Current Trends in Multiaxial Fatigue Research and Assessment

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ABSTRACT. *Within the more general field of structural durability research, multiaxial fatigue has been one of the most active research topics over the past 30 years. The current paper presents a limited review of the current research trends in multiaxial fatigue research based on examination of a small fraction of the nearly 1500 international scientific papers published over the past seven years and the nearly 150 abstracts and extended abstracts submitted for presentation at the 9th International Conference on Multiaxial Fatigue and Fracture (ICMFF9). While any number of topics could be selected, this review considers developments with respect to multiaxial fatigue testing of new materials, multiaxial effects of notched components, multiaxial fatigue of welds and developments in the implementation of multiaxial fatigue assessment in design.*

INTRODUCTION

At the 7th International Conference on Biaxial and Multiaxial Fatigue and Fracture (ICBMFF7) , held in Berlin, a paper similar to this one, i.e., a summary of the state-of-the-art and recent developments in the field of multiaxial fatigue and fracture, was prepared [1]. Having completed that previous task, it was only after significant deliberation that it was agreed to undertake a similar responsibility for the ICMFF9 Conference. Note that it was during the Berlin event that the letter "B" was dropped from the Conference title acronym, since multiaxial fatigue and fracture also encompasses biaxial cases. A recent search via ScienceDirect [2] revealed that since the 2003 review, which, even at the time, was admittedly limited, has shown that more than 1500 journal articles have appeared in international scientific publications dealing with the topic of multiaxial fatigue and fracture. Even on the significantly narrower topic of multiaxial fatigue of welded structures, there have been more than 200 articles. Numerous other articles on topics like fretting fatigue, rolling contact fatigue, and thermo-mechanical fatigue, which can be considered specialized topics involving multiaxial fatigue failure, have also been published in major journals.

In addition to the 8th International Conference on Multiaxial Fatigue and Fracture (ICMFF8) which was held in 2007 in Sheffield [3], there have been two more international conferences devoted to the growth rate and direction of crack paths due to complex fatigue and fracture loading: Crack Paths (CP 2006) [4] and Crack Paths (CP 2009) [5].

In 2003 the Elsevier Science Encyclopaedia Comprehensive Structural Integrity was published and included a chapter on multiaxial fatigue [6]. This volume is intended to be a first point of entry to the key literature and background material for those planning research, teaching, learning and writing about structural integrity and includes aspects relevant to mechanics, materials and applications. The books *Spectral Method in Multiaxial Random Fatigue* by Macha and Nieslony [7] and *Multiaxial Notch Fatigue* by Susmel [8] were published in 2007 and 2009, respectively.

The large number of new publications and the nearly 150 abstracts that were received in preparation for this conference clearly indicate that interest in the field of multiaxial fatigue and fracture shows no sign of waning. The wealth of published information following the 2003 review makes it impossible for a single conference paper to provide a comprehensive overview. For this reason the current paper devotes most attention to those abstracts and extended abstracts which were submitted in preparation for this ICMFF9 Conference.

NEW MATERIALS

Early multiaxial fatigue studies were primarily conducted using aluminium alloys, stainless steels, super alloys and steels (low alloy, bearing and QT). These choices were governed by the extensive use of these materials in fatigue critical components, e.g., pressure vessels, turbine blades, axles, bearings, and crankshafts, which are subject to multiaxial cyclic stresses and strains. Initially, damage models for multiaxial fatigue were developed primarily empirically, using numerical methods to fit data from two or more stress states. As more knowledge has been gained about the complex growth mechanisms of cracks in uniaxial and complex strain fields, newer damage models were devised which attempted to capture the essential loading features which lead to fatigue damage. These models therefore seek to describe the complex crack nucleation and microcrack growth mechanisms using load or strain values available in most engineering applications [10].

Depending on the loading mode, stress level and environment, one or more failure mechanisms may be observed for a given material: shear-dominated crack growth, tensile-dominated crack growth, or fatigue life dominated by crack nucleation. Crack branching, coalescence and closure are also strongly dependent on the material microstructure and loading mode. No single mechanism is responsible for the nucleation and growth of fatigue cracks. As a result, no single damage model will be applicable to all materials and loading situations [10].

