Residual Fatigue Life Estimation under Mixed Mode Loading

Slobodanka Boljanović[1], Stevan Maksimović[1]

[1] VTI – Aeronautical Institute, Ratka Resanovića 1, Belgrade, Serbia
e-mail: slobodanka.boljanovic@gmail.com, s.maksimovic@open.telekom.rs

ABSTRACT. *In the present paper, a computational model for crack growth analysis of mixed mode configurations with and without overload is proposed. Stress intensity factors for mixed mode I/II are calculated by applying analytical and numerical approaches. Six-node singular finite elements are employed in order to simulate the crack growth path. Moreover, the residual life evaluation is investigated using crack growth law based on the equivalent stress intensity factor and a two-parameter driving force model. The proposed crack growth model is validated with experimental data. Fatigue life calculations and the crack path simulation are in a good agreement with crack growth observations.*

INTRODUCTION

During exploitation engineering structures are subjected to random loadings. Presence of such loadings could cause different load interaction effects due to complex fatigue process. As a consequence of continued impact of cyclic variable amplitude loadings, it leads to retardation, acceleration and interrupted retardation phenomena.

The nature of fatigue process is such that it could initiate failure, so the significant engineering aspect is to develop reliable mathematical models including adequate mechanisms and effects. A computational crack growth model is proposed, in this paper, for investigation of retardation phenomenon caused by single overload.

In fracture mechanics, different mechanisms are introduced to describe the effect of retardation such as: compressive residual stresses acting at the crack tip [1], changes in the crack tip plastic zone [2], yield zone interaction [3], crack blunting, strain hardening [4], crack closure generated by the crack tip deformation [5].

Moreover, for safety design and maintenance it is important to take in the consideration defects that appear as a consequence of either in-homogeneities of materials or effects of external environment. The detected cracks of this type on structural components are arbitrary oriented and known as mixed mode configurations. Crack growth analysis of mixed mode problems should include adequate criteria [6-8] as well as different crack growth laws [9-10]. Furthermore, fatigue life estimation of these problems which due to the complexity involve in the analysis two or three modes of loading, could be investigated either by equivalent stress intensity factor [9-10] or J-integral [11].

The authors of this paper formulate a computational model in order to describe crack growth process under mixed mode conditions. In proposed model, cyclic loadings with and without overloads are examined. Single overload phenomenon is investigated, thanks to the yield zone interaction mechanism. Considered mixed mode crack growth law for life estimation is based on the equivalent stress intensity factor. The reliability of the developed model is discussed by comparing the presented evaluations with available crack growth data.

CRACK GROWTH ANALYSIS

In the context of fracture mechanics, fatigue process for the crack growth phase can be theoretically investigated due to existence of different crack growth laws. Various crack growth models are proposed in order to estimate residual fatigue life of components such as: models based on adequate empirical relationships [12], models where considered are damage accumulation ahead of the crack [13], the two-parameter driving force models [14-15] or crack closure models [5].

Due to the fact that single overload phenomenon is considered in this paper, it is important to include the effect of mean stress. In the two-parameter driving force model, the combination of maximum stress intensity factor and stress intensity factor range enables introduction of the mean stress in crack growth estimations. Furthermore, as far as, mixed mode configurations are examined, relationship for crack growth rate must be formulated as a function of maximum equivalent stress intensity factor and equivalent stress range:

$$\frac{da}{dN} = C\,K_{\max eq}^{2}\Delta K_{eq}\,. \tag{1}$$

The maximum equivalent stress intensity factor and equivalent stress intensity factor range are parameters which include adequate modes of loading. In the present computational model, the relationship for equivalent stress intensity factor range (Mode I and Mode II) based on the dislocation model [16], is used:

$$\Delta K_{eq} = \left(\Delta K_{I}^{4} + 8\Delta K_{II}^{4}\right)^{0.25}. \tag{2}$$

The relationship for crack growth rate (Eq.1) describes fatigue crack growth of mixed mode configurations under constant amplitude loadings. In order to include single overload effect it is necessary to perform an adequate modification of the expression. The crack growth retardation appears since the currant plastic zone lies within the plastic zone created by overload [3]. This phenomenon happens until the boundary of the current plastic zone reaches the boundary of the plastic zone created by overload. Single overload effect could be mathematically modeled by introducing the retardation scaling parameter C_{pi} in the relationship for crack growth rate:

$$\frac{da}{dN} = C_{pi}\,C\,K_{\max eq}^{2}\Delta K_{eq}\,. \tag{3}$$

Due to the nature of the crack growth process under single overload, the retardation parameter could be expressed in the following form [3]:

$$C_{pi}=\begin{cases}\left(\dfrac{r_{pi}}{a_{ol}+r_{po}-a_i}\right)^p & ;\ a_i+r_{pi}\le a_{ol}+r_{po}\\ 1 & ;\ a_i+r_{pi}\ge a_{ol}+r_{po}\end{cases} \tag{4}$$

where r_{pi} is the current plastic zone size due to the ith loading cycle, a_i presents the current crack length at the ith loading cycles, r_{po} denotes overload plastic zone size, a_{ol} is crack length at overload, p represents empirically determined shaping parameter.

