Benefit of Taking Account of the Yield-Point Phenomenon in a Multiaxial Constitutive Model of Cyclic Plasticity

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ABSTRACT. *Annealed low-alloy steels and some related alloys exhibit a typical sharp yield point and an immediate stress drop followed by a yield plateau and further hardening. Most existing constitutive models of cyclic plasticity do not take account of the aforementioned phenomenon. As a result, these models evaluate a degree of plastic strain inaccurate. The paper presents the benefit of taking account of the yield-point phenomenon in constitutive models of cyclic plasticity when evaluating the development of a multiaxial stress-strain state and accumulated plastic strain of the surface-hardened raceway of a large-dimension rolling bearing. Raceways of such bearings are usually made of low-alloy steel 42 CrMo 4 (ISO 683/1), which distinctly exhibits the yield-point phenomenon in normalized and even tempered state. The results indicate that taking account of the yield-point phenomenon results in a reduction of the evaluated plastic strain size, while simulated growth of plastic strain accumulation is much faster. To determine material parameters and examine the compatibility of stress-strain state in comparison to the real-life state, monotonous and cyclic tests were carried out.*

INTRODUCTION

At first transition from the elastic into the elasto-plastic region, annealed low-alloy steels and some related alloys exhibit the yield-point phenomenon. The phenomenon is not taken account of by most existing constitutive models of cyclic plasticity [1]. In most cases of cyclic loading, the plastic zone is covered by the elastic zone. Since the yield-point phenomenon is not taken account of, the models evaluate a degree of plastic deformation different from the real one. Precise stress-strain analyses require a model which accurately describes the yield-point phenomenon as well as the subsequent cyclic plasticity.

Large-dimension rolling bearings are one of many groups of machine elements which are cyclically loaded and where local plasticity occurs. Due to large dimensions and a complex form of the bearing raceways with a teeth drive and some attachment drill holes, a material is required which can be bent, welded, mechanically treated and inductively surface hardened. The most frequently used material is low-alloy steel 42 CrMo 4 (ISO 683/1). Due to a required high surface hardness of a raceway (58-62 HRc), which prevents rolling elements from being pressed in, the surface is quenched. A surface-hardened raceway layer which is sufficiently hard and thick gives resistance to wear and fatigue, leaving the core of the ring soft. In case of overloading on the boundary between the hardened raceway layer and the soft core, local cyclic plasticity occurs. The paper presents the benefit of the cyclic plasticity model by taking account of the yield-point phenomenon to enable a better forecast of the stress-strain state and the growth of the accumulated plastic strain of material.

CONSTITUTIVE MODELS

In order to present how important the yield point phenomenon in cycle plasticity is, numerical analyses of cyclic loading of the rolling contact were carried out by means of two constitutive models (a model which takes account of the yield-point phenomenon (model with YPP) and a model which does not take account of the yield-point phenomenon (model without YPP)). The effects of cyclic plasticity such as the Bauschinger effect, cyclic hardening or softening and the changing of the mean stress value are in both cases defined by constitutive equations of kinematic and isotropic hardening or softening [1 - 8]. The only difference between the two constitutive models is reflected in the description of the first transition from the elastic into the elasto-plastic region.

The formulation presented below is a significant equation of the model which takes account of the yield-point phenomenon [8]. It is based on the phenomenological approach of the small-strain theory. The von Mises yield surface is taken account of, while the equation defining the size of the yield surface is supplemented by the coefficients of the yield-point phenomenon and cyclic and kinematic hardening:

$$F = \tilde{\sigma}_{eq} - \left(R + M + \sigma_{Ypre}\right) + \sigma_{Ycor} = 0\,, \tag{1}$$

where σ_{Ypre} presents the stress of the upper yield point and R is the coefficient of cyclic hardening or softening. σ_{Ycor} represents the correction of the yield stress and denotes the change in the size of the elastic region within the region of the yield-point phenomenon. $\tilde{\sigma}_{eq}$ represents the equivalent stress. The sudden stress drop immediately after upper yield point (M) is defined by the equation denoting the isotropic changing of the elastic region size in the stress space:

