An Extrapolation Method applied to a Constitutive Material Model to Recalculate Creep Fatigue Experiments

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ABSTRACT. *The cyclic behaviour of steam turbine components is currently a key issue in terms of the discontinuity in the power supply from regenerative energy systems like wind-mills. These systems call for a combined and flexible use of conventional steam power plants. The influence on fatigue behaviour with superimposed creep on lifetime of large plant components is still unknown. Fatigue loading of such components is induced by temperature transients during start-up and shut-down processes. This requires an anisothermal inspection of the thermo-mechanical fatigue (TMF) loading at design. Hence a viscoplastic constitutive material model of type Chaboche was adapted to a 10%Cr forged steel. As a verification of the constitutive material model, uniaxial and biaxial TMF experiments were performed. These experiments were derived from a temperature cycle on the surface of a turbine rotor during a hot start. In order to recalculate deformation and to estimate lifetime the constitutive material model was implemented in a FEM application. The constitutive material model of type Chaboche introduced here demonstrates the applicability for the recalculation of multiaxial deformation with implicit evolution of damage under service-type conditions. Further efforts are necessary for the investigation of innovative parameter identification methods and routines for the extrapolation of the inner variables of the constitutive material model under cyclic loading.*

INTRODUCTION

High temperature transients at start-up and shut-down processes lead to multiaxial strains and stresses at the heated surface of rotors (Figure 1a). The difference between the surface temperature and the average core temperature causes compression strains during the start-up (Figure 1b), while at shut-down tension strains occur. At stationary service condition of the turbine temperature equibalance usually leads to tension stresses which relax during long hold times. Notches in component structure lead to superimposed plastic strains and contribute to a decrease of lifetime.

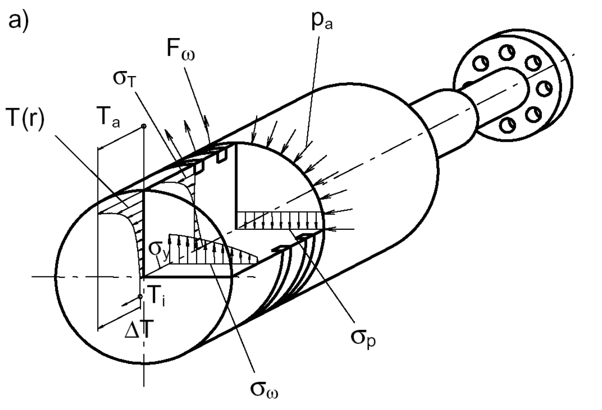
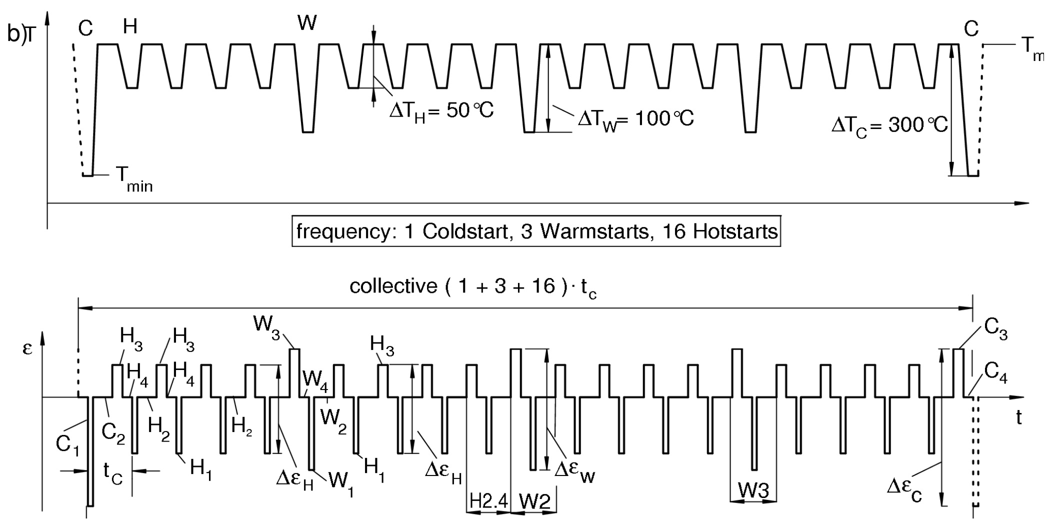