The plastic zone sizes are possible to calculate using model suggested by Irwin [17]. The equation for plastic zone diameter under cyclic loading can be expressed as:

$$r_{pi}=\frac{1}{\pi}\left(\frac{\Delta K_{eq}}{2\sigma_{ys}}\right)^2 \tag{5}$$

and the relationship for monotonic overload plastic zone size can be written in the form:

$$r_{po}=\frac{1}{\pi}\left(\frac{K_{ol}}{\sigma_{ys}}\right)^2 \tag{6}$$

where K_{ol} is the stress intensity factor at overload and σ_{ys} denotes yield stress of the material.

Final number of loading cycles for crack growth process under overload – mixed mode loading combination can be estimated if the expression for crack growth rate (Eq.4) is integrated:

$$N=\int_{a_0}^{a_f}\frac{da}{C_{pi}\,C\,K_{\max eq}^2\Delta K_{eq}} \tag{7}$$

where a_0 is the initial crack length and a_f presents the final crack length.
Due to the fact that the complex function is under integral (Eq.7), the number of loading cycles up to failure can be calculated only if numerical integration is applied.

STRESS INTENSITY FACTOR EVALUATION

The crack growth process is caused by the stress field around the crack tip that appeared due to the effect of external cyclic loadings. For the fracture strength estimation, stress fields i.e. loading, geometry and material are examined by parameter known as the stress intensity factor.

In this paper, the crack growth analysis is performed for an Arcan specimen and the SEN specimen subjected mixed mode loading. In general, when mixed mode configurations are considered, for each mode it is necessary to calculate adequate stress intensity factors. The expressions for stress intensity factor of the Arcan specimen (Fig.1.a) subjected to Mode I and Mode II are:

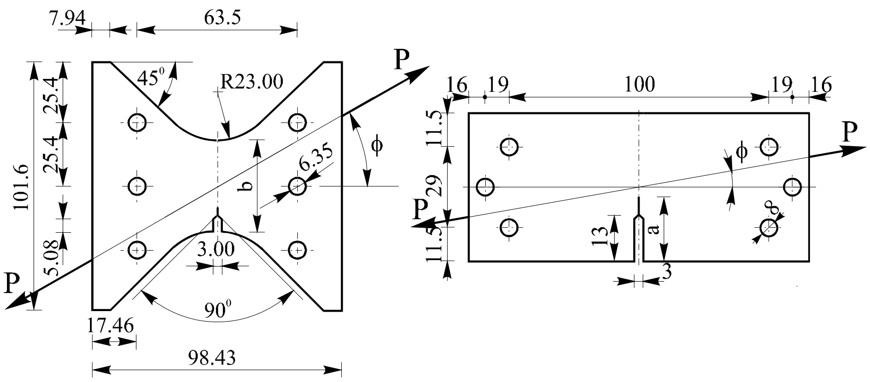
$$\Delta K_I=\frac{\Delta P}{bt}\cos\phi\, f_I\left(\frac{a}{b}\right)\sqrt{\pi a} \tag{8}$$

$$\Delta K_{II} = \frac{\Delta P}{bt} \sin\phi \, f_{II}\left(\frac{a}{b}\right)\sqrt{\pi a} \tag{9}$$

where a is the crack length, t presents the thickness of the specimen, P is external load, ϕ denote the loading angle, f_I and f_{II} are corrective functions. The relationships for corrective functions for the Arcan specimen can be expressed as:

$$f_I\left(\frac{a}{b}\right) = 1.12 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.27\left(\frac{a}{b}\right)^3 + 30.39\left(\frac{a}{b}\right)^4 \tag{10}$$

$$f_{II}\left(\frac{a}{b}\right) = \frac{1.122 - 0.561\left(\frac{a}{b}\right) + 0.085\left(\frac{a}{b}\right)^2 + 0.180\left(\frac{a}{b}\right)^3}{\left(1 - \left(\frac{a}{b}\right)\right)^{1/2}}. \tag{11}$$



a b

Figure 1. a) Arcan specimen; b) SEN specimen.

The same relationships for stress intensity factors can be used in the case of the SEN specimen (Fig.1.b). However, there is difference for the SEN specimen related to corrective functions. For stress intensity factor calculations, if the SEN specimen is considered, as corrective function only one expression is used instead of two (for Mode I and Mode II) given by:

$$f\left(\frac{a}{b}\right) = 1.12 - 0.231\left(\frac{a}{b}\right) + 10.55\left(\frac{a}{b}\right)^2 - 21.72\left(\frac{a}{b}\right)^3 + 30.39\left(\frac{a}{b}\right)^4. \tag{12}$$

Moreover, the stress analysis and the stress intensity factor evaluation are performed employing numerical approach. Quarter-point (Q-P) singular finite elements are applied in the framework of the program package MSC/NASTRAN.