For ductile structural steels and stainless steels, significant reduction in fatigue life under nonproportional multiaxial fatigue loading has been observed. In contrast, an increase in fatigue life is often observed under nonproportional loading for brittle materials like cast aluminium, cast iron and sintered steels [11]. There are also semi-ductile materials which reveal no difference between in- and out-of-phase multiaxial loading, e.g., cast steels and forged or wrought aluminium alloys. Experiments have shown that some materials show additional cyclic hardening during nonproportional loading that is not observed in uniaxial or for any proportional loading path. The 90º out-of-phase loading path has been found to produce the largest degree of nonproportional hardening.

In looking through the abstracts of the ICMFF9 papers, it is easy to observe the wide variety of materials that has been studied (see Table 1). This is perhaps an indication that *a priori* decisions about what type of damage model should be selected cannot be made. Thus, in order to incorporate these materials into designs, observations of the cracking and fracture behaviour must be made and a suitable damage parameter must be defined.

Table 1 Examples of the variety of materials presented at ICMFF9

Material	Reference
Non metallic materials	
Asphalt mixes	[12]
Concrete	[13]
Nuclear graphite	[14]
Cortical bone	[15]
White (Harsin) marble	[16]
Synthetic rubber	[17]
Thermoplastics	[18]
Composite materials and structures	
Hybrid aluminium and glass fibre panel	[19]
E-glass and polyester matrix	[20-22]
Hybrid steel polymer joint	[23]
Bonded joints	[24]
Light alloys	
Aluminium alloy	[25-28]
Titanium alloys	[29-33]
Magnesium alloy	[34-35]
New steels	
Sintered steel	[36]
Cast trip steel	[37]
Maraging steel	[29]
Other metals	
Sn-Ag solder	[38]
Copper and alpha brass	[39]
Ni-based single crystal	[40]

NOTCH EFFECTS

Estimation of the fatigue life of a component requires consideration of how and where the external loads combine to produce high stresses, as well as what the state of stress is at these locations. This comprises the study of stress concentrations. Stress concentrations cannot be avoided in the design of structures and engineering components. Fatigue cracks will nucleate at stress raisers such as holes, fillets, welds or keyways. Full understanding of the role of notches must take into account not only the stress concentration factors, K_T, but also the stress gradient near the notch and statistical size effects. Even during uniaxial loading, the state of stress in a notch is often multiaxial because of geometric constraints. During multiaxial loading, the issues are compounded in that different load components have different K_T and the location of greatest stress or strain may even change depending on the magnitude and phase of the load components.

Consider the example of a notched shaft as given in Fig. 1. Four independent loads may be applied to this type of geometry: tension, P, torsion M_T and two bending moments, M_X and M_Y. The maximum stresses due to tension and torsion loading are located in an annular ring at the base of the stress concentration. The location of the maximum bending stress depends on the magnitude and phasing of the two bending moments M_X and M_Y. Bending and tension loads will produce a stress σ_z. A stress σ_θ will also be produced due to notch constraint. Torsion moments result in a shear stress $\tau_{\theta z}$. The shear and normal stresses will combine in tension and torsion loading and the resultant principal stress direction will not be in the plane of the notch. Stresses due to tension and torsion will be constant around the circumference of the shaft. Bending stresses will be a maximum at only one location.

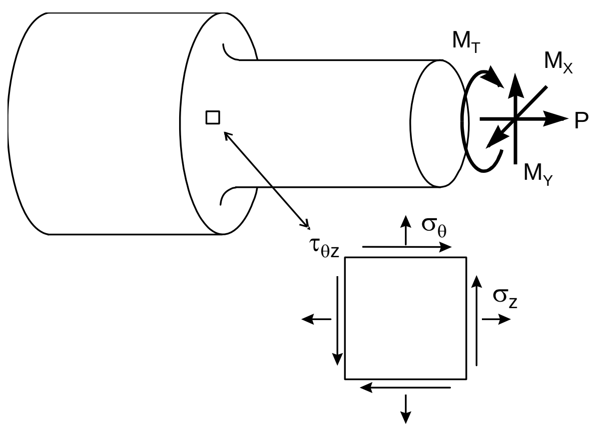

Figure 1 Alternate load components on a notched shaft [10].

