Multiaxial Life Estimations Based on Tensile Properties

N. Shamsaei[1] and S. A. McKelvey[2]

[1] Chrysler Group LLC, 800 Chrysler Dr, Auburn Hills, MI 48326, ns745@chrysler.com
[2] Altair Engineering, 1820 E. Big Beaver Rd., Troy, MI 48083, sm1453@chrysler.com

ABSTRACT. *This study aims to estimate fatigue life of steels under multiaxial loading based on commonly available tensile properties or hardness. Multiaxial fatigue models have been developed to predict fatigue behavior under multiaxial state of loading. These models relate multiaxial stress/strain components to uniaxial fatigue properties to predict fatigue life. Generating fatigue properties is a significant cost and time consuming process; hence, it would be beneficial to estimate fatigue properties for life prediction. In this study, Roessle-Fatemi, Muralidharan-Manson, and Bäumel-Seeger prediction methods are employed to predict uniaxial fatigue properties of steels based on usually available or easily measurable tensile properties in the absence of any fatigue data. Appropriate multiaxial fatigue models representing the damage mechanism are then used along with the estimated uniaxial fatigue properties to predict fatigue lives under in-phase and out-of-phase multiaxial loading.*

INTRODUCTION

Multiaxial loading can be categorized as in-phase (IP) and out-of-phase (OP) loading. For in-phase loading, the ratio of torsion to axial loading and principal directions remain fixed. However, under out-of-phase loading, principal directions and consequently maximum shear directions rotate in time.

Fatigue lives under out-of-phase loading are usually shorter than in-phase loading with the same equivalent strain level. Kanazawa et al. [1] related the shorter fatigue lives under out-of-phase (non-proportional) loading to the non-proportional cyclic hardening phenomenon caused by the change in slip plane from one crystallographic slip system to another resulting from the rotation of maximum shear plane under non-proportional loading. The intersection of active slip systems then may cause an additional hardening under non-proportional cyclic loading.

Multiaxial fatigue models should be used to relate multiaxial state of loading to uniaxial fatigue properties. Classical models were first proposed in the early twentieth century for yielding under static or monotonic loading. These hypotheses were later extended to cyclic loading and fatigue strength. However, these models may only work for proportional or in-phase loading. For the case of non-proportional or out-of-phase loadings, using classical models often leads to significant errors.

Critical plane models which reflect the damage mechanism on the specific critical plane(s) within the material have been studied over the last few decades [2]. These models can be used for fatigue life estimations under both IP and OP loading and also for predicting the direction of crack initiation. Strain-stress-based models include both a strain component as the driving parameter and a secondary stress component taking into account the cyclic hardening due to non-proportionality of loading as well as mean and residual stresses. Smith-Watson-Topper (SWT) [3] and Fatemi-Socie (FS) [4] damage parameters are two examples of strain-stress-based critical plane approaches for tensile and shear failure mode materials, respectively.

The SWT critical plane model for tensile failure mode materials considers the maximum principal strain amplitude, $\Delta\varepsilon_1$ /2, as the primary parameter driving the crack and the maximum stress on the principal plane , $\sigma_1^{\max}$, as the secondary parameter opening the crack and expediting the failure process if tensile, as presented below:

$$\sigma_1^{\max}\frac{\Delta\varepsilon_1}{2}=\frac{\sigma_f'^{\,2}}{E}(2N_f)^{2b}+\sigma_f'\varepsilon_f'(2N_f)^{b+c} \tag{1}$$

where E is modulus of elasticity, $2N_f$ is the number of reversals to failure, and σ_f', ε_f', b, c are the uniaxial strain-life fatigue properties.

