 after 50 cycles. A comparison between the results of the two tests are shown in figures 6 to 9. We can verify that the accelerated procedure achieves the steady state much faster than the standard incremental procedure.

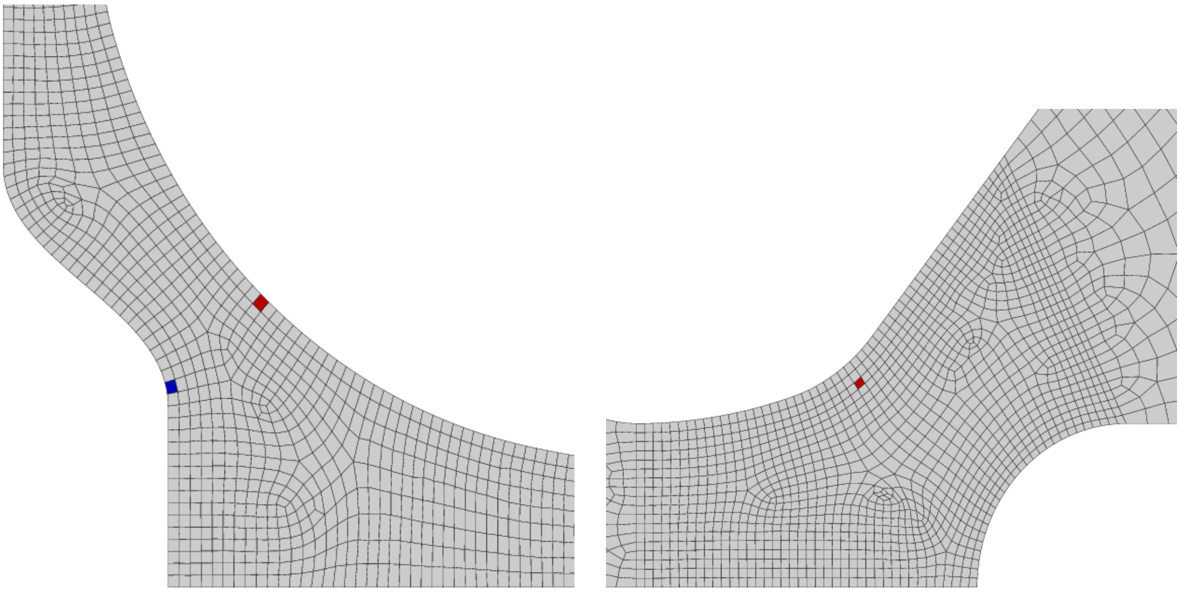

Figure 3: Mesh of turboengine rotating disk.

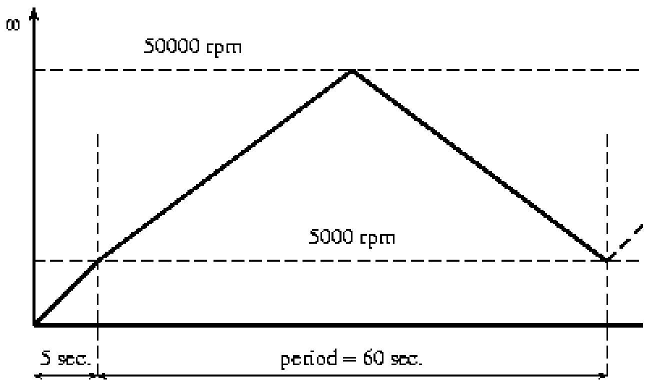

Figure 4: Centrifugal applied loading.

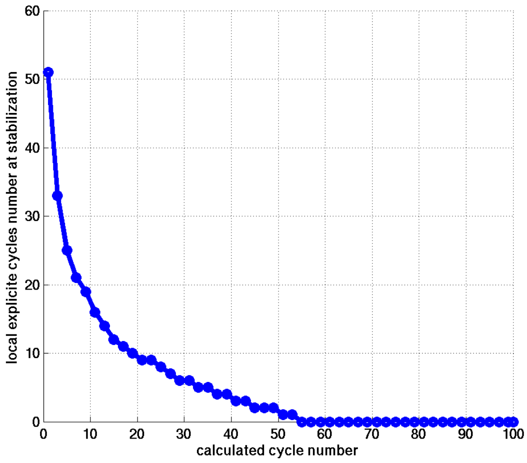

Figure 5: Number of explicit local cycles required during the incremental cyclic calculation.

