Cohesive zone model for the simulation of fatigue crack growth in adhesive joints under mixed mode I/II loading conditions

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ABSTRACT. *This work deals with the simulation of the fatigue crack growth [FCG] in bonded joints. In particular a cohesive damage model is implemented in the commercial software Abaqus, in order to take into account for the damage produced by fatigue loading. The crack growth rate is evaluated with a Paris-like power law expressed in terms of strain energy release rate. The crack growth rate is then translated into a variation of the damage distribution over the cohesive zone setting an equivalence between the increment of crack length and the increment of damage. The model takes also into account mixed mode I/II conditions, using the Kenane e Benzegaggh theory. In this work the validity of the model is tested by comparison with theoretical trends for conditions of pure mode I, pure mode II and mixed mode ratio. The results of the model are in very good agreement with the expected trends, therefore the model is adequate to simulate the fatigue crack growth behaviour of bonded joint.*

INTRODUCTION

It is largely established that fatigue is at the basis of most structural failures. Especially in the case of damage tolerant or fail safe design, it is necessary to know how cracks, or in generally defects, propagate during the component service life. A relationship between the loading state of the component and the fatigue crack growth (FCG) rate of a defect is therefore necessary. This relationship was given by Paris [1] in terms of crack growth rate as a function of the stress intensity factor, K. In the case of polymers and adhesives it is traditionally written as a function of the range of applied strain energy release rate (ΔG) using the equation

$$\frac{dA}{dN} = C\Delta G^m \tag{1}$$

where C and m are parameters depending on the material and loading mixity ratio and A is the defect area. This method allows to predict the growth of a defect under fatigue

loading, but it can be easily used only in the case of simple geometries. In real applications instead, the strain energy release rate can only be computed numerically with the help of, for example, Finite Element (FE) simulation. The prediction of the crack growth can be carried out by dividing the analysis in steps, each corresponding to user-defined crack growth increment. This method generally requires a remarkable amount of time.

On the opposite, the cohesive zone model is extensively used for the prediction of fracture propagation under quasi-static conditions, especially in the case of bonded joints and delamination in composites [2,3].

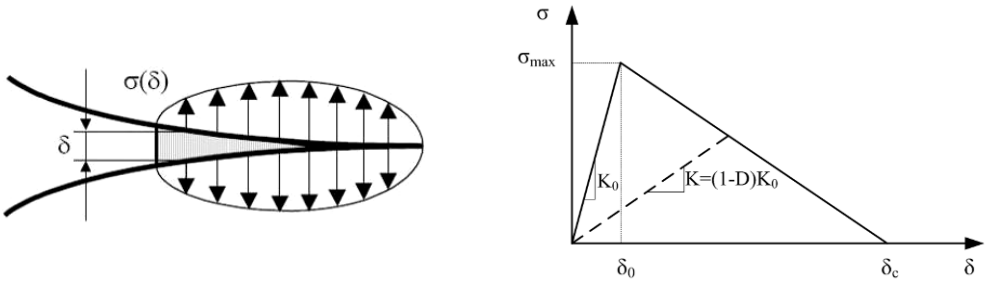

Figure 1. Example of a triangular cohesive law.

This model consists simply of a relationship between the normal (tangential) stress and the opening (sliding) of crack faces over a region ahead of the physical crack tip, as shown in Fig. 1. The relationship can be assumed in several different ways: in the example of Fig. 1, the relationship has a triangular shape. Initially it is linearly increasing with a stiffness K_0 until a value of opening equal to δ_0, then decreasing until the critical opening δ_C in order to represent damage evolution up to failure. In this phase the stiffness K is degraded with respect to the initial one, K_0, in the following way:

$$K = (1 - D)K_0 \tag{2}$$

where D is a state variable representing damage. In literature, a certain number of works can be found dealing with the simulation of the fatigue crack propagation using the cohesive zone model [4-8]. In particular the traditional definition is modified in order to take into account for the damage due to the fatigue loading: in other words, the damage is not only dependent by the opening, but also by the number of cycle. For example Maiti and Geubelle [6] defined a relationship between the stiffness and the number of cycles in terms of a two-parameters power law. Roe and Siegmund [5] instead, simulated the crack growth at an interface, using a cycle by cycle simulation where damage is incremented according to the stress level reached in the previous cycle. In both cases, model parameters have always to be identified by comparison of FCG simulation and experiments, with possible limitations on the transferability to cases different from those on which identification was made. The model proposed by Turon et al. [4] instead does not require parameter identification: in this case only the cohesive

law identified from quasi-static fracture tests and the coefficients of the Paris law of FCG tests are needed. However this model was implemented only in the case of geometries where the strain energy release rate could be computed analytically and it was not dependent on the crack length.