$$\dot{M} = c(\sigma_{Ydrop} - M)\dot{\lambda}\,. \tag{2}$$

In the upper equation, σ_{Ydrop} represents total yield stress drop, c is the basic parameter which determines the stress drop velocity and λ is a plastic multiplicator denoting the presence of plastic strain. Studies have proven that the steels with the yield-point phenomenon exhibit the change of the elastic region size in the stress space (Fig. 1) at first transition from the elastic into the elasto-plastic region. In accordance with the measurement results, two types of yield stress should be implemented: the upper yield stress $\left(\sigma_{Ypre}\right)$ when the material has not yet been subject to plastic strain and the yield stress when the material has already been subject to inhomogeneous plastic strain $\left(\sigma_{Ypost}\right)$. The correction of the yield stress denoting the change of the elastic region size in the stress plateau which is taken account of in the yield equation (1) is as follows:

$$\sigma_{Ycor} = \begin{cases} X_{eq} & \text{for stress plateau region,} \\ \sigma_{Ypre} - \sigma_{Ypost} & \text{for hardening region.} \end{cases} \tag{3}$$

X_{eq} represents the effective value of the entire displacement of the elastic region centre and is defined by kinematic hardening equations.

To determine material parameters and examine the compatibility of stress-strain state in comparison to the real-life state, monotonic and cyclic tests were carried out. The proposed model is in accordance with the findings of experimental work (Fig. 1) and provides an accurate description of the yield-point phenomenon and cyclic plasticity.

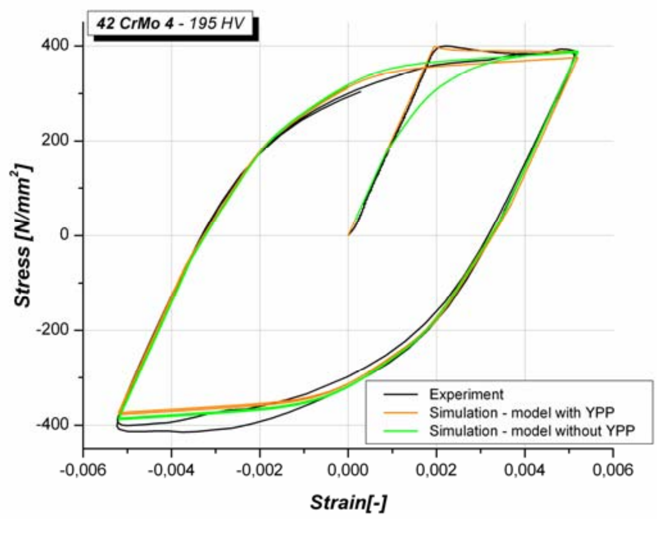

Figure 1: Comparison of hysteresis loops of the model with YPP and the model without YPP for the 1st and 2nd cycle

CYCLIC PLASTICITY OF BEARING RACEWAY

With the purpose of comparing the models in a real-life situation, the models are included in the finite element code based on which numerical simulations of cyclic loading of the bearing raceway were carried out. It is evident that despite the uniaxial loading the stress-strain state of the raceway is multi-axial due to the shape of the raceway.

Generated was a FE model of the rolling contact of the raceway and the ball bearing element which is by means of a finite element applied in the AceFem software (Fig. 2) [9]. The software enables examination of the cyclic course of the stress-strain state and material hardening or softening as well as the accumulation growth of the material plastic strain. The rolling contact FE model includes measured geometric and material properties of the test raceway [10 - 12].

Since the bearing raceway is surface quenched, material properties of the bearing change in line with the raceway depth. Based on the results of the hardness-change measurements, the numerical model takes account of various material parameters defined in accordance with the results of uniaxial monotonic and cyclic tests [10 - 12] (Fig. 2). Numerical simulation of the rolling contact was carried out for repeated loading with load amplitude 6 kN.

