A Multiaxial Fatigue Approach to Estimate Crack Initiation and Stress Gradient Effect in Fretting

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ABSTRACT. *Fretting tests are performed using a cylinder plane contact configuration under partial slip to investigate the stress gradient effect on crack initiation. Three different cylinder radii are tests to sweep a large range of stress gradient. The experimental results confirm the stress gradient effect on crack initiation. Three different crack initiation boundaries are identified for each contact configuration. The smaller the cylinder radius, the higher the stress gradient. A non local multiaxial fatigue approach is introduced to estimate the stress gradient effect observed experimentally. The proposal is based on the Crossland criterion and a weight function depending on the stress gradient. The proposal compared to other non local approaches already applied to fretting show good capabilities to estimate the stress gradient effect in a general multiaxial fatigue framework.*

INTRODUCTION

The lifetime computation in fatigue is well adressed by multiaxial fatigue criteria in case of smooth specimens. Reliable multiaxial fatigue criteria [1] have already been proposed and validated on smooth specimens under multiaxial fatigue loadings. However, the same approaches applied to components featuring stress gradients due to either notches or contacts in fretting tests produce conservative results. This is known as the stress gradient effect which is likely due to the fail of the homogenization assumption in case of stress gradient due to the crystalline structure of metallic materials [2].

This paper aims to study the stress gradient effect on crack initiation, especially in the field of fretting. Fretting is characterized by small oscillatory motion between two parts in contact resulting in either earlier crack initiation or wear depending on the displacement amplitude. This paper only focussed on small oscillatory motions resulting in cracking. The sliding regime associated to cracking is called partial slip.

First, results of fretting experiments on Inconel 718 using a cylinder plane contact configuration under partial slip condition are introduced. Three cylinder radii are tested to sweep a large range of stress gradients. The crack initiation conditions as a function of loading conditions are identified for each configuration.

Next, a proposal based on a multiaxial fatigue approach is developed. The approach is based on the Crossland multiaxial fatigue criterion modified with a weight function depending on the stress gradient. It is aimed to propose a model able to compute the stress gradient effect on crack initiation observed experimentally using a general multiaxial fatigue framework in order to apply in next studies this model on various applications.

STRESS FIELD DESCRIPTION IN FRETTING

This part consists in a short description of the stress field induced by a cylinder plane contact under varying tangential load remaining in partial slip condition. The subsurface stress field is fully described by the surface tractions p and q, respectively the surface pressure and the shear surface traction. This problem was solved by Mindlin [5] and Cattaneo [6] simultaneously. The Figure 1 shows some key parameters of a cylinder plane contact under varying tangential load.

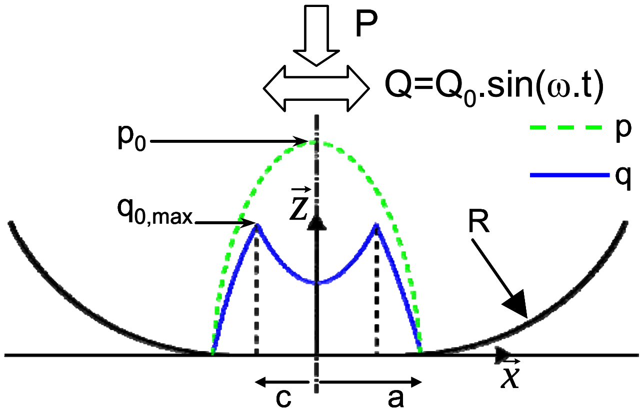

Figure 1: Surface traction distribution p and q for $Q(t)=Q_0$ in a cylinder plane contact and definition of the maximal pressure p_0, the maximal surface shear traction $q_{0,max}$, the semi contact width a and the semi sticking zone width c.

Next, the subsurface stress field is computed using equations available in [4]. These computations enable a short study of the stress gradient induced by a cylinder plane contact under partial slip. The gradient of two usual parameters in the field of multiaxial fatigue i.e. $\sigma_{H,max}$ and $\sqrt{J_{2,a}}$ are illustrated in Figure 2. First, it shows that the stress gradients for the cylinder plane contact are very high, compared for instance to those encountered in bending **[3]**.

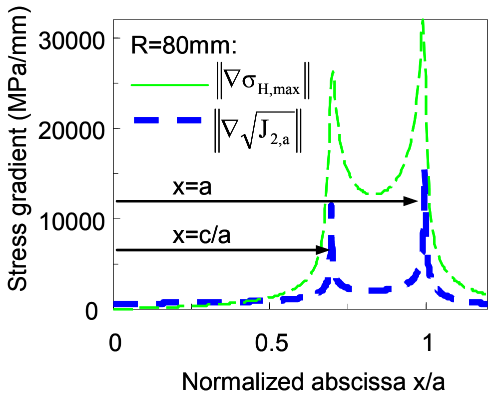

Figure 2: Comparison of the gradients of $\sigma_{H,max}$ and $\sqrt{J_{2,a}}$ for a cylinder plane contact using two cylinder radii.