Figure 1 Loading scenario of a steam turbine rotor according to primary loading caused by pressure p_a and rotation ω, secondary loading caused by temperature change ΔT, schematic (a), temperature at the surface T and mechanical strain ε during a loading sequence of cold, warm and hot starts (b), p_a steam pressure at the surface, F_ω centrifugal blade force, σ_p stress caused by pressure, σ_ω stress caused by centrifugal forces, T_a metal temperature at the surface, T(r) temperature profile in the cross-section of the core, σ_T stress caused by constrained thermal expansion



MATERIAL

The examined 10%Cr-steel (Table 1) of type 10CrMoWVNbN represents a modern ferritic-martensitic steel which is suitable for high temperature applications up to 600°C. The basic material characterization was investigated earlier in [1], which demonstrates the complex creep deformation and relaxation behaviour.

Table 1. Chemical composition and heat treatment of a 10%Cr-steel

10CrMoWVNbN	C	Cr	Mo	W	Ni	V	Nb	N
weight %	0.12	10.0	1.0	1.0	0.8	0.2	0.05	0.05
heat treatment	1050°C 7h / oil + 570°C 10.25h / air + 690°C 10h / air							

CONSTITUTIVE MATERIAL MODEL

The viscoplastic constitutive material model of Type Chaboche (Table 2) enables the description of viscoplastic material behaviour for small deformations, which was introduced by Tsakmakis [2]. A key feature is the combination of the effective stress and strain, which represents the damaged material by the generalised energy equivalence principle [2].
An undamaged fictitious material is described by means of effective variables which are the basis of the constitutive material model. A damage variable D is defined with an approach proposed by Lemaitre [3] additionally to another set of variables for the damaged real material. The known behaviour of the undamaged fictitious material is then mapped to the unknown behaviour of the real material with damage. This step is done by substitution of the effective variables using relations which implicate the damage variable D.
The material model for a damaged material given by Table 2 describes a linear combination of the strain tensor **E** with an elastic part $\mathbf{E}_e$ and plastic part $\mathbf{E}_p$ (eq. 1). The stress tensor **T** is associated with the elastic strain Tensor by Hooke's law (eq. 2). Kinematic hardening $\boldsymbol{\xi}$ and isotropic hardening R are considered in eqn. 3 and 4. The equilibrium stress f is given by the von Mises norm of the difference between stress and kinematic hardening tensor (eq. 5). The evolution of the strain valued kinematic hardening **Y** and isotropic hardening r, consider a term of dynamic and static recovery (eqn. 9 and 10). Further, the function B(s) describes the cyclic softening or hardening (eq. 9) by governing the term of dynamic recovery. Cyclic softening is described with values of B(s) > 1 and cyclic hardening with values of B(s) < 1. The viscous overstress F represents the difference between the equilibrium stress and the material resistance (eq. 6). It is assumed that the function g(D) (eq. 6) depends only on the damage variable D. The evolution of the plastic arc length s (eq. 7) is given by a modified Norton approach. This modification deals with a Norton exponent m_1 for overstresses $F < F_{12}$ and $m_1 + m_2$ for overstresses $F_{12} < F$. The scalar damage variable D is splitted into two parts representing fatigue damage D_f and creep damage D_c (eq. 11). While fatigue damage increases linear, with the plastic arc length (eq. 12), the creep damage follows the evolution proposed by Rabotnov [3] (eq. 13). Here, the evolution of the creep damage is governed by the stress valued variable σ^*, which is derived from the modified strain-energy release rate Y*. The variable σ^* represents a scalar combination of the deviatoric and hydrostatic parts of the stress condition.

Table 2. Constitutive material model of type Chaboche [2], $\mathbf{A}^D$ deviatoric part of tensor **A**, $\mathbf{A}^T$ transpose of **A**, tr(**A**) trace of **A** and **A**·**B** scalar product of tensors **A** and **B**

1	$\mathbf{E} = \mathbf{E}_e + \mathbf{E}_p$
2	$\mathbf{T} = (1-D)\cdot \boldsymbol{C}[\mathbf{E}_e]$ and $\boldsymbol{C} = 2\mu \cdot \boldsymbol{I} + \lambda \cdot \mathbf{1} \otimes \mathbf{1}$
3	$\xi = (1-D)\cdot c\mathbf{Y}$
4	$R = (1-D)\cdot \gamma(r_0 + r)$