The FS critical plane model for shear failure mode materials is expressed as a function of maximum shear strain amplitude, $\Delta\gamma_{\max}/2$, as the primary parameter driving the crack and maximum normal stress acting on the maximum shear strain plane, $\sigma_n^{\max}$, as the secondary parameter, as presented by Eq. (2). The maximum normal stress on the maximum shear plane opens the crack and expedites the failure process if tensile and closes the crack and retards the failure process if compressive.

$$\frac{\Delta\gamma_{\max}}{2}\left[1+k\left(\frac{\sigma_n^{\max}}{\sigma_y}\right)\right]=\left[(1+\nu_e)\frac{\sigma_f'}{E}(2N_f)^b+(1+\nu_p)\varepsilon_f'(2N_f)^c\right]\left[1+k\frac{\sigma_f'}{2\sigma_y}(2N_f)^b\right] \tag{2}$$

where σ_y is the material monotonic yield strength, ν_e and ν_p are elastic and plastic Poisson's ratios, and k is a material constant found by fitting fatigue data from uniaxial tests to fatigue data from torsion tests.

Fatigue data are not always available and generating fatigue properties is an expensive process. Therefore, developing predictive techniques for fatigue properties based on simple and commonly available material properties has been of great interest. Kim et al. [5] evaluated seven methods for estimation of fatigue properties for steels and found the Roessle-Fatemi hardness method [6], the Muralidharan-Manson modified universal slopes method [7], and the Bäumel-Seeger uniform material law [8] to yield better fatigue life predictions under uniaxial or torsion loading.

Shamsaei and Fatemi [9] recently extended the Roessle-Fatemi [6] hardness method to multiaxial loading by combining it with FS parameter for shear failure mode materials. They examined this method for five different steels with Brinell hardness as the only material property used and reported satisfactory predictions [9]. They also used this method for fatigue life predictions under variable amplitude multiaxial loading [10].

In this study, Roessle-Fatemi, Muralidharan-Manson, and Bäumel-Seeger prediction methods based on hardness and tensile properties are used to predict multiaxial fatigue life of steels in the absence of any fatigue data. Classical criteria as well as critical plane models are then used to predict fatigue lives for shear and tensile failure mode materials under IP and OP multiaxial loading. In this paper, first the experimental data from literature are reviewed. Next, multiaxial fatigue models are combined with Roessle-Fatemi, Muralidharan-Manson, and Bäumel-Seeger approximation methods to predict fatigue lives for shear and tensile failure mode materials under multiaxial loading and results are compared with experimentally observed fatigue lives. This is followed by a brief discussion on stress response predictions under multiaxial loading in absence of experimental cyclic data. Finally, some conclusions are made.

EXPERIMENTAL DATA

In this study multiaxial fatigue data from eleven shear failure mode materials and five tensile failure mode materials were investigated. Most of the data was found in literature [12-23], but some data originates from testing performed by the coauthor [9, 11]. The eleven shear failure mode materials include 1050 (normalized, quenched and tempered, and induction hardened) [9], S45C [21], 1045 [16, 17], SNCM630 [15], 304L [11], AL6XN [22], 1Cr-18Ni09Ti [19], Hastelloy-X (at room temperature) [24], and Inconel 718 [20]. The tensile failure mode materials include SA333Gr6 [18], 304 [12, 13], 310 [23], Haynes 188 (at 760°C) [14], and Hastelloy-X (at 649°C) [24].

LIFE PREDICTIONS BASED ON TENSILE PROPERTIES

Roessle-Fatemi Hardness Method

Roessle and Fatemi [6] proposed a method for estimation of uniaxial strain-life fatigue properties as a function of Brinell hardness (*HB*) as follows:

$$\sigma'_f = 4.25(HB) + 225,\ \varepsilon'_f = \frac{1}{E}\left[0.32(HB)^2 - 487(HB) + 191000\right],\ b = -0.09,\ c = -0.56 \quad (3)$$

Using von Mises criterion and estimated fatigue properties based on hardness (i.e. Eq. (3)), 78% of data are within scatter bands of 5. However, 59% of out-of-phase data fall within scatter bands of 5. Due to the limited space, all figures related to classical models will be presented later.