This work starts from the framework presented by Turon et al. [4], but it is extended for any 2D geometry. Moreover, mixed mode criteria were introduced respectively for the cohesive zone in terms of traction-separation law and for the fatigue crack growth rate using the Kenane and Benzegaggh theory [9].

THEORETICAL APPROACH

First of all is necessary to define the significance of the damage D in terms of real phenomenon. Referring to the Continuum Damage Mechanics theory [10], the damage D can be written as the ratio

$$D = \frac{A_d}{A_e} \tag{3}$$

where, taking as reference a representative interface element, A_e is the effective area of the element, while A_d is the damaged area (produced by voids or cracks). The increment of D associated with an increment of A_d, can be therefore written as:

$$\frac{dD}{dA_d} = \frac{1}{A_e} \tag{4}$$

In a FE simulation of the adhesive layer using cohesive elements, A_e is the area associated with an integration point (IP) of these elements. An increment of the crack extension (dA) can be written as the sum of the increment of A_d of all the IPs lying in the cohesive zone (A_{CZ}), yielding

$$dA = \sum_{i \in A_{CZ}} dA_d^i \tag{5}$$

therefore, the increment of the crack length with the number of cycle can be written as

$$\frac{dA}{dN} = \sum_{i \in A_{CZ}} \frac{dA_d^i}{dN} = \sum_{i \in A_{CZ}} \frac{dA_d}{dN} = n_{CZ} \frac{dA_d}{dN} = \frac{A_{CZ}}{A_e} \frac{dA_d}{dN} \tag{6}$$

where dA_d is the average of the dA_d^i over A_{CZ} and n_{CZ} is the number of IPs lying in A_{CZ}, therefore it can be written as the ratio between the dimension of A_{CZ} and the average area associated to the IPs (A_e).

Using Eqns. 1, 4 and 6 the increment of damage with the number of cycles is:

$$\frac{dD}{dN} = \frac{dD}{dA_d}\frac{dA_d}{dN} = \frac{1}{A_e}\frac{dA}{dN}\frac{A_e}{A_{CZ}} = \frac{1}{A_{CZ}}\frac{dA}{dN} = \frac{1}{A_{CZ}} C\Delta G^m \tag{7}$$

The Eq. 7 allows to write the increment of damage at an IP as a function of the range of applied strain energy release rate. This procedure was demonstrated to give good result in pure mode I and pure mode II FCG analysis [11].

With the aim to extend this model to mixed mode I/II conditions, some more considerations have to be done. First of all a mixed mode cohesive law has to be defined.

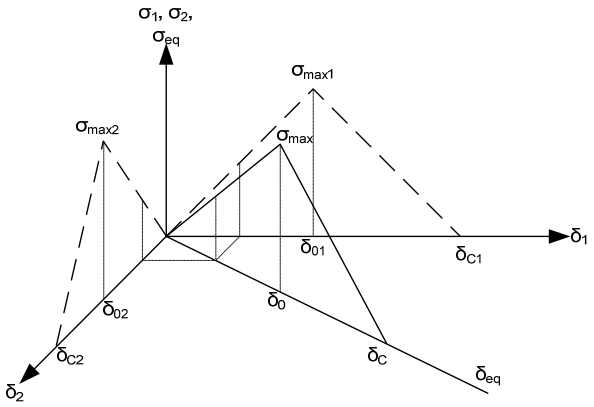

Figure 2. Representation of the mixed mode cohesive law.