The terms nonproportional *loading* and nonproportional *stressing* are frequently used interchangeably, but the concepts are not identical. Nonproportional loading is used to describe the loads acting on a structure or component, while nonproportional stressing is used to describe the resulting stresses acting on the material. Nonproportional loading of a notched structure frequently results in proportional or uniaxial loading of the material at a stress concentration. Consider the notched shaft in Fig. 1, subjected to the two bending moments M_X and M_Y, as shown in Fig. 2.

There are four loading segments designated 1 through 4. First, an in-phase loading is applied. This is followed by an out-of-phase loading. Finally, the two moments are applied independently. The resulting bending moments at several locations A, B, C, and D, around the circumference of the shaft are given in Fig. 3. Stresses and bending moments at A' have the same magnitude but opposite sign to those at A. These bending moments will produce a stress σ_z, the magnitude of which can be computed from simple beam theory.

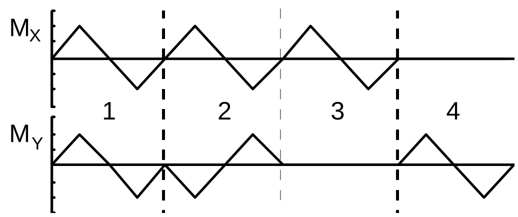

Figure 2 Bending moments applied to a notched shaft of Fig. 1 [10].

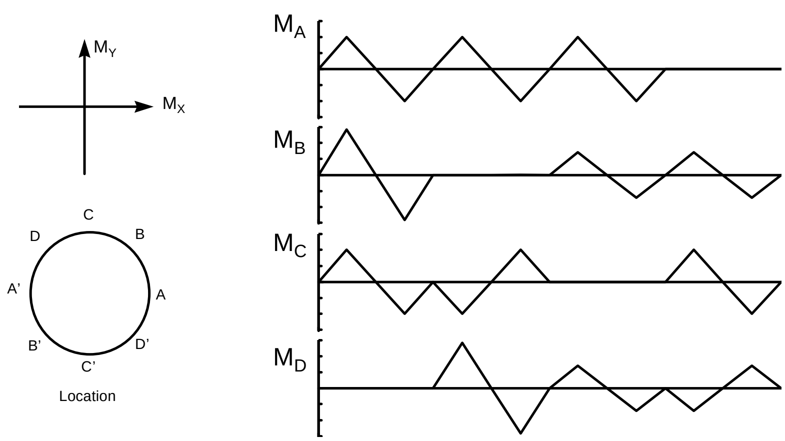

Figure 3 Bending moments at various locations around the circumference [10].

Stresses at location A will be directly proportional to the bending moment M_X because point A lies on the neutral axis for M_Y loading. Similarly, the stresses at C will be directly proportional to the bending moment M_Y. Both M_X and M_Y will combine at locations B and D to produce a larger or smaller bending moment. Point B has the highest bending moment and stress for in-phase loading. Stresses at D will be zero during in-phase loading. Out-of-phase loading produces the largest stress at point D because both bending moments combine. The magnitude of these stresses is $M_B = M_D = \sqrt{2}\, M_X$. Rainflow counting identifies the cycles listed in Table 2 for each location.

In this example, nonproportional loading determines the magnitude of the stresses and the location of highest fatigue damage. However, the state of stress remains uniaxial and is unaffected by the nonproportional loading (note: that transverse constraint may produce a proportional biaxial stress state). Principal stress directions remain fixed with respect to the axis of the shaft. Principal stress directions are the same for both M_X and M_Y loading. Nonproportional loading is very important in all structures because it determines the magnitude and location of the highest stresses.

Table 2 Cycles for nonproportional loading history

ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

Now consider an example where the loading M_Y in Fig. 2 is replaced by a torsional moment M_T as in Fig. 4. The stresses at all points on the circumference of the shaft will be the same for the torsion loading. Torsion and bending will combine to produce the largest stress at point A. Stresses in the plane of the notch are shown for four points in time. The normal stress due to the torsion loading is shown as σ_T and has the same magnitude as the shear stress $\tau\theta z$. At times t_1 and t_2, both σ_T and σ_Z combine to produce the principal stress whose magnitude and direction are shown in the figure. The magnitude of the principal stress is the same at t_1 and t_2 but the direction is changed. Both magnitude and direction change at t_3 and t_4. Out-of-phase torsion and bending produce both *nonproportional loading* and *nonproportional stressing*.