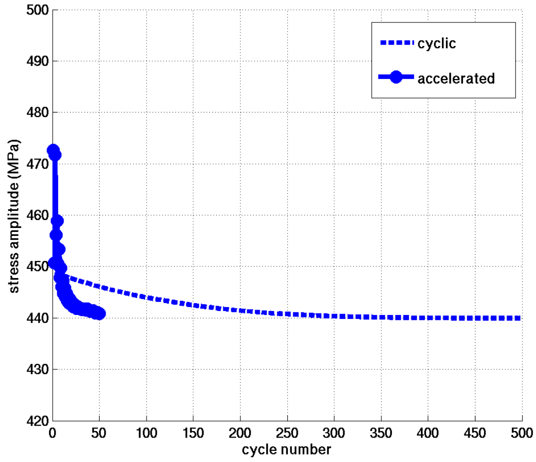

Figure 6: Comparison of the stress amplitude evolution during cycles for a surface node obtained by standard calculation and the accelerated procedure.

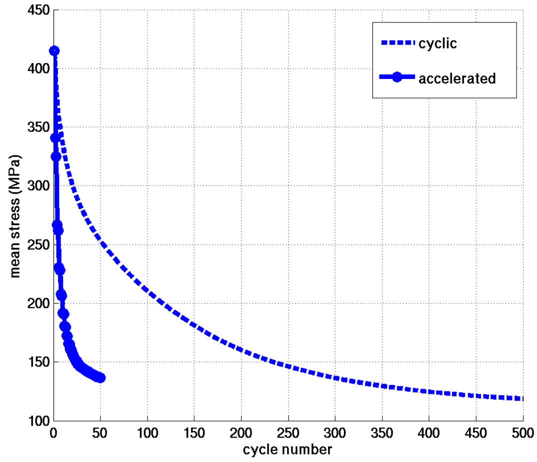

Figure 7: Comparison of the mean stress evolution during cycles for a surface node obtained by standard calculation and the accelerated procedure.

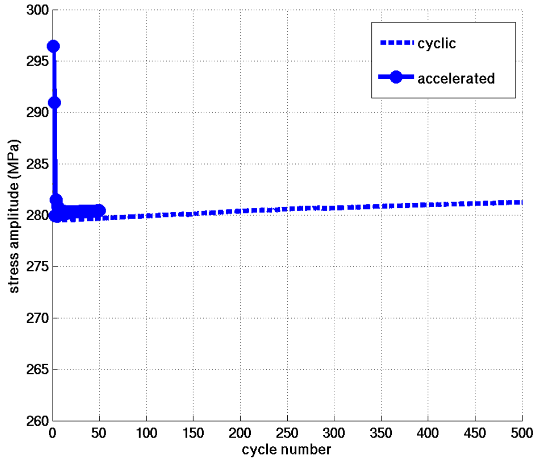

Figure 8: Comparison of the stress amplitude evolution during cycles for a sublayer node obtained by standard calculation and the accelerated procedure.

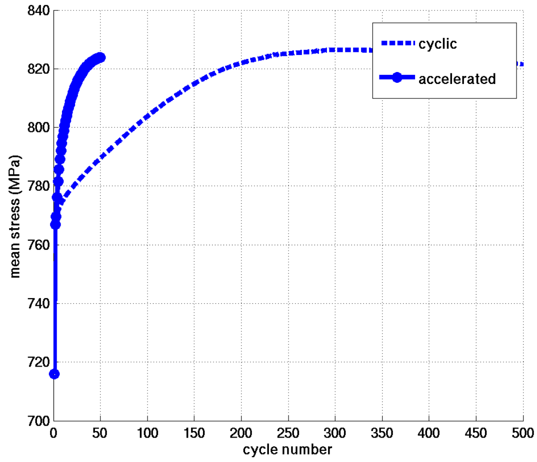

Figure 9: Comparison of the mean stress evolution during cycles for a sublayer node obtained by standard calculation and the accelerated procedure.