5	$f = \sqrt{\frac{3}{2} \frac{(\mathbf{T}-\xi)^D}{\sqrt{1-D}} \cdot \frac{(\mathbf{T}-\xi)^D}{\sqrt{1-D}}}$
6	$F = g \cdot f - g \cdot \frac{R}{\sqrt{1-D}} - k_0$ and $g := g(D) = \frac{\sqrt{1-D}}{(1-D)^n}$
7	$\dot{s} = \frac{\langle F \rangle^{m_1}}{\eta} \cdot (1 + \langle \langle F \rangle / F_{12} - 1 \rangle)^{m_2}$
8	$\dot{\mathbf{E}}_p = \frac{3}{2} \frac{(\mathbf{T}-\xi)^D}{\sqrt{1-D} \cdot f} \dot{s}$
9	$\dot{\mathbf{Y}} = \dot{\mathbf{E}}_p - B \cdot b\sqrt{1-D}\mathbf{Y}\dot{s} - p\left\| c\sqrt{1-D}\mathbf{Y}\right\|^{w-1} \frac{\mathbf{Y}}{\chi}$ and $\chi := \chi(D) = (1-D)^l$, $B(s) = B_1 + (1-B_1)e^{-B_2 \cdot s}$
10	$\dot{r} = \dot{s} - \beta\sqrt{1-D}r\dot{s} - \pi(\gamma\sqrt{1-D}r)^{\omega-1} \frac{r}{\chi}$
11	$\dot{D} = \dot{D}_f + \dot{D}_c$
12	$\dot{D}_f = \dot{s} / A_f$
13	$\dot{D}_c = (\sigma^* / A_c)^{k_c} \cdot (1-D)^{-r_c}$ with $\sigma^* = (1-D) \cdot \sqrt{2 \cdot E \cdot -Y}$, $-Y^* = -\rho \frac{\partial \psi_e}{\partial D} \cdot \frac{1 + a_c \cdot (R_\nu - 1)}{R_\nu} - \rho \frac{\partial \psi_p}{\partial D}$ and $R_\nu = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_V}\right)^2$
14	Material parameter: Deformation (11 + 9): E, (ν = 0,3), k_0, η, m_1, m_2, F_{12}, c, b, B_1, B_2, p, (w = 1, γ, β, π, ω, r_0, n = 1, l = 0) / Damage (5): A_f, A_c, k_c, r_c, a_c

PARAMETER IDENTIFICATION

Within the research work on creep-fatigue [5], the material parameters of the constitutive model were determined by a two-step approach with a combination of the Neural Networks method (Figure 2) and the optimisation method by Nelder-Mead. The Neural Networks method is already established in similar non-linear problems and can deliver a "global" solution. The method by Nelder-Mead is a direct search method without the need of numerical or analytical gradients and leads only to a "local" solution. This method is commonly referred to as unconstrained non-linear optimisation. In the first step, Neural Network identifies a parameter vector close to the global solution within a parameter interval. This result is used subsequently in the second step as an initial parameter vector in the Nelder-Mead method for further improvement of the solution. In order to identify the parameters of the subjected model for 1%CrMoNiV steel by means of Neural Networks, 1D calculations depending on parameter variations were performed [5].

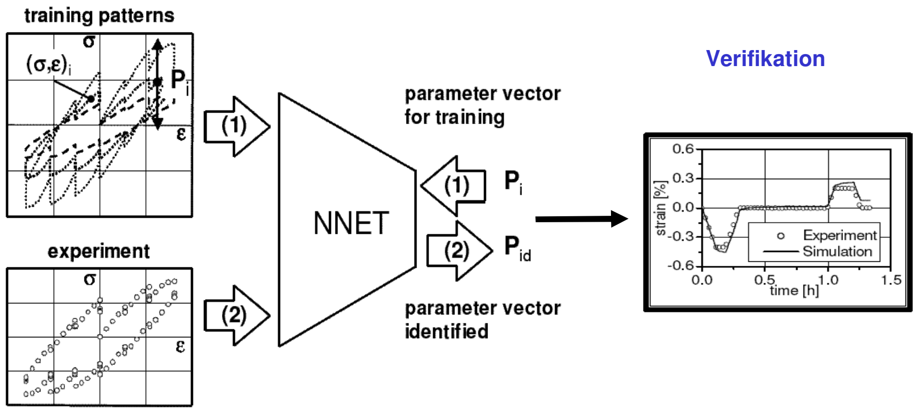


Figure 2. Two step procedure to identify material parameters by Neuronal Network Method

In terms of parameter identification, uniaxial data from tensile tests, creep tests and low cycle fatigue tests [5] were used. In the first step the parameters describing the deformation (k_0, η, m_1, m_2, F_{12}, c, b, B_1, B_2, p) are inserted into the tensile data, up to a value of total strain of 1.5% and the creep data of primary creep, secondary creep and minimum creep rate (Figure 3 a, b and c).