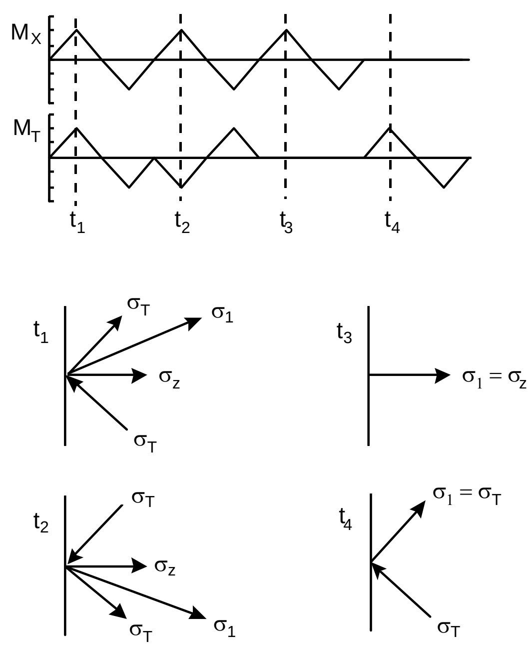

Figure 4 Stresses during nonproportional loading [10].

Modern FE analysis software makes it possible to directly compute the stresses and strain at the root of notches for relatively short fatigue histories. For longer load histories, however, more efficient routines are needed. Stresses and strains at notches are most often computed using elastic FE analysis and/or Neuber's rule. For multiaxial loading, equivalent stresses can be computed from the von Mises yield criterion. Here eσeq and eεeq are defined as the elastically calculated notch stress and strain and σ^{eq} and ε^{eq} are the elastic-plastic notch stress and strain. Neuber's rule becomes

$$^{e}\sigma^{eq} \cdot {}^{e}\varepsilon^{eq} = \sigma^{eq} \cdot \varepsilon^{eq} \tag{1}$$

The material's cyclic stress strain curve provides a relationship between σ^{eq} and ε^{eq}

$$\varepsilon^{eq} = \frac{\sigma^{eq}}{E} + \left(\frac{\sigma^{eq}}{K'}\right)^{\frac{1}{n'}} \tag{2}$$

In multiaxial stress states, the material constants, K' and n', can be used to relate the individual stress and strain components shown here for plane stress.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} f(E, K', n') \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{3}$$

Equations 1-3 provide five relationships to resolve the six unknown stress and strain components, i.e. three unknown components of stress (σ_x, σ_y, τ_{xy}) and three unknown components of strain (ε_x, ε_y, γ_{xy}). Thus, one additional equation is required to obtain a solution. Several strain-based approaches have been developed for analyzing multiaxial stressed notches when plasticity occurs. Each of the methods has a different assumption that provides the extra equation needed to solve the elastic-plastic analysis [10].

Based on the abstracts submitted to ICMFF9, it can be argued that multiaxial fatigue of notches is the single most dominant theme of the Conference. At least 15 papers address topics such as elastic plastic stress-strain analysis at stress concentrations [38, 41-43], critical distance and the influence of notch gradients [44-46], crack growth at notches [13, 47-50], geometrc optimization [51], gradient effects in fretting fatigue [52], and nonproportional loading [53-54].

WELDED JOINTS

As recently as ten years ago, the number of experimental studies and test data for multiaxial fatigue of welded joints was relatively limited. In a survey of data prepared 10 years ago, less than 400 experimental data points from 10 studies were reported [55]. At the 5th ICBMFF Conference, only five papers dealing with welded structures were presented [56], while at the 6th and 7th ICBMFF Conferences in 2001 and 2004, that number had doubled to ten [57-58]. The current Conference features eight papers concentrating on this topic [59-66].

Fatigue assessment of welded joints subjected to multiaxial spectrum loading is very complex. Scientific critical plane-oriented or integral shear stress hypotheses have been developed [67-70]; however, these methods require significant expertise and are not well suited for design guidance documents. The design recommendations of the IIW for multiaxial fatigue have recently been revised [71], using a modified form of the stress interaction algorithm originally proposed by Gough and Pollard [72]. Especially in the case of variable amplitude multiaxial loading, very little published data is available and significant engineering judgement was needed to formulate the rules. The recommendations assume a bi-linear SN curve with a knee point at $N_f = 10^7$ cycles. The Miner's rule is applied, but the design damage summation is reduced to $D = 0.5$ for variable amplitude loading. For out-of-phase multiaxial loading, an additional reduction is introduced via a so-called "comparison value". The damage equation has the form of a Gough-Pollard ellipse quadrant

