A study on notch fatigue based on multiaxial gradient dependent fatigue limit criterion

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ABSTRACT: An analytical formula of K_F for U and V shaped blunt notches has been derived on the basis of a multiaxial gradient dependent fatigue criterion. The critical distance method has also been assessed. Both approaches have been applied in experimental data collected from the literature.

Keywords: Multiaxial fatigue, V-notches, gradient.

1. INTRODUCTION

The fatigue assessment of notched components under uniaxial remote cyclic loading is traditionally based on the concept of the fatigue strength reduction factor, denoted as K_F. Many empirical formulas for K_F have been proposed over decades of research [2-3]. However, none of these approaches takes into account the multiaxiality of the stress state usually occurring around a notch root even under remote uniaxial loading.
More recent approaches in this field address the problem of the stress multiaxiality; see for instance the revival of the critical distance approach in a recent paper by Susmel [4] or the thorough work of Taylor [5]. The scope of the present work is to propose an approach to assess the fatigue limit of notched components through the application of a gradient dependent criterion. In particular an analytical formula of the fatigue notch factor K_F for plane specimens with V and crack like U blunt notches submitted to remote axial load is derived. A comparative analysis of the predictions obtained by the proposed approach and by the critical distance method [4], against experimental results collected from the literature, is also provided.

2. GRADIENT DEPENDENT MULTIAXIAL FATIGUE LIMIT CRITERION

The following multiaxial fatigue limit criterion is introduced to assess the fatigue strength:

$$\max \tau_a + \alpha \sigma_{H,\max} - \beta G \leq \gamma \qquad (1)$$

According to the above formula, the fatigue strength for infinite life is determined by the maximum shear stress amplitude, $\max\tau_a$, corrected for the effect of the maximum hydrostatic stress $\sigma_{H,max}$ occurring in a load cycle and for the effect of the stress gradient. In Eq.(1), the reasoning developed by Papadopoulos and Panoskaltsis [6], is also adopted. Hence, the experimentally observed beneficial effect of the stress gradient is solely attributed to the gradient of the hydrostatic stress:

$$\mathbf{G} = \left[\frac{\partial \sigma_H}{\partial x}, \frac{\partial \sigma_H}{\partial y}, \frac{\partial \sigma_H}{\partial z} \right] \tag{2}$$

The quantity G appearing in the criterion Eq.(1) is the modulus of the above vector **G**, calculated at the moment at which the hydrostatic stress reaches its maximum. The material parameters α and γ, appearing in Eq.(1), can be determined through the application of the fatigue limit criterion in the case of smooth specimens loaded in two different stress states, e.g. fully-reversed tension-compression and fully reversed torsion:

$$\alpha = 3\left(t_{-1} / s_{-1} - 1/2\right), \qquad \gamma = t_{-1} \tag{3}$$

The determination of β requires test results from stress states where a gradient of the hydrostatic stress is present. For instance, one can use fatigue limits obtained on smooth cylindrical specimens of different section radii, submitted to four point fully reversed bending. In this case the criterion Eq.(1) leads to the following formula for the fatigue limit f_{-1} of a specimen of radius R:

$$f_{-1}(R) = \frac{s_{-1}}{1 - \dfrac{\beta(s_{-1}/3t_{-1})}{R}} \tag{4}$$

The above relationship correctly captures the general trend of experimental results [1], i.e. f_{-1} is a decreasing function of the specimen radius R and tends asymptotically to the tension-compression limit for an infinite radius, $R\rightarrow\infty$. Eq. (4) can be used to fit four point bending fatigue limits allowing thus the identification of β. Optimization techniques applied to fatigue limit test data of notched specimens can also be used for the identification of parameter β.