Shamsaei and Fatemi [9] combined FS critical plain multiaxial model and Roessle-Fatemi hardness method as presented with Eq. (4) to predict multiaxial fatigue lives for several steels and reported satisfactory results [9, 25].

$$\Delta\gamma_{\max}/2\left[1 + k\left(\sigma_n^{\max}/\sigma_y\right)\right] = \left[A\left(2N_f\right)^{-0.09} + B\left(2N_f\right)^{-0.56}\right]\left[1 + kC\left(2N_f\right)^{-0.09}\right]$$

$$A = \frac{2.53(HB) + 293}{200{,}000},\ B = \frac{0.48(HB)^2 - 731(HB) + 286{,}500}{200{,}000},\ C = \frac{1}{0.0022(HB) + 0.38} \quad (4)$$

In Eq. (4), $\sigma_y = 0.0044(HB)^2 + 1.33(HB)$ [6] and $k = [0.0003(HB) + 0.0585]\left(2N_f\right)^{0.09}$ [9], or $k = 1$ can simply be used; therefore, hardness is the only required material property. It should be noted that the modulus of elasticity is approximated as 200,000 MPa for steels and this value was used in all life predictions in this study. Eq. (4) is also employed here to predict fatigue lives for shear failure mode materials and results are presented in Fig.

1(a). $k = 1$ was used since the effect of k is not significant for the uniaxial form of the FS equation. However, a more accurate k value may be necessary when the shear form of the FS equation is used. From this figure, 94% of total data and 91% of OP data are within scatter bands of 5. Better OP fatigue life predictions using the FS critical plane approach can be explained by the fact that this model represents the failure mechanism and takes into consideration the constitutive behavior of material including non-proportional cycle hardening.

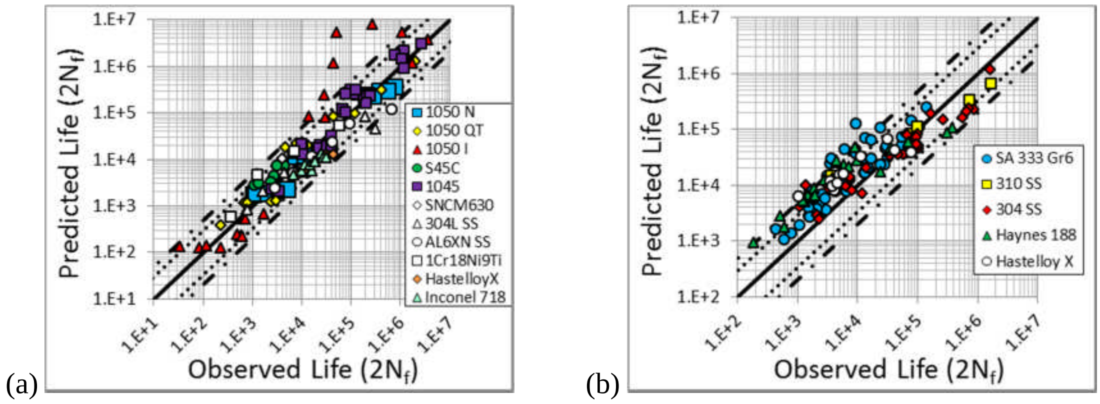

Figure 1. Predicted versus experimental lives using Roessle-Fatemi hardness method with (a) FS for shear failure mode and (b) SWT for tensile failure mode materials.

For tensile failure mode materials, Maximum Principal criterion has been commonly used to predict fatigue lives under multiaxial loading. Comparing the experimental fatigue lives with predicted ones using Maximum Principal criterion and fatigue properties estimated from Roessle-Fatemi hardness method (Eq. (3)), 82% of data are within scatter bands of 5.

The SWT critical plane model (i.e. Eq. (1)) for tensile failure mode materials takes into account material constitutive behavior including mean and residual stresses as well as non-proportional cycle hardening. The SWT multiaxial model can be rewritten based on only hardness using Roessle-Fatemi hardness method (i.e. Eq. (3)) as follows:

$$\sigma_1^{\max}\frac{\Delta\varepsilon_1}{2} = P\left(2N_f\right)^{-0.18} + Q\left(2N_f\right)^{-0.65}, \tag{5}$$

$$P = 0.0097(HB) + 0.25, \quad Q = -0.0013(HB)^2 + 0.22(HB) + 567$$

Predicted fatigue lives based on Eq. (5) for five tensile failure mode materials are compared with experimental lives in Fig. 1(b). As can be seen from this figure, 91% of data are predicted within scatter bands of 5 based on only hardness level of material. Comparing the results, 52% and 61% of OP data are predicted within scatter bands of 5 using Maximum Principal criterion and SWT critical plane model, receptively. Therefore, better fatigue life predictions are obtained using critical plane approaches.