Fatigue life calculation

After calculating "asymptotic" conditions, we obtain the necessary input to evaluate the durability of the turboengine rotatig disk : stress and plastic strain history during stabilized cycle. The life estimation is performed using a new multiaxial fatigue model developed by Onera [3]. The model is classified as an equivalent effective stress model including the influence of the mean stress. The equations used are recalled bellow.

$$\sigma_{a_{eff}} = \sigma_{a_{eq}} \left(1 + b\, \sigma_{D_0}\, t_{eq}\right) \tag{6}$$

The triaxiality factor t_{eq} takes the following form:

$$t_{eq} = \xi \frac{S_F\, t_F}{1 + |t_F|} + (1 - \xi)\,(S_F - 1) \tag{7}$$

which results from the three following forms associated respectively with Sines, Crossland and Gonçalvès criteria (see [3] for more details) :

$$t_{eq} = t_F = \frac{(Tr\sigma)_{mean}}{\sigma_{a_{eq}}},\quad t_{eq} = \frac{(Tr\sigma)_{max} - \sigma_{a_{eq}}}{\sigma_{a_{eq}}},\quad t_{eq} = S_F - 1 = \frac{\sigma_{p_{max}} - \sigma_{a_{eq}}}{\sigma_{a_{eq}}} \tag{8}$$

where $\sigma_{a_{eq}}$ is the octahedral shear stress amplitude, $(Tr\sigma)_{mean}$ the mean value of the first stress invariant during the cycle. The term σ_{D_0} represents the fatigue limit for reversed cycle at 10^7 cycles. Here b and ξ are a material parameters. For uniaxial loadings $t_{eq} = \dfrac{1 + R}{1 - R}$ where R is the stress ratio. This creterion is correlated to the lifetime by using a simple Basquin type function $\sigma_{a_{eff}} = BN_f^{-\beta} + \sigma_{D_0}$. In this expression, B and β are a material parameters, that are identified on LCF (strain or stress control) and HCF tests (stress control).
The isocontours number of cycles to failure calculated with the above fatigue model are shown in figures 10 and 11 respectively for the accelerated test and the standard incremental test.

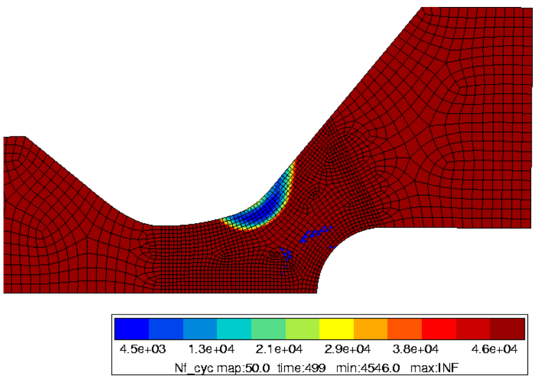

Figure 10: Isocontours of number of cycles to failure, accelerated method.

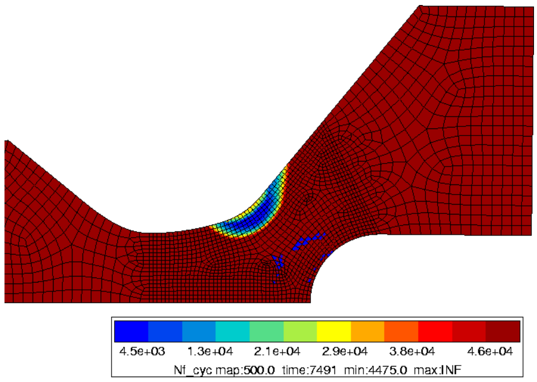

Figure 11: Isocontours of number of cycles to failure, standard incremental case.

CONCLUSION

With cyclic behavior, due to the modeling of cyclic hardening and relaxation of mean stress until a non zero stabilized value, it may be necessary to achieve a significant number of cycles before reaching a stabilize response. The explicit local method is an effective solution to this problem. Tests on different calculations have shown that the method achieves a stabilized response (or quasi-stable) in a reduced number of cycles. On the other hand, the stabilized solution is very close to the one obtained in an incremental cyclic calculation performed up to stabilization after several hundred cycles.

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