The proportionality of the stress field in a cylinder-plane is studied through the comparison of the Crosland and the Papadopoulos criteria (Figure 3). It appears that the non proportionality of the stress field is localized in the sliding zone. However it is worth pointing out that the maximum value of the criteria are the same and are located at the border of the contact (x=a). The proportionality of the stress field is also investigated using the ratio $\left|\frac{\sqrt{J_{2,a}}-\sqrt{\langle T_a^2\rangle}}{\sqrt{J_{2,a}}}\right|$. The spatial distribution of the amount is shown in Figure 4. It appears that the ratio is up to 20%. Moreover, adding to the previous fact illustrated in Figure 3, this difference is very localised. These two results explain the choice to use the Crossland criterion instead of the Papadopoulos criterion in the following since the proposal only use the maximum value of the multiaxial fatigue criterion. This allows a significant saving regarding computation costs.

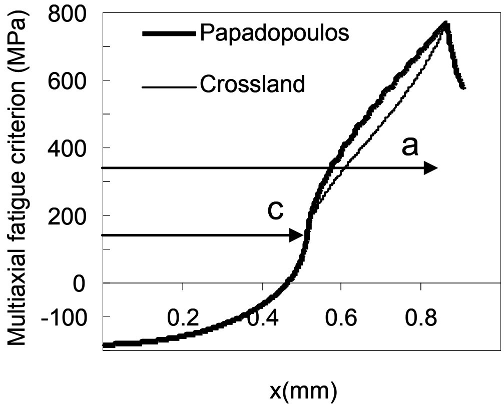

Figure 3: Comparison of the Crossland and the Papadopoulos distribution at the surface of the plane.

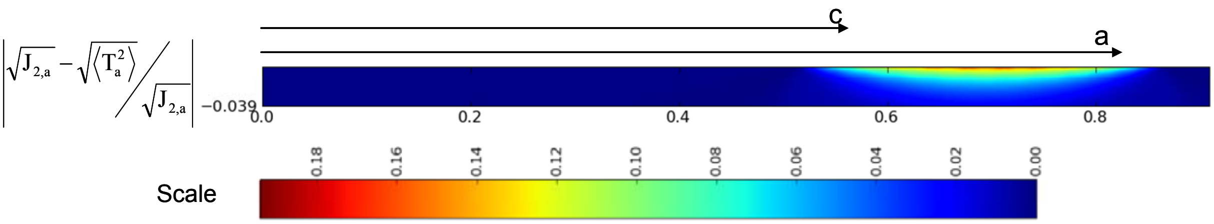

Figure 4: Estimation of the proportionality of the stress field induced by a cylinder plane contact under varying tangential load.

EXPERIMENTAL RESULTS

Fretting experiments on Inconel 718 are performed using an in-house set up mounted on a hydraulic fatigue test system. In order to identify the crack initiation conditions for each contact configuration, several tests with different tangential loads are performed for a given normal load to bracket the initiation condition. All tests are performed for 10^5 cycles. After each test, several cross sections of the plane are investigated using an optical microscope to check if cracks initiated or not. A crack is considered as initiated when it reaches a depth of 10 µm. This threshold is chosen in connection with the grain size of the Inconel 718. The smallest grains are about 10 µm. Thus, a crack is assumed to be initiated when a grain is fully failed. The results are introduced in Figure 5.

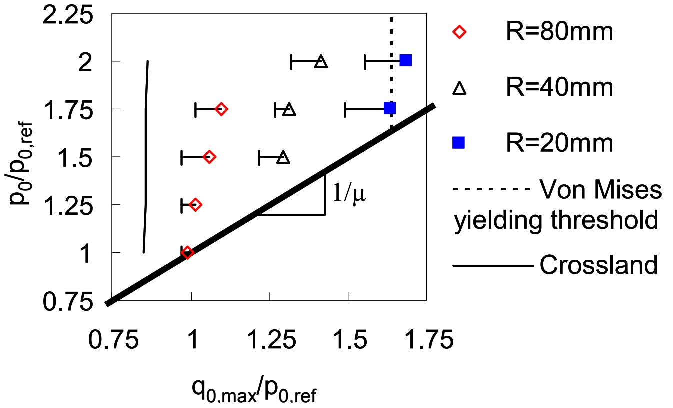

Figure 5: Experimental results showing crack initiation conditions for fretting tests under partial slip regime on Inconel 718 at 10^5 cycles. Three crack initiation boundaries are clearly identified for each contact configurations tested. The experimental results are compared to the computations done using the local Crossland multiaxial fatigue criterion.