Referring to 4, the equivalent opening is defined as

$$\delta_{eq} = \sqrt{\left(\frac{\delta_1 + |\delta_1|}{2}\right)^2 + (\delta_2)^2} \tag{8}$$

and starting from the cohesive laws in pure mode I (defined using the parameters σ_{max1}, δ_{01} and δ_{C1}) and pure mode II (defined using the parameters σ_{max2}, δ_{02} and δ_{C2}) the mixed mode law can be expressed making use of Eqs. (9-11).

In Eq. 9 the equivalent strain energy (U_{eq}) is defined as the sum of the strain energies (U_1 and U_2) associated to the single components

$$U_{eq} = U_1 + U_2 = 0.5 \cdot \delta_{eq}^2 \cdot K_{0eq} = 0.5 \cdot \left(\delta_1 + |\delta_1|\right)^2 \cdot K_{01} + 0.5 \cdot \delta_2^2 \cdot K_{02} \tag{9}$$

In this equation appear the initial stiffness values (K_{0i}) for the mode 1 (i=1) mode II (i=2) and the mixed mode (i=eq).

Another equation is needed for damage nucleation: this condition is obtained when

$$\left(\frac{\sigma_1}{\sigma_{\max 1}}\right)^2+\left(\frac{\sigma_2}{\sigma_{\max 2}}\right)^2=1 \tag{10}$$

The last equation is needed for the calculation of the mixed mode cohesive energy, that is the area underling the mixed mode cohesive law (Γ_{TC}), computed using the Kenane and Benzeghagg theory [12] as a function of the areas of the pure mode I (Γ_I) and pure mode II (Γ_{II}),

$$\Gamma_{TC}=\Gamma_I+\left(\Gamma_{II}-\Gamma_I\right)\cdot MM^{m_m} \tag{11}$$

In the last equation *MM* is the ratio between the strain energy in mode II and the total strain energy for each IP, and m_m is a parameter depending on the material. Eqns. 9, 10 and 11 allow to completely define the shape of the mixed mode cohesive law.

It is necessary also to define a mixed mode Paris-like law. Using the Kenane and Benzegaggh theory concerning the mixed mode fatigue propagation, the fatigue crack growth can be in general written as

$$\frac{dA}{dN}=B\Delta G^d \tag{12}$$

where the parameters B and b depend from the mixed mode ratio. In particular they can be written using the following equations:

$$d=d_I+\left(d_{II}-d_I\right)\cdot\left(MM\right)^{md} \tag{13}$$

$$\ln B=\ln B_{II}+\left(\ln B_I-\ln B_{II}\right)\left(1-MM\right)^{mB} \tag{14}$$

where d_I, B_I and d_{II}, B_{II} are respectively the parameters of the Paris-like law in pure mode I and pure mode II, respectively, and m_B and m_d are material parameters.

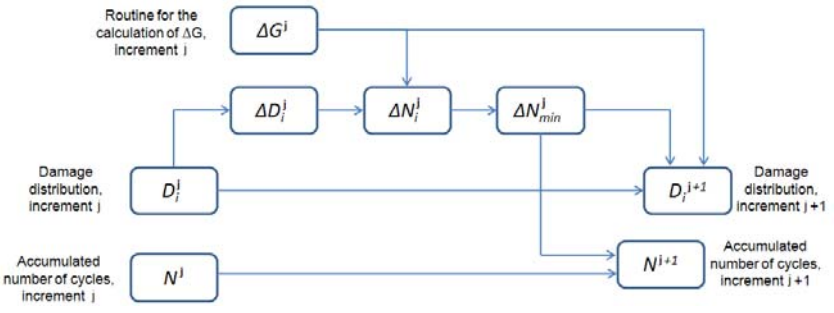

Figure 3. Schematic representation of the algorithm used for the crack growth simulation.

IMPLEMENTATION INTO THE FE SOFTWARE

The procedure written previously is implemented in the commercial software Abaqus, using external subroutines interacting with the analysis solver. The analysis is divided in increments and each increment is assigned a number of cycles using the algorithm shown in Fig. 3.