$$\left(\frac{\Delta\sigma_{eq,S,d}}{\Delta\sigma_{R,d}}\right)^2 + \left(\frac{\Delta\tau_{eq,S,d}}{\Delta\tau_{R,d}}\right)^2 \leq CV \tag{4}$$

where $\Delta\sigma_{eq,S,d}$ and $\Delta\tau_{eq,S,d}$ are the design values for the characteristic equivalent stress ranges for normal and shear stress and are calculated using

$$\Delta\sigma_{eq,S,d} = \sqrt[m_1]{\frac{1}{D} \cdot \frac{\Sigma\left(n_i \cdot \Delta\sigma_{i,S,d}^{m_1}\right) + \Delta\sigma_{L,d}^{(m_1-m_2)} \cdot \Sigma\left(n_j \cdot \Delta\sigma_{j,S,d}^{m_2}\right)}{\Sigma n_i + \Sigma n_j}} \tag{5}$$

where D and CV are the respective Miner sum and "comparison value" given in Table 3. These values depend on the material and loading type. Computation of $\Delta\tau_{eq,S,d}$ follows the same form as Eq. (5).

m_1 is the slope of the bi-linear SN curve above the knee point
m_2 is the slope of the SN curve below the knee point
$\Delta\sigma_{i,S,d}$ design stress ranges for cycles above the knee point
$\Delta\sigma_{j,S,d}$ design stress ranges for cycles below the knee point
$\Delta\sigma_{L,d}$ design stress range at the knee point
n_i is the number of cycles belonging to $\Delta\sigma_i$
n_j is the number of cycles belonging to $\Delta\sigma_j$

Table 3 Recommended Miner sum, *D*, and “comparison values”, *CV*, for multiaxial fatigue analysis [71]

Type of load	Phase of stresses	Verification procedure	Miner-sum D or comparison value CV	
Constant amplitude	proportional	verification of maximum principal stress or $\left(\frac{\Delta\sigma_{S,d}}{\Delta\sigma_{R,d}}\right)^2 + \left(\frac{\Delta\tau_{S,d}}{\Delta\tau_{R,d}}\right)^2 \le CV$	CV=1.0	
	non-proportional	$\left(\frac{\Delta\sigma_{S,d}}{\Delta\sigma_{R,d}}\right)^2 + \left(\frac{\Delta\tau_{S,d}}{\Delta\tau_{R,d}}\right)^2 \le CV$	steel	CV=0.5
			aluminium	CV=1.0
Variable amplitude	proportional	Verification of maximum principal stress and Miner sum D, or $\left(\frac{\Delta\sigma_{eq,S,d}}{\Delta\sigma_{R,d}}\right)^2 + \left(\frac{\Delta\tau_{eq,S,d}}{\Delta\tau_{R,d}}\right)^2 \le CV$	D=0.5 CV = 1.0	
	non-proportional	$\left(\frac{\Delta\sigma_{eq,S,d}}{\Delta\sigma_{R,d}}\right)^2 + \left(\frac{\Delta\tau_{eq,S,d}}{\Delta\tau_{R,d}}\right)^2 \le CV$	steel	D=0.5 CV=0.5
			aluminium	D=0.5 CV=1.0

ENGINEERING APPLICATIONS

Estimation of the fatigue life of a component requires consideration of:

- How and where the external loads combine to produce high stresses,
- What the state of stress/strain is at these locations,
- How fatigue damage is accumulated for the computed stress state and, for complex loading,
- How individual cycles are counted for a variable amplitude and multiaxial history.

These are the topics of the plenary lecture to be delivered by Prof. Ali Fatemi [73]. In this paper, recent experimental results are presented and some simple approximations for capturing a few of the effects discussed in multiaxial fatigue life estimations are also provided. The significance of understanding and capturing the observed damage mechanisms during multiaxial fatigue stressing for proper life assessment is emphasized. Critical plane approaches have been shown to be most robust. Among them, those with both stress and strain terms are the most appropriate, due to their general applicability to LCF and HCF and their ability to capture material constitutive response under nonproportional loading. Such loading often results in shorter life, even for materials without nonproportional hardening. A simple life estimation method for steels based on hardness is presented, for situations when fatigue properties are not available.