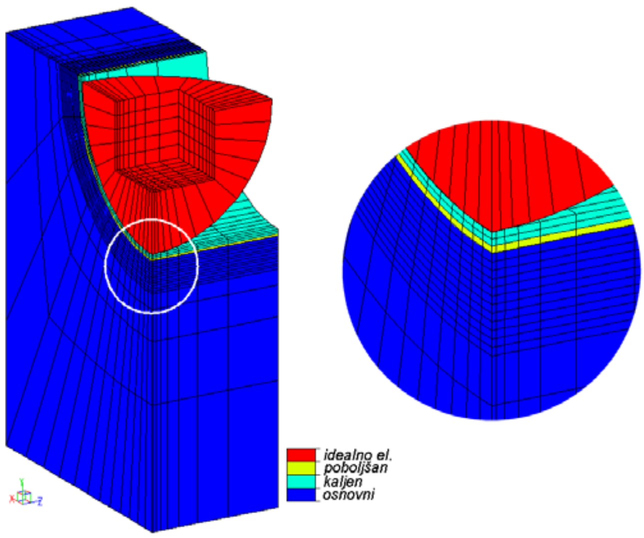

Figure 2: Numerical model of rolling contact

Numerical simulation results

During the numerical simulation of uniaxial tension-compression tests with strain control, the benefit of the model is evident in the first hysteresis loop (Fig. 1), where the curve of the model with YPP corresponds to the experiment results.

When carrying out numerical simulations of cyclic plasticity of real structures, local plasticity usually occurs with the elastic zone enclosing the plastic one. In these examples, it is necessary to take a proper account of the first transition from the elastic into the elasto-plastic region and the change of the elastic region size in the stress space.

Fig. 3 shows a comparison of the accumulated plastic strain after 200 cycles. The simulation results on the left side of the figure show the accumulated plastic strain calculated by the model without YPP. However, on the right side is the accumulated plastic strain calculated by the model with YPP. Comparison of the simulation results shows that taking into account the yield-point phenomenon decreases the size of the plastic strain zone, while the size of maximum accumulated plastic strain when taking account of the yield-point phenomenon increases.

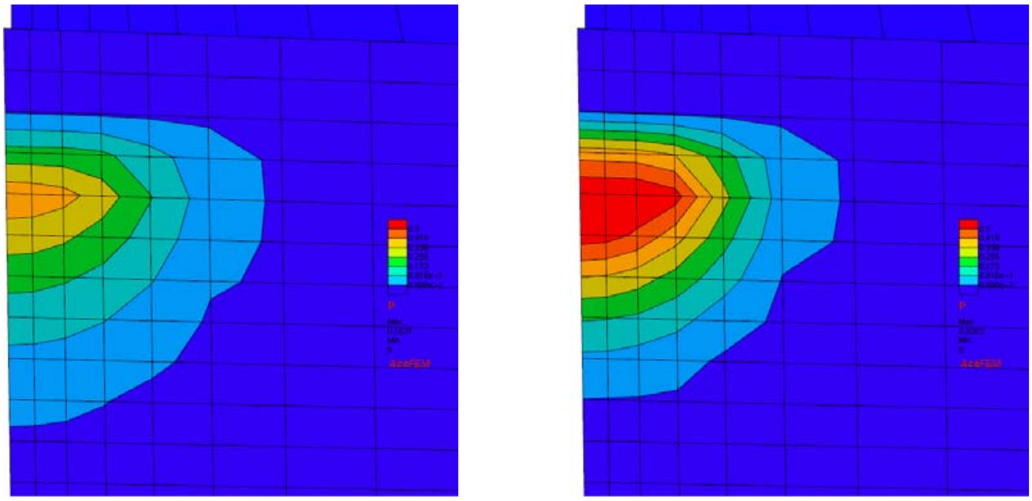

Figure 3: Accumulated plastic strain of raceway for the model without YPP (left) and the model with YPP (right) after 200 cycles

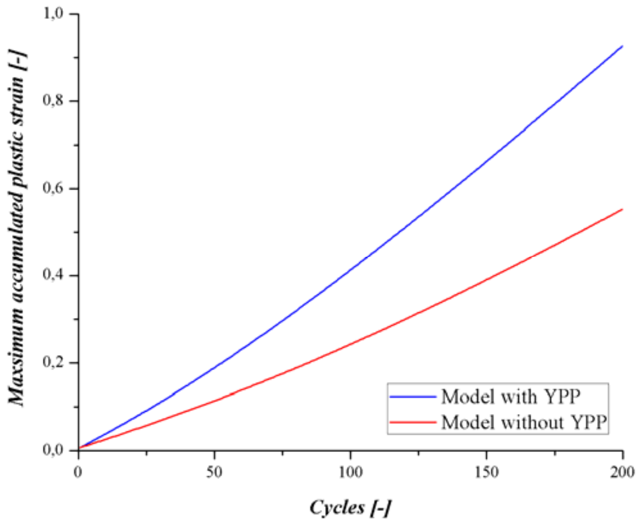

Figure 4: Maximum accumulated plastic strain of raceway

Increase of the maximum accumulated plastic strain during cyclic loading for a model with and model without the yield-point phenomenon is shown in Fig. 4. It can be seen that the maximum accumulated plastic strain increases much faster for the model with the yield-point phenomenon.