For a generic increment (j), the number of accumulated cycle is equal to N^j, and the damage for a generic IP (i) is equal to D_i^j. Afterwards a maximum value of increment of the damage within the analysis increment (ΔD_i^j) is assigned for each IP as the minimum between the increment needed to reach D=1 (failure) and a user-defined value $\Delta D_{\max}$.

$$\begin{aligned} \Delta D_i^j &= \Delta D_{\max} \qquad \text{if } 1 - D_i^j > \Delta D_{\max} \\ \Delta D_i^j &= 1 - D_i^j \qquad \text{if } 1 - D_i^j < \Delta D_{\max} \end{aligned} \tag{15}$$

In the same increment a dedicated subroutine calculates ΔG as the contour integral over a path surrounding the cohesive zone (in Abaqus the contour integral is not available for cohesive elements meshes).

From $\Delta D_{\max}$ and ΔG, a number of cycle ΔN_i^j is computed for each IP using Eq. (7). The subroutine looks for the minimum among the ΔN_i^j of the IPs within the cohesive zone, $\Delta N_{\min}^j$, which is then set to be the number of cycle of the increment. Finally, the number of cycle (N^{j+1}) and the damage distribution (N^{j+1}) are updated.

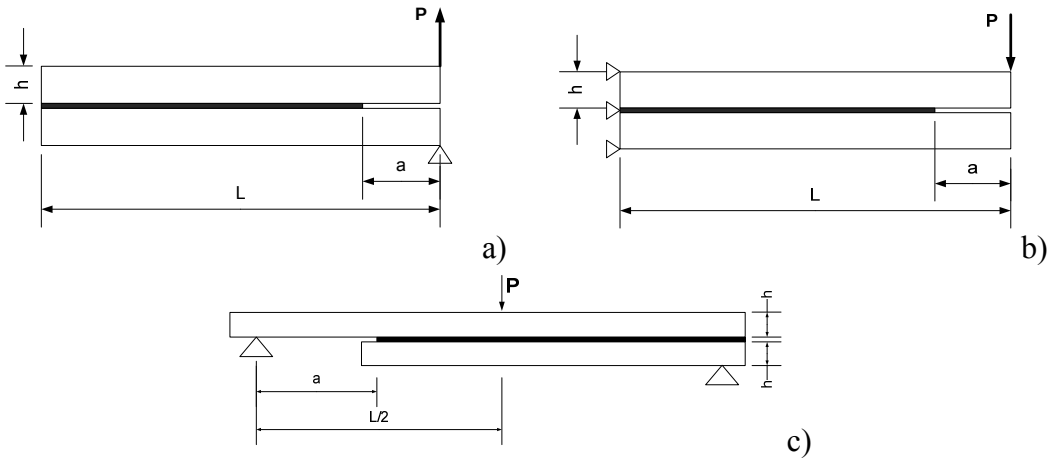

Figure 4. Tested geometries: a) DCB, b) ELS and c) MMENF.

FINITE ELEMENT SIMULATION

In order to verify the accuracy and the robustness of the model, it is tested for different joint geometries characterized by different mixed mode ratios. In particular, pure mode I condition are simulated with a Double Cantilever Beam (DCB) geometry, pure mode II conditions with an End Loaded Split (ELS) geometry and mixed mode I/II conditions with a Mixed Mode End Notched Flexure (MMENF) geometry as shown in Fig. 4. The adherends material is supposed to be aluminium with a Young modulus equal to 70'000MPa and a Poisson's ratio equal to 0.3.

The material parameters are taken from literature [4], for both the cohesive law and fatigue crack propagation. These are shown in Tab. 1 together with the applied load and the specimens dimensions.

Table 1. Parameters of the cohesive model and Paris-like law for pure mode I and pure mode II, along with the mixed mode parameters [4], together with the specimens dimensions and the applied load for unity of thickness.