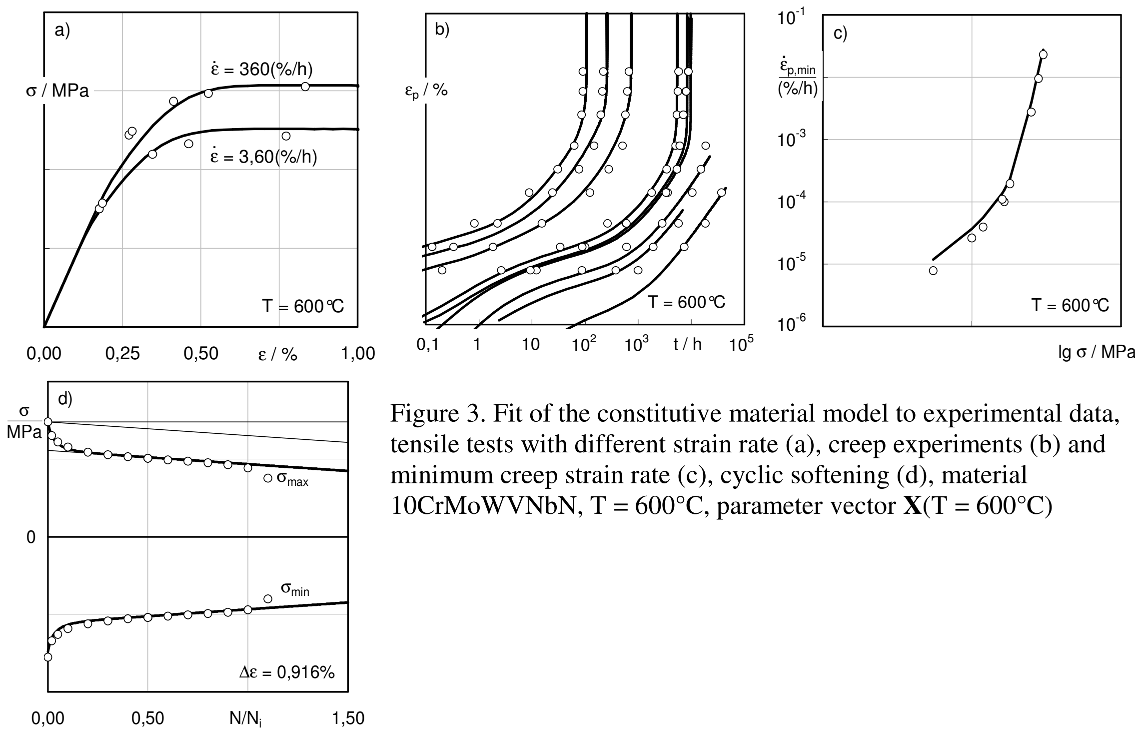


Figure 3. Fit of the constitutive material model to experimental data, tensile tests with different strain rate (a), creep experiments (b) and minimum creep strain rate (c), cyclic softening (d), material 10CrMoWVNbN, T = 600°C, parameter vector **X**(T = 600°C)

The parameters B_1 and B_2 fit into the phase of exponential softening, at the beginning of low cycle fatigue tests. The next step deals with the parameters describing the creep damage (A_c, k_c, r_c), to the evolution of tertiary creep (Figure 3 b). With respect to fatigue, the parameter A_f describes damage evolution during linear cyclic softening at low cycle fatigue (Figure 3 d). The parameter a_c governs the sensitivity of the creep damage due to multiaxial states of stress. This parameter can be derived from creep tests of notched specimens with high hydrostatic stress values in the notched root. A detailed description of the approximation procedure of the material model to the experimental data and a verification is given in [6].At each temperature (T_i = (300), 550, 600 and 625°C) the material behaviour has been described by the constitutive material model with one optimal set of parameters given in the vector $\mathbf{X}_i = (k_0, \eta, m_1, m_2, F_{12}, c, b, B_1, B_2, p, A_f\ A_c, k_c, r_c, a_c)_i$. For anisothermal calculation the parameter vector $\mathbf{X}(T)$ at the temperature T has to be interpolated between the underlying data points (T_i, $\mathbf{X}(T_i)$). In conclusion, the description of the examined 10%Cr steel with the constitutive material model is appropriate and the specific high temperature effects of deformation and damage evolution are described sufficiently.