Muralidharan-Manson Modified Universal Slopes Method

In order to estimate strain-life fatigue properties based on tensile properties, Muralidharan and Manson [7] modified the universal slopes method, previously proposed by Manson [26] as follows:

$$\sigma'_f = 0.623E\left(\frac{\sigma_u}{E}\right)^{0.832}, \ \varepsilon'_f = 0.0196\varepsilon_f^{0.155}\left(\frac{\sigma_u}{E}\right)^{-0.53}, b = -0.09, \ c = -0.56 \tag{6}$$

where σ_u is ultimate strength and ε_f is true fracture ductility. Using von Mises criterion and estimated fatigue properties based on Muralidharan-Manson method, 80% of data are within scatter bands of 5. However, only 66% of out-of-phase data fall within scatter bands of 5.

To account for non-proportional cyclic hardening under out-of-phase loadings, FS critical plane model is used here and fatigue properties are estimated from Muralidharan-Manson method based on tensile properties as presented by Eq. (7):

$$\Delta\gamma_{\max}/2\left[1+k\left(\sigma_n^{\max}/\sigma_y\right)\right] = \left[A\left(2N_f\right)^{-0.09} + B\left(2N_f\right)^{-0.56}\right]\left[1+kC\left(2N_f\right)^{-0.09}\right]$$

$$A = \frac{\sigma_u^{0.832}}{32{,}000}, \ B = 19\varepsilon_f^{0.155}\sigma_u^{-0.53}, \ C = \frac{2.42\sigma_u^{0.832}}{1.17\sigma_u - 426}, \ k = \frac{0.075\sigma_u - 32}{\sigma_u^{0.832}\left(2N_f\right)^{-0.09}} \ or \quad k = 1 \tag{7}$$

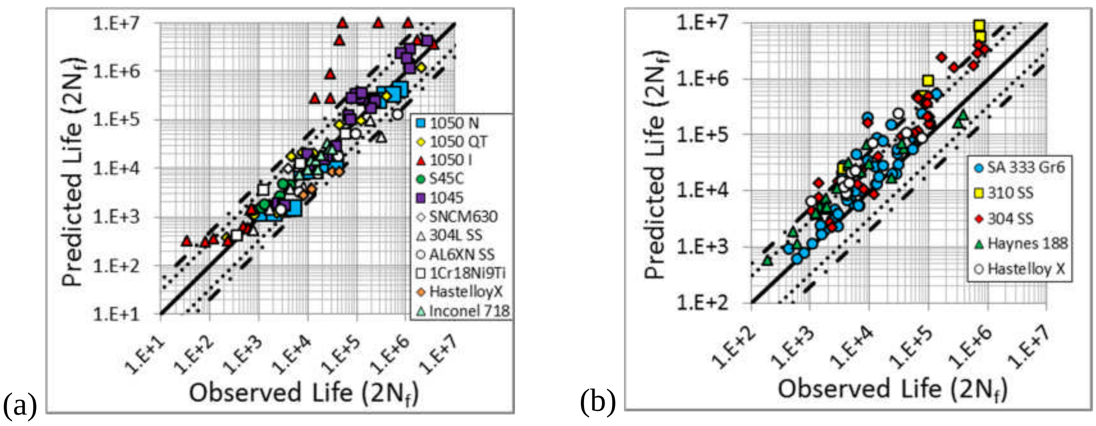

Figure 2. Predicted versus experimental lives using Muralidharan-Manson modified universal slopes with (a) FS for shear and (b) SWT for tensile failure mode materials.