The results clearly show the beneficial effect of the stress gradient since the smaller the cylinder radius, the higher the loads required to initiate a crack. It results that the error on the experimental results and the computations made using the Crossland criterion increases when the cylinder radius decreases i.e. when the stress gradient increases. Figure 6 shows another illustration of the experimental results which comes more within a multiaxial framework whereas the graphic on Figure 5 is specific to the fretting field. The error between the estimation of the multiaxial fatigue criterion and the experimental results is supposed to be explained by the stress gradient which results in the failed of the homogenization assumption. Therefore, the stress field computed under this assumption is no more representative of stresses bearing by the material close to high stress concentrations.

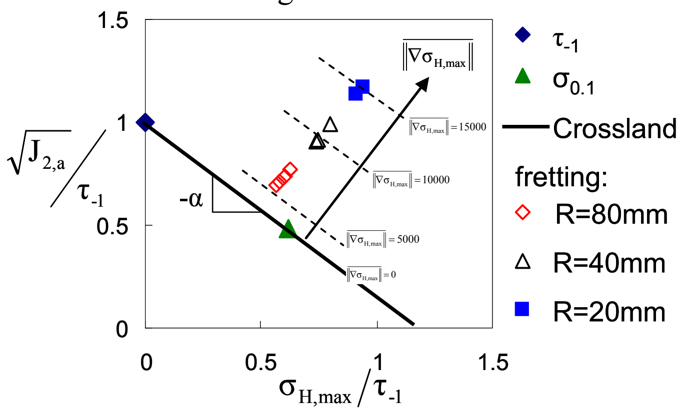

Figure 6: Experimental results plotted in a $\sqrt{J_{2,a}} - \sigma_{H,max}$ diagram compared to the fatigue strength computed using to the Crossland criterion.

A NON LOCAL APPROACH TO ACCOUNT FOR STRESS GRADIENT

The proposal of Proudhon et al. in [8] is faced to the previous experimental results. This approach called the "variable process volume" is an evolution of the basic approach consisting in computing the mean value of a multiaxial fatigue criterion over a basic shape representative usually associated to the grain [9]. The "variable process volume" is defined by:

$$d = \frac{h}{a - c} \tag{1}$$

where d is side width of the square where the multiaxial fatigue criterion is computed, a-c is the width of the sliding zone in partial slip, and h a constant. The width of the sliding zone a-c is taken as an input data in this model since the sliding zone width decreases when the stress gradient increases. Thus, this is a way to quantify the stress gradient.

The “variable process volume” produces good results since the error on experimental results is between -10% to +5%. The computed fretting map is plotted on Figure 7. The errors are computed using this relation:

$$E(\%) = \frac{q_{0,exp} - q_{0,num}}{q_{0,exp}} \times 100 \tag{2}$$

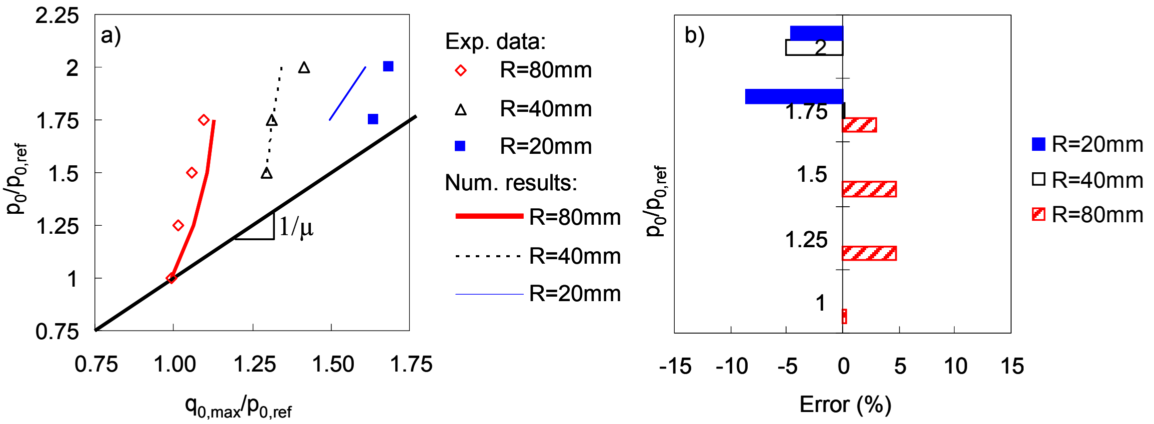

Figure 7: Variable process volume approach [8], h=6.2 a) Comparison between experimental and numerical initiation boundaries in a p_0-$q_{0,max}$ diagram. b) Error between experimental and numerical results.