Parameter	Mode I	Mode II
Γ [N/mm]	0.266	1.002
σ_{max} [MPa]	30	30
δ_0 [mm]	0.003	0.003
δ_C [mm]	0.0173	0.066
B	0.0616	4.23
d	5.4	4.5
ΔG_{th} [N/mm]	0.06	0.1

Parameter	Value
m_m	2.6
m_d	1.85
m_B	0.35

	DCB	ELS	MMENF
P [N/mm]	20	20	25
a [mm]	60	92	17
h [mm]	10	10	5
L [mm]	335	335	130

RESULTS

The results of the simulations are compared with the analytical trends obtained by Eq. 12-14, where MM=0 in case of DCB, MM=1 in case of ELS and MM=0.4 in case of MMENF (this value is computed from G_I and G_{II} obtained using the virtual crack closure technique for the initial crack length). The comparison is shown in Fig. 5 for the three geometries. It can be noticed that for pure mode I or pure mode II the two sets of data are in very good agreement with each other over the entire range considered. For the mixed mode I/II condition instead, the simulation seems to overestimate slightly the reference trend. This occurs because the reference trend considers a constant mixed mode ratio over the entire range, while during the propagation, when the crack approaches the midpoint, the value of MM slightly increases, and this variation is captured by the simulation.

CONCLUSIONS

In this work a procedure based on the cohesive damage model is developed for the simulation of fatigue crack growth in bonded joints for different mixed mode I/II

conditions. It is completely automated, i.e. the fatigue crack propagation simulation is performed in a unique. The procedure is validated by comparison with analytical trends for different mixed mode conditions, giving very good result. Further enhancement will concern the extension of the procedure to 3D models.

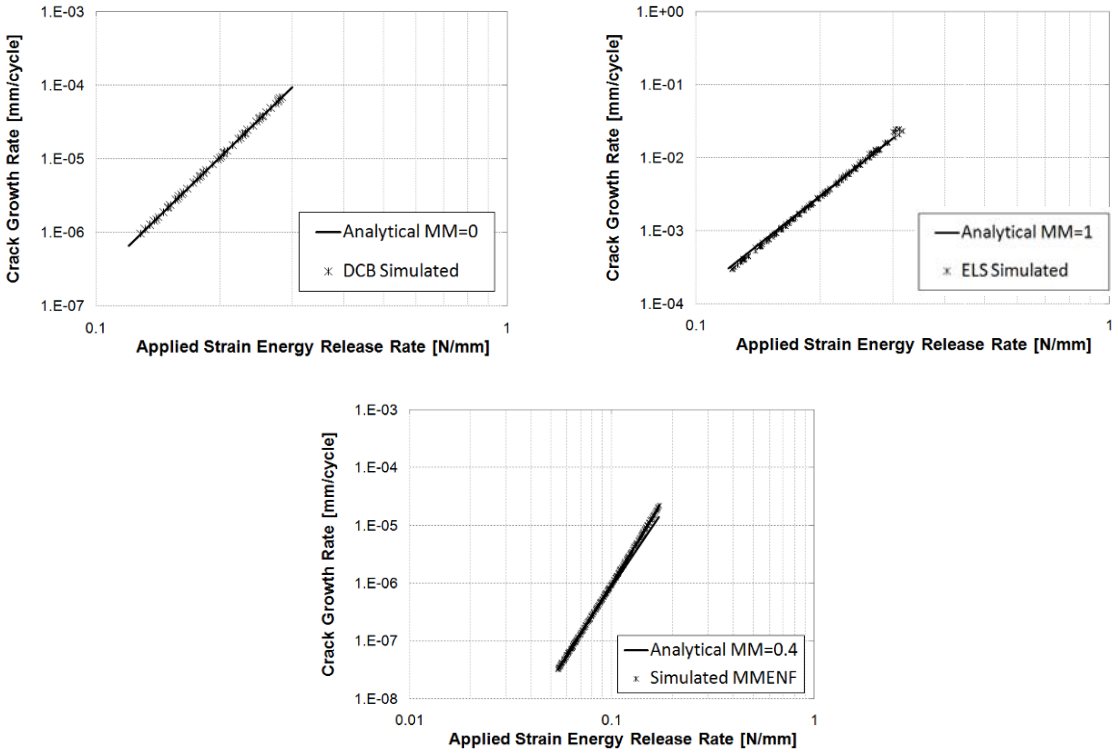

Figure 5. Comparison between analytical and simulated trends for the tested geometries.

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