In Eq. (7), yield strength can be estimated from $\sigma_y = 1.17\sigma_u - 426$. Predicted lives employing Eq. (7) versus experimental ones for shear failure mode materials are presented in Fig. 2(a). About 91% of data are within scatter bands of 5 and 88% of out-of-phase data are also within scatter bands of 5. Thus, OP life predictions are significantly improved using the FS critical plane multiaxial fatigue model.

For the five tensile failure mode materials in this study, Maximum Principal strain criterion with strain-life fatigue properties estimated from Muralidharan-Manson modified universal slopes (i.e. Eq. (6)) is used to predict fatigue lives resulting in 89% of total data and 57% of OP data are within scatter bands of 5.

The SWT critical plane multiaxial model modified based on Muralidharan-Manson method as presented by Eq. (8) is also used here to predict fatigue lives for tensile failure mode materials and results are presented in Fig. 2(b). Approximately 79% of total data and 43% of OP data are within scatter bands of 5 and the only material properties required are ultimate strength and true fracture ductility. Comparing these results with predictions from Maximum Principal theory, the SWT critical plane model did not improve the predictions.

$$\sigma_1^{\max}\frac{\Delta\varepsilon_1}{2}=P\left(2N_f\right)^{-0.18}+Q\left(2N_f\right)^{-0.65},\ P=0.00012\sigma_u^{1.664},\ Q=61\varepsilon_f^{0.155}\sigma_u^{0.302} \tag{8}$$

Bäumel-Seeger Uniform Material Law Method

Bäumel-Seeger [8] also proposed a uniform material law method to estimate fatigue properties based on ultimate strength, σ_u, and modulus of elasticity, E, as follows:

$$\sigma_f'=1.5\sigma_u,\ \varepsilon_f'=0.59\psi,\ b=-0.087,\ c=-0.58,$$
$$\psi=1\ \text{ for }\ \frac{\sigma_u}{E}\le 0.003,\ \psi=1.375-125\left(\frac{\sigma_u}{E}\right)\ \text{ for }\ \frac{\sigma_u}{E}>0.003 \tag{9}$$

Using von Mises criterion and estimated fatigue properties using Bäumel-Seeger method for the shear failure mode materials, approximately, 71% of total data and 59% of OP data are within scatter bands of 5. To take into consideration the non-proportional hardening and improve the predictions, FS critical plane model is used and fatigue properties are estimated from Bäumel-Seeger uniform material law method, as presented below:

$$\Delta\gamma_{\max}/2\left[1+k\left(\sigma_n^{\max}/\sigma_y\right)\right]=\left[A\left(2N_f\right)^{-0.087}+B\left(2N_f\right)^{-0.58}\right]\left[1+kC\left(2N_f\right)^{-0.087}\right]$$
$$A=\frac{\sigma_u}{103{,}000},\ B=0.885\psi,\ C=\frac{\sigma_u}{1.56\sigma_u-570},\ k=\frac{0.248\sigma_u-88}{\sigma_u\left(2N_f\right)^{-0.087}}\quad or\quad k=1 \tag{10}$$

In Eq. (10), yield strength can be estimated from $\sigma_y=1.17\sigma_u-426$; therefore, ultimate strength is the only required material property in this model. Using this equation, predicted and experimentally observed fatigue lives for eleven different shear failure mode steels are compared in Fig. 3(a). Approximately, 86% of total data and 82% of OP data are within scatter bands of 5; hence, life predictions for OP loading are significantly improved using FS critical plane approach.

For tensile failure mode materials, Maximum Principal criterion with fatigue properties estimated from Bäumel-Seeger method (Eq. (9)) are employed in this study and 89% of total data and 57% of OP data fall within scatter bands of 5. To account for non-proportional cyclic hardening effects, SWT critical plane model is also used and fatigue properties are estimated from Eq. (9) as follows:

$$\sigma_1^{\max}\frac{\Delta\varepsilon_1}{2}=P\left(2N_f\right)^{-0.174}+Q\left(2N_f\right)^{-0.667},\ P=\frac{\sigma_u^2}{89{,}000},\ B=0.885\sigma_u\psi \tag{11}$$

Experimental fatigue lives for tensile failure mode materials are compared with predictions from Eq. (11) in Fig. 3(b). As can be seen from this figure, 88% of total data and 74% of OP data are within scatter bands of 5, respectively. Therefore, fatigue life predictions for OP loading are improved by using SWT critical plane model as compared to the Maximum Principal theory.