However, this approach is very specific to fretting issue since it uses data specific to fretting i.e the width of the sliding zone. Therefore, such an approach cannot be applied to notched specimens or bending for instance. This leads to propose another approach able to deal with experimental results in a general fatigue framework.

The non local approach proposed to estimate the stress gradient effect observed in fretting is based on the Crossland multiaxial fatigue criterion. A weight function is applied to the multiaxial fatigue criterion which depends on the mean value of the gradient of the maximum value of the hydrostatic pressure over a period $\overline{\|\nabla\sigma_{H,max}\|}$. The mean value of $\|\nabla\sigma_{H,max}\|$ is computed over a volume corresponding approximately to the smallest grains found in the Inconel 718, i.e. about 10 µm width. Thus, the formulation is:

$$\max\left(\sqrt{J_{2,a}} + \alpha.\sigma_{H,max}\right) \times w = \beta \tag{3}$$

with $w = 1 - k.\overline{\|\nabla\sigma_{H,max}\|}$.

The shape of the weight function is found by plotting the ratio $w_{exp} = \dfrac{\max\left(\sqrt{J_{2,a}}_{\ exp} + \alpha.\sigma_{H,max\ exp}\right)}{\beta}$ against the mean value of the gradient of the hydrostatic pressure $\overline{\|\nabla\sigma_{H,max}\|}$ for each experimental condition inducing the crack initiation (Figure 8). It results that the gap between the Crossland criterion and the experimental results follows a linear relation of the mean value of the stress gradient. The error on the experimental results is between -5% to 5%. The proposal produces a better estimation of the crack initiation conditions than the previously introduced approach. Moreover it lies within a more general multiaxial fatigue framework.

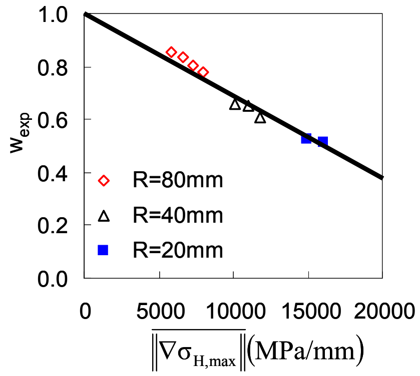

Figure 8: Evolution of the weight function w according to the mean stress gradient ($k=31.10^{-6}$)

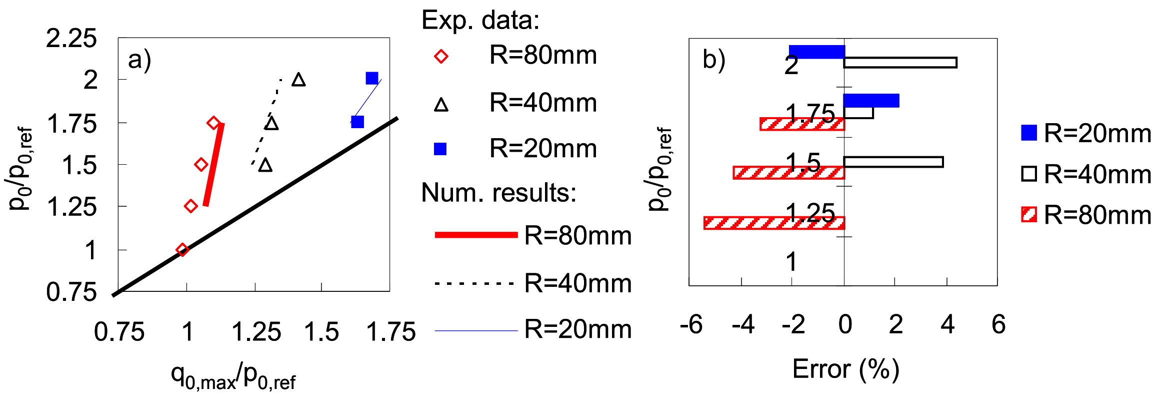

Figure 9: Proposal a) Comparison between experimental and numerical initiation boundaries in a p_0-$q_{0,max}$ diagram. b) Error between experimental and numerical results.

DISCUSSION AND OUTLOOK

The experimental results of fretting tests confirm the failure of the homogenization assumption in case of stress gradients. It results that the stress gradients have a beneficial impact on crack initiation. Thus, usual multiaxial fatigue criteria requires a modification to deal with the gap between the stress field computed based on the homogeneous assumption and those really faced by materials. A proposal is developed in a multiaxial fatigue framework using the Crossland multiaxial fatigue criterion and a weight function depending on stress gradients. The proposal show good capabilities to estimate the stress gradient effect observed in fretting. In order to check the robustness of the proposal, it should be applied to various applications like notched specimens for instance.

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