Fig. 5 shows a comparison of the equivalent von Mises stress of loaded raceway after 200 cycles. The simulation results on the left side of the figure show the equivalent von Mises stress calculated by the model without YPP and on the right side is the equivalent von Mises stress calculated by the model with YPP. Comparison of the simulation results shows that taking into account the yield-point phenomenon increases the equivalent von Mises stress on the boundary between elastic and plastic zone.

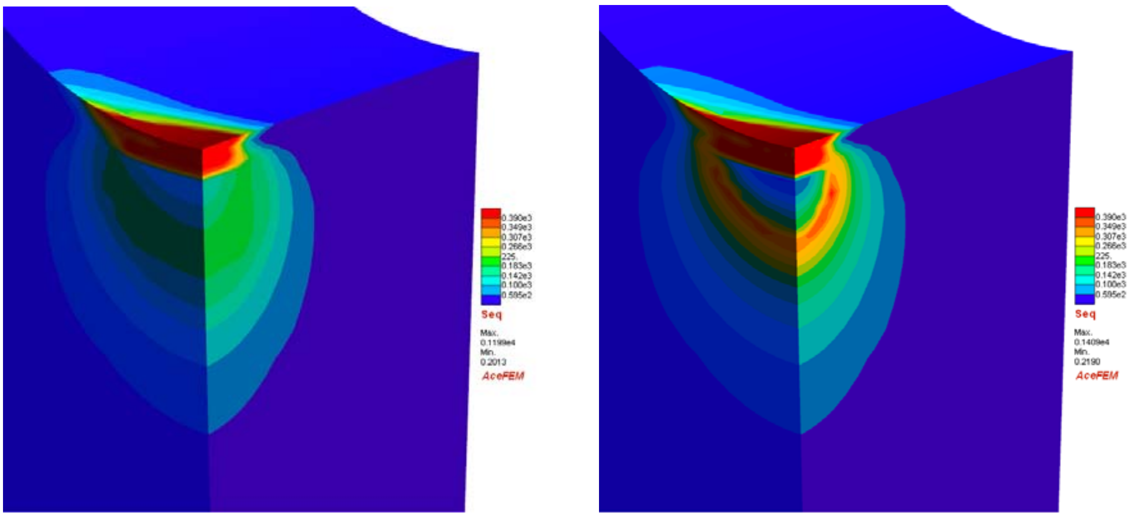

Figure 5: Equivalent stress of loaded raceway for the model without YPP (left) and the model with YPP (right) after 200 cycles

From numerical simulations it is evident that the model with the yield-point phenomenon evaluates a much faster increase of the maximum accumulated plastic strain during cyclic plasticity, which is significant for the forecast of the continuum damage growth and the lifetime of a product [13].

CONCLUSIONS

With the purpose of presenting the benefit of taking account of the yield-point phenomenon in cyclic plasticity, the finite element codes were generated with non-linear material models which provide different descriptions of the first transition from the elastic into the elasto-plastic region. The applied equations of isotropic and kinematic hardening or softening are the same. An FE model of the rolling contact was generated based on which numerical simulations of cyclic loading were carried out, by using the aforementioned models. Presented are the results of the stress state development and accumulated plastic strain of the material during cyclic loading. Since the hardened surface is too thin and the core soft, on the boundary between the hardened surface and the soft core of a bearing ring, local cyclic plasticity occurs, which can lead to material damage.

The results presented clearly indicate that taking account of the yield-point phenomenon during cyclic plasticity simulation reduces the size of the plastic strain zone and increases the accumulated plastic strain. Based on the results of uniaxial cyclic simulations a conclusion can be made that taking account of the yield-point phenomenon in constitutive models of cyclic plasticity also enables a more realistic evaluation of the size, location and accumulation of the plastic strain which is significant for the forecast of the continuum damage growth and the lifetime of a product.

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