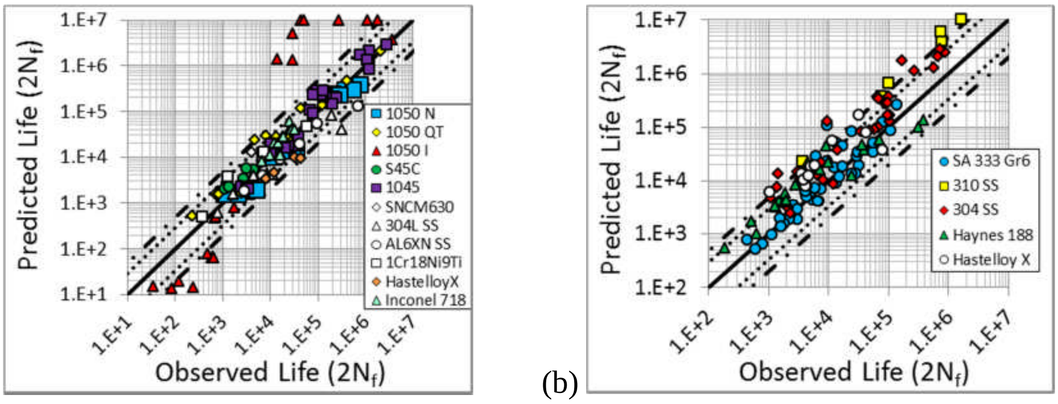

Figure 3. Predicted versus experimental lives using Bäumel-Seeger uniform material law with (a) FS for shear failure mode and (b) SWT for tensile failure mode materials.

STRESS RESPONSE PREDICTIONS FROM TENSILE PROPERTIES

One of the challenges of using strain-stress-based critical plane approaches such as FS and SWT is requiring the stress response of the material under multiaxial loading. Lopez and Fatemi [27] recently developed a model to predict uniaxial cyclic deformation of steels (i.e. Romberg-Osgood type equation) employing common tensile properties and hardness. In order to predict stress response under multiaxial loading, Shamsaei and Fatemi [28] proposed a predictive model for non-proportional cyclic hardening coefficient based on the comparison of the monotonic and uniaxial cyclic deformation curves. Therefore, only knowing commonly available or easily obtainable tensile properties and hardness, stress response of steels under multiaxial loading can be predicted. The effect of employing the above multiaxial stress response estimation method on fatigue life predictions will be discussed in the longer version of this article.

CONCLUSIONS

The following conclusions can be made from the analyses performed in this study:

1. Acceptable fatigue life predictions were obtained employing strain-stress-based critical plane approaches (i.e. Fatemi-Socie and Smith-Watson-Topper) and fatigue properties estimated from Roessle-Fatemi, Muralidharan-Manson, or Bäumel-Seeger method. Stress response of steels under multiaxial loading for such critical plane models can be also estimated from tensile properties and hardness. More details regarding the derivation of related equations will be presented in the longer version of this article.
2. FS critical plane model always provided significantly better predictions for shear failure mode materials compared to von Mises criterion no matter which method was used to predict uniaxial fatigue properties.
3. SWT critical plane approach also improved fatigue life predictions for tensile failure mode materials compared to Maximum Principal criterion when it was combined with Roessle-Fatemi and Bäumel-Seeger methods. However, when fatigue properties

were estimated from Muralidharan-Manson modified universal slopes method, Maximum Principal criterion provided better fatigue life predictions than SWT.

4. Fatigue properties estimated from Roessle-Fatemi hardness method, when used in the appropriate critical plane multiaxial model yield in better fatigue life predictions compared to other estimation methods for fatigue properties. In addition, hardness can be measured nondestructively even for in-service components, whereas measuring ultimate strength is a destructive test requiring a sample specimen.

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