3. STRESS FIELD AROUND THE NOTCH ROOT

The elastic stress field around the notch tip for U shaped crack-like notches under mode-I remote traction has been established by Creager and Paris [7] in the late sixties. More recently Lazzarin and Tovo [8] and Filippi, Lazzarin and Tovo [9] addressed the same problem for V-shaped blunt notches for mode-I and mode-II loading. The solution for mode-I loading will be used in the present work and is briefly reviewed here. The geometry of the notch is depicted in Fig. 1. The co-ordinate system has the *x*-axis directed along the bisector of the notch angle 2α and its origin is placed at a distance r_0 from the notch tip, as shown in Fig. 1. The distance r_0 varies as a function of the notch geometry and is related to both the notch tip radius ρ and the opening angle 2α, Fig. 1:

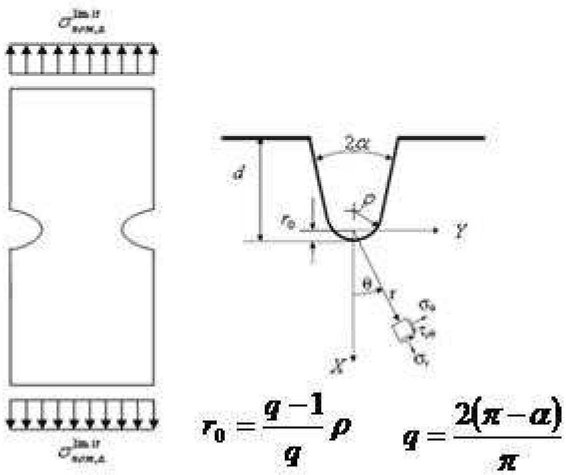

Figure 1: Notch geometry and associated co-ordinate axes

Following [9] the stress field ahead of a V-shaped blunt notch for mode-I remote traction is expressed as:

$$\begin{aligned}
\sigma_{rr} &= \lambda_1 r^{\lambda_1-1} a_1 \{ (3-\lambda_1)\cos(1-\lambda_1)\theta - \chi_{b1}(1-\lambda_1)\cos(1+\lambda_1)\theta + \\
&\quad + (r/r_0)^{\mu_1-\lambda_1} [q/4(q-1)][\chi_{d1}(3-\mu_1)\cos(1-\mu_1)\theta - \chi_{c1}\cos(1+\mu_1)\theta] \} \\
\sigma_{\theta\theta} &= \lambda_1 r^{\lambda_1-1} a_1 \{ (1+\lambda_1)\cos(1-\lambda_1)\theta + \chi_{b1}(1-\lambda_1)\cos(1+\lambda_1)\theta + \\
&\quad + (r/r_0)^{\mu_1-\lambda_1} [q/4(q-1)][\chi_{d1}(1+\mu_1)\cos(1-\mu_1)\theta + \chi_{c1}\cos(1+\mu_1)\theta] \} \\
\sigma_{r\theta} &= \lambda_1 r^{\lambda_1-1} a_1 \{ (1-\lambda_1)\sin(1-\lambda_1)\theta + \chi_{b1}(1-\lambda_1)\sin(1+\lambda_1)\theta + \\
&\quad + (r/r_0)^{\mu_1-\lambda_1} [q/4(q-1)][\chi_{d1}(1-\mu_1)\sin(1-\mu_1)\theta + \chi_{c1}\sin(1+\mu_1)\theta] \}
\end{aligned} \tag{4}$$

Table 1 summarises the values of exponent λ_1 and exponent μ_1 along with the values of the coefficients χ_{b1}, χ_{c1}, χ_{d1}, for three different values of the opening angle 2α, [9].

Table 1: Parameters λ_1, μ_1, χ_{b1}, χ_{c1}, χ_{d1} as a function of the opening angle of the notch

2α[deg]	q	r_0	λ_1	μ_1	χ_{b1}	χ_{c1}	χ_{d1}
0°	2	$\rho/2$	0.5	-0.5	1	4	0
60°	5/3	$2\rho/5$	0.512	-0.406	1.312	3.283	0.096
90°	3/2	$\rho/3$	0.544	-0.345	1.841	2.506	0.105