EXTRAPOLATION METHOD

Further with respect to the general goal in terms of creep fatigue lifetime calculation it is of interest to develop methods for the extrapolation of inner variables (e.g. eq. (3) and (4)) of the material model). Here the idea arises to use the information of current calculated stresses and strains and the corresponding values of the inner variables to conduct an extrapolation of number of cycles (Figure 4a). This extrapolation method contributes to a reduction of efforts for lifetime calculation of multiaxial loaded structures by Finite Element Method.

For monotone or constant loadings (e.g. creep load) it is typical that the inner variables alter monotonously for a wide range of time. Therefore the numeric integration of the evolution equations needs a small number of increments and the calculation time is shorted. Contrarily cyclic loadings cause much bigger changes of the inner variables during a single cycle. Some variables even change the algebraic sign during a cycle.

Looking at the inner variables of various cycles at periodic time points (e.g. the end of cycle), a monotonous behaviour can be observed.

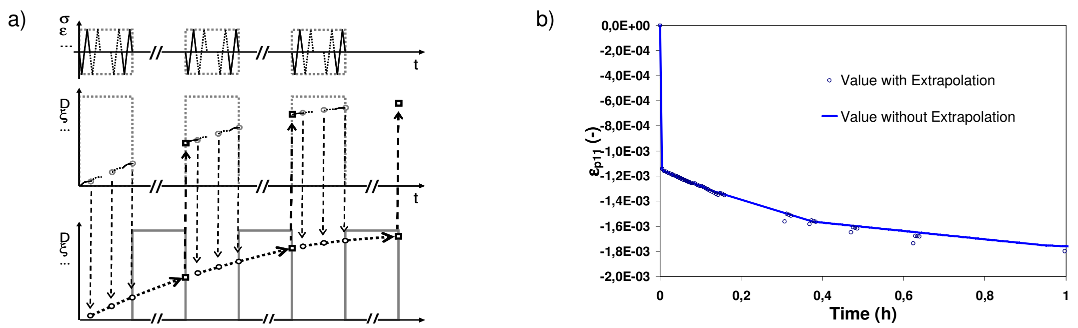


Figure 4. Principle of extrapolation method (a) and evolution of the inner variable ε_{p11}

In principle, this behaviour of the inner variables during the whole lifetime is addressed to three sections:

1) The initial phase is characterised by major changes of all inner variables.

2) In the following phase, the changes are very small. Some variables show appropriate constant behaviour. This phase takes up to 99% of the lifetime.
3) The final phase leads to the failure of the structure. In this phase, the damage variable D grows extremely due to the high nonlinearity of its evolution equation.

Here the idea arises to use numerical extrapolation methods, especially at the second phase, to save high amounts of calculation time (Figure 4 b).

The extrapolation of each inner variable is realised by a linear basis function with a predefined time increment using the integration results of the last three steps. The result of the extrapolation is used as initial condition for the integration of the next three steps of the evolution equations. This ensures that the equilibrium conditions of the material model are fulfilled.

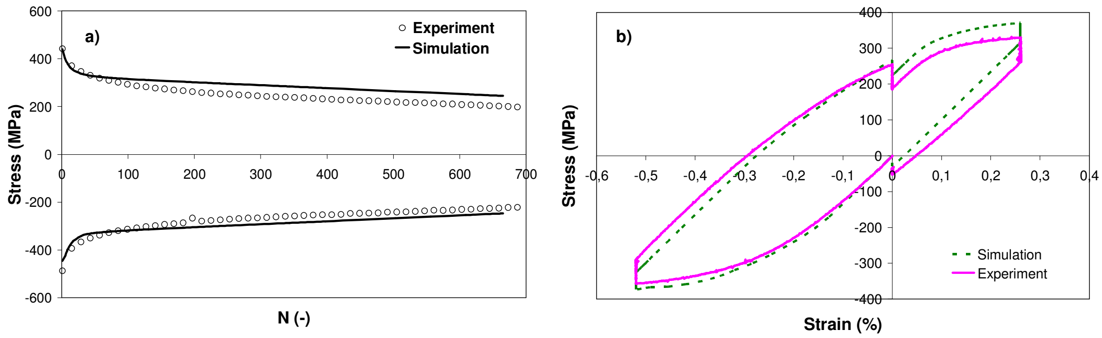


Figure 5. Recalculation of maximum stress values (a) and corresponding hysteresis loop (b)