In addition to the plenary lecture, numerous other papers in these proceedings present concepts of multiaxial fatigue in relation to specific load-bearing components. These include: crane runway welds [65], spiral welded gas pipelines [60], pressure vessel nozzles [63, 74], pressure barrier components [75], diesel engine components [76], turbine components [77-81], a helicopter tail rotor [82] and aircraft fuselage panels [83].

In recent years, there has been a significant development in software available for evaluating multiaxial fatigue problems. In France, the mesoscopic (or macro- micro) group of fatigue damage models first proposed by Dang Van [84] has received considerable attention. These critical plane fatigue limit models are based on the concept of micro-stress within a critical volume of material. The microscopic stresses and strains within critical grains are different from the macroscopic stresses and strains commonly employed for fatigue analysis. These models have a reputation both for accuracy and for ease of programming. For more than a decade FE post-processors based on the Dang Van method have been used in the French automobile industry [85-86].

Recent years have seen a rapid development in commercial software tools that are equipped to handle multiaxial fatigue assessment [87-90]. Capabilities of these packages are not evaluated here, but different software packages include features like multiaxial rainflow counting algorithms, various kinematic hardening models, critical plane analysis based on one or more damage parameters and assistance in determining proper data properties for multiaxial fatigue assessment, etc. Analysis tools are commonly linked to commercial FE packages. These impressive tools will help make multiaxial fatigue assessment more-or-less routine for design engineers. Unfortunately, it also provides the ability to make more sophisticated errors and continuing education should not be neglected.

DISCUSSION AND CONCLUSIONS

This paper has presented a limited survey of the state of the art related to multiaxial fatigue assessment of structures and components. A small percentage of the nearly 1500 international scientific papers published over the past seven years has been examined, but the primary source has been the nearly 150 abstracts and extended abstracts submitted to ICMFF9.

The overall impression has been the rather practical orientation of ICMFF9. The wide variety of materials which has been the subject of many studies could indicate the desire to incorporate multiaxial fatigue and fracture assessment concepts for numerous practical situations. The role of understanding material behaviour is highlighted in the plenary paper by Sonsino.

> *"The multiaxial fatigue behaviour of components seems to depend mainly on the ductility of the material used. The ductility steers the damage mechanisms. While, in the case of low-ductility (brittle) materials, the normal stress (strain) is the decisive parameter, in the case of ductile materials, it is the shear stress (strain), and, for semi-ductile materials, a combination of normal and shear stresses (strains). Critical plane oriented hypotheses can consider these different parameters, but the*

difficulty lies in the definition of ductility and, based on this, the selection of the appropriate hypothesis. Therefore, especially for the evaluation of safety parts, experimental verifications are still necessary, because of the lack of a general multiaxial fatigue hypothesis".[59]

Other major topics identified include multiaxial fatigue of welded structures and multiaxial fatigue and fracture of notched components. Both of these topics are of immense practical importance for industrial applications of multiaxial fatigue assessment. In his plenary lecture, Sakane points out the difficulties in using Neuber's rule for assessing stresses and strains at notches for loading modes other than that for which it was originally conceived.

"The Neuber's rule overestimated the strain concentration in tension loading because the Neuber's rule was theoretically derived in prismatic shear loading. The Neuber's rule underestimated crack initiation and failure lives of SUS stainless steel round notched specimens fatigued under push-pull loading but properly estimated them under torsion loading ... The applicability of the Neuber's rule for estimating the crack initiation and failure lives of the stainless steel under combined tension and torsion loading. The estimation was appropriate in torsion loading but went worse as the tension component increased. This trend was discussed in relation with the strain constraint under tension loading."[41]

In his plenary paper, Fatemi points out several of the most critical issues involved in order to implement multiaxial fatigue life assessment. The fatigue damage process consists of nucleation, growth and coalescence of fatigue cracks on specific planes in a solid. Therefore, the damage parameter used must capture the physical nature of the fatigue damage process. Stress-strain analyses and constitutive models must also be able to capture the true material hardening behaviour that occurs during nonproportional loading. Cycle counting and damage accumulation are challenges that still require some attention. He concludes:

"Although large amount of experimental data and research over the last four decades has significantly advanced the understanding of multiaxial fatigue, additional work is still needed for reliable and robust multiaxial fatigue life estimations."[73]

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