In the case of a U shaped notch, i.e. opening angle $2\alpha=0$, $r_0=\rho/2$, Eqs (4), coincides with the Creager-Paris solution of a crack-like blunt notch, provided that $a_1=K_I/\sqrt{2\pi}$, where K_I is the usual mode I stress intensity factor of a crack of length equal to the depth of the U-shaped notch. Hence, the parameter a_1 has the function of a stress intensity factor for V-shaped sharp notches. It is noted that the stress $\sigma_{\theta\theta}$, at the position ($r=r_0$, $\theta=0$), is the maximum local stress at the tip of the notch and is usually expressed as the product of the stress concentration factor K_t by the remote nominal stress σ_{nom}. Keeping this in mind and using the second of Eqs.(4) one obtains:

$$\sigma_{\theta\theta}(r_0,0)=K_t\sigma_{nom}\Rightarrow a_1=\frac{K_t\sigma_{nom}}{\lambda_1 r_0^{\lambda_1-1}\left[(1+\lambda_1)+\chi_{b1}(1-\lambda_1)+\frac{q}{4(q-1)}(\chi_{d1}(1+\mu_1)+\chi_{c1})\right]} \tag{5}$$

Clearly, for a given V-shaped blunt notch the above relationship allows to determine the stress intensity factor a_1, provided K_t of the given notch is known.

4. A NEW FATIGUE STRENGTH REDUCTION FACTOR FORMULA

A new formula of K_F for blunt V- and U-shaped notched specimens loaded under a remote axial stress amplitude, is here derived from the gradient dependent multiaxial fatigue criterion, Eq.(1).

The instantaneous maximum shear stress at a position (r,θ), denoted as $\tau(r,\theta,t)$, can be derived from the stress field, Eqs.(4), as the semi-difference of the maximum and minimum principal stresses. Moreover, since the loading considered is proportional, the maximum shear stress amplitude at the position (r,θ) is equal to:

$$\tau_a(r,\theta)=a_{1,a}\lambda_1 r^{\lambda_1-1}\left\{(1-\lambda_1)^2\left[1+\chi_{b1}^2-2\chi_{b1}\cos 2\theta\right]+\frac{q^2}{16(q-1)^2}\left[\chi_{d1}^2(1-\mu_1)^2-2\chi_{c1}\chi_{d1}(1-\mu_1)\cos 2\theta+\chi_{c1}^2\right]\left(\frac{r}{r_0}\right)^{2(\mu_1-\lambda_1)}+\right.$$

$$\left.+\frac{q(1-\lambda_1)}{2(q-1)}\left[(\chi_{d1}-\mu_1\chi_{d1}+\chi_{b1}\chi_{c1})\cos(\mu_1-\lambda_1)\theta-\chi_{c1}\cos(2+\mu_1-\lambda_1)\theta-\chi_{b1}\chi_{d1}(1-\mu_1)\cos(2+\lambda_1-\mu_1)\theta\right]\left(\frac{r}{r_0}\right)^{\mu_1-\lambda_1}\right\}^{1/2} \tag{6}$$

The position (r,θ) where $\tau_a(r,\theta,t)$ reaches its maximum value, $\max\tau_a$, turns out to be $r=r_0$ and $\theta=0$:

$$\max\tau_a=\lambda_1 r_0^{\lambda_1-1}a_{1,a}\left[(1-\lambda_1)(\chi_{b1}-1)+[q/4(q-1)][\chi_{c1}-(1-\mu_1)\chi_{d1}]\right] \tag{7}$$

Upon introducing in the above relation the value of the stress intensity factor a_1 from Eq.(5) one has:

$$\max\tau_a=\frac{\left[(1-\lambda_1)(\chi_{b1}-1)+[q/4(q-1)][\chi_{c1}-(1-\mu_1)\chi_{d1}]\right]}{\left[(1+\lambda_1)+(1-\lambda_1)\chi_{b1}+[q/4(q-1)][\chi_{d1}(1+\mu_1)+\chi_{c1}]\right]}K_T\sigma_{nom,a} \tag{8}$$

The hydrostatic stress at any point (r,θ) in the neighbourhood of the notch tip and time t during a loading cycle is:

$$\sigma_H(r,\theta,t) = (1/3)\left\{\lambda_1 r^{\lambda_1-1} a_1 [4+[q/(q-1)](r/r_0)^{\mu_1-\lambda_1}\chi_{d1}]\cos(1-\lambda_1)\theta\right\} \qquad (9)$$