Numerical studies show that polynomials of grade 2 or higher as basis functions do nott lead to more accurate results but to higher calculation times. The length of the extrapolated increment is calculated for each step. For this purpose, the inner variables are extrapolated backwards and are compared to the values of the last extrapolation step. If the differences in the values are higher than a predefined failure threshold, the step time is reduced. This extrapolation procedure is repeated stepwise until the damage variable D exceeds a predefined value (figure 5).

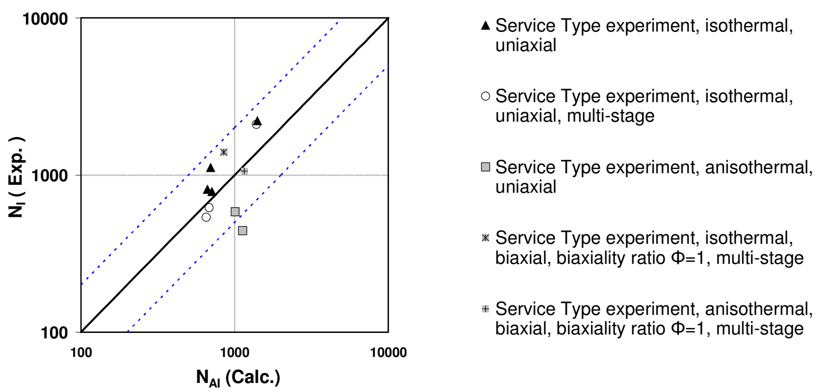


Figure 6. Recalculation of number of cycles to crack initiation

The damage variable D is derived from experimental exercises at complex multiaxial creep fatigue experiments [6]. On the underlying material a value of D=0.26 has been derived from creep-fatigue experiments. Recalculations of lifetime of various isothermal and anisothermal service-type loadings (e.g. Fig. 1) compared to experimental data (uniaxial and biaxial) show acceptable results (Figure 6).

CONCLUDING REMARKS

The recalculation of uniaxial and multiaxial service-type verification experiments partly under thermo-mechanical loading leads to satisfying results of the underlying material model. Deformation and fatigue life can be recalculated by a new extrapolation method which results in a reduction of computing time by a factor of one decade. For parameter identification, a new method using the Neural Network Method constitutes benefit for design purposes. Potential of improvement is addressed to the evolution of creep & fatigue and their complex superposition at thermo mechanical loading.

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REFERENCES

[1] Berger, C., Scholz, A. Müller, F., Schwienheer, M.: Creep fatigue behaviour and crack growth of steels, In: Abe, F., Kern, T.U., Viswanathan R. (Eds): Creep-resistant steels. (2008, First Ed.), Cambridge, Woodhead Publishing Limited, ISBN 978-1-84265-129-2, p. 446-471.

[2] Tsakmakis, C., Reckwerth, D.: The Principle of Generalized Energy Equivalence in Continuum Damage Mechanics, 2003, Deformation and Failure in Metallic Materials, Springer Verlag, ISBN: 3-540-00848-9

[3] Lemaitre J.: A course on damage mechanics (1996, Second Ed.), Berlin, Springer Verlag New York, ISBN: 3-540-60980-6

[4] F. Colombo, B. Masserey, E. Mazza and S.R. Holdsworth.: Simple modeling of the constitutive behaviour of a 1%CrMoV rotor steel in service-like thermo-mechanical fatigue tests, 2002, Mater. High Temp. 19(4), p. 225-233.

[5] Samir, A., A. Simon, A. Scholz and C. Berger: Service-type creep-fatigue experiments with cruciform specimens and modelling of deformation, International Journal of Fatigue, Volume: 28, Issue: 5-6, 2006, pp.643-651

[6] A. Simon, A. Scholz, C. Berger: Validation of a Constitutive Material Model with Anisothermal Uniaxial and Biaxial Experiments, MP Materials Testing, 09/2009, pp 532-541