Introducing in the above relation a_1 from Eq.(5), the maximum value of the hydrostatic stress in a loading cycle at the position ($r=r_0$, θ=0), where $max\tau_a$ occurs, is equal to:

$$\sigma_{H,\max}(r{=}r_0,\theta{=}0) = \frac{[4+q\chi_{d1}/(q-1)]}{\left\{(1+\lambda_1)+(1-\lambda_1)\chi_{b1}+[q/4(q-1)][\chi_{d1}(1+\mu_1)+\chi_{c1}]\right\}}\frac{K_T\sigma_{nom,a}}{3} \qquad (10)$$

The modulus of the gradient of σ_H at the notch root and at the instant of the loading cycle where the hydrostatic stress becomes maximum is:

$$G(r=r_0,\theta=0) = \frac{\lfloor 4(1-\lambda_1)+q\chi_{d1}(1-\mu_1)/(q-1)\rfloor}{\left\{(1+\lambda_1)+(1-\lambda_1)\chi_{b1}+[q/4(q-1)][\chi_{d1}(1+\mu_1)+\chi_{c1}]\right\}}\frac{K_T\sigma_{nom,a}}{3r_0} \qquad (11)$$

Let us remind that K_F is defined as the ratio of the smooth specimen fatigue limit over the nominal fatigue limit of the notched specimen. Keeping this in mind and introducing in Eq.(1) the quantities calculated in Eqs.(8), (10), (11) along with the values of the parameters α and γ from Eq.(3), one derives the following analytical expression of the fatigue notch factor K_F for blunt V- and U-shaped notched specimens loaded under a remote axial stress amplitude:

$$K_f = \frac{s_{-1}}{\sigma_{nom,a}^{limit}} = K_t\left[\left(A-\frac{B}{2}\right)\frac{s_{-1}}{t_{-1}}+B-C\frac{\beta}{3\rho}\frac{s_{-1}}{t_{-1}}\right] \qquad (12)$$

The constants A, B and C depend on the notched specimen geometry:

$$A=\{(1-\lambda_1)(\chi_{b1}-1)+q/[4(q-1)](\chi_{d1}(\mu_1-1)+\chi_{c1})\}/Y, \quad B=\{4+\chi_{d1}\,q/[4(q-1)]\}/Y$$
$$C=q\{4(1-\lambda_1)+q\chi_{d1}(1-\mu_1)/[4(q-1)]\}/[Y(q-1)] \qquad (13)$$
$$with \quad Y=\{(1+\lambda_1)+(1-\lambda_1)\chi_{b1}+[q/4(q-1)][\chi_{d1}(1+\mu_1)+\chi_{c1}]\}$$

The values of the constants A ,B, C for the opening angles reported in table 1 are summarised in table 2.

Table 2: Values of the constants A, B, and C for the opening angles considered

2α deg	A	B	C
0	0.5	1	1
60	0.5	0.957	1.2
90	0.5	0.945	1.342

It is of interest to notice that the K_F formula, Eq.(12) achieves the following simple form for U-shaped blunt notches:

$$K_F = K_T\left[1-(\beta/3\rho)(s_{-1}/t_{-1})\right] \quad (14)$$

5. COMPARISON WITH EXPERIMENTAL DATA AND DISCUSSION

The theoretical formula of K_F, Eq.(12), has been used to analyse experimental results for V shaped blunt notches [10-11] . The results are provided as (ρ, K_F) couples. The critical distance approach [4], both point and line methods, has also be applied in the same test data.

The material of the notched specimens was a 0.22% C steel with ultimate tensile strength R_M=432 MPa and tension-compression (R=-1) fatigue limit s_{-1}=201 MPa. The theoretical stress concentration factor K_T has been calculated with Peterson's analytical expression [3]. The geometry of the notched specimens is reported in Table 3.

Table 3: V-shaped notched specimens geometries [10-11]

notch radius, ρ[mm]	notch depth [mm]	2α [deg]	gross section width [mm]
7.62	5.08	55	63.5
1.27	5.08	55	63.5
0.51	5.08	55	63.5
0.25	5.08	55	63.5

In Fig(2) the analysis of the test data with the proposed K_F formula is shown. In Figs (3) and (4) the test data have been analysed by the point and line methods respectively, of the critical distance approach.

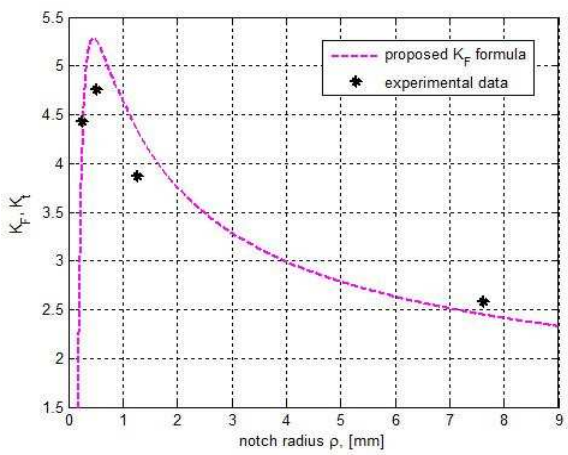

Figure 2: Test data analysed by the K_F formula based on the gradient dependent fatigue criterion

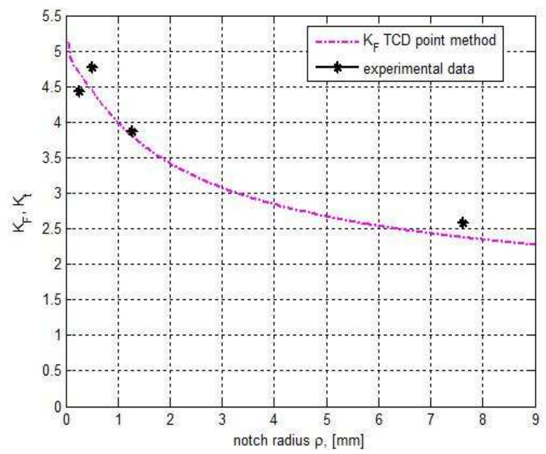

Figure 3: Test data analysed by the critical distance approach (point method)

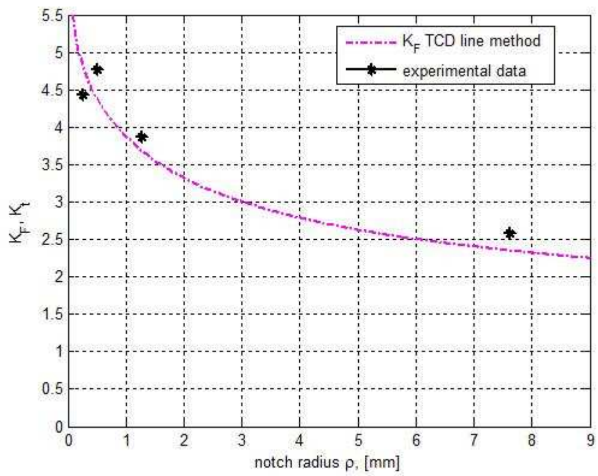

Figure 4: Test data analysed by the critical distance approach (line method)

Examining Figures (2), (3) and (4) it appears that both the proposed approach and the critical distance method interpolate rather well the experimental points. However, it is noticed that the critical distance proposals predict a continuous increasing K_F as ρ decreases, Figs (3) and (4), whereas from the test results the point most on the left corresponding to the smallest ρ exhibits a lower K_F with the respect to the second point from left which corresponds to a higher ρ. Observing Fig.(2) it seems that the proposed approach is able to capture this behaviour, since the predicted theoretical evolution of K_F against the notch tip radius, exhibits a local maximum at a small value of ρ, behind which K_F decreases. Such a finite value maximum of the K_F-ρ curve has already been reported by some researchers [12-13] and could be captured by the application of a gradient dependent fatigue criterion at the stress field around the notch tip, as has been shown in the present work.

6. REFERENCES

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