NUMERICAL RESULTS

Now the proposed computational crack growth model is verified through a few numerical examples. The presented examples tackle stress analysis and fatigue life calculation under either mixed mode loading or overload – mixed mode loading combination.

Crack Growth Estimation under Mixed Mode Loading

The first example deals with the fatigue life calculation of an Arcan specimen subjected to mixed mode loading. The specimen is made of 2024 T351 Al Alloy whose geometry and material characteristics are as follows: a_0 = 2.997 mm, b = 38.1 mm, t = 1.6 mm, C= 2.22 x 10^{-10}. The Arcan specimen is subjected to cyclic mixed mode loading (ϕ=60°) with a constant amplitude (P_{max} = 2758 N, R = 0.1) [18-19]. The evaluated number of loading cycles up to failure is compared with available experimental results [18].

Since geometry, material and type of loading are known, the equivalent stress intensity factor and number of loading cycles up to failure can be evaluated by Eq.1 together with different Eq.2 and Eqs.8-11.

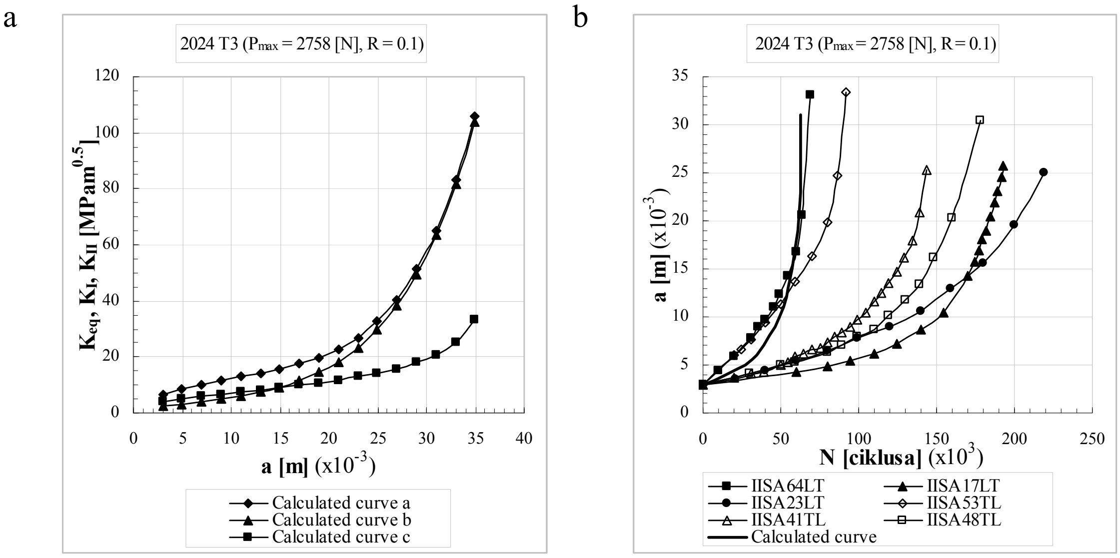


Figure 2. a) Stress intensity factor versus crack length (a – K_{eq}-a, b – K_I -a, c – K_{II}-a); b) Crack length versus number of loading cycles (experiment from Ref. [18]).

Computed values of equivalent stress intensity factors for several crack increments are presented in Fig.2.a. Moreover, calculated results for number of loading cycles are compared (see Fig.2.b) with available experimental data [18]. The evaluated number of loading cycles is in a very good correlation with experimental observation.

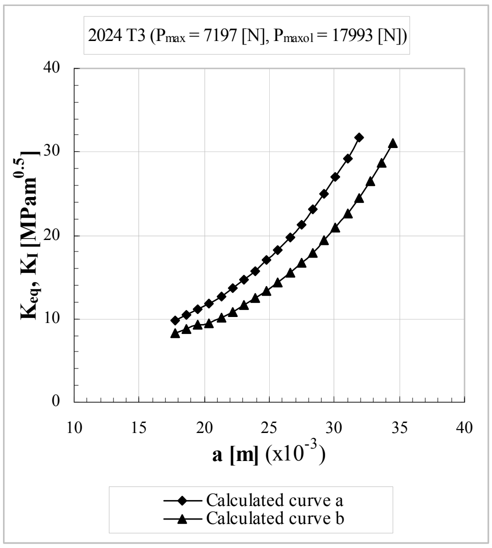
Fatigue Life Analysis of Loading Combination Overload-Mixed Mode

In this example, the residual life estimation of the SEN specimen is carried out. The SEN specimen is subjected to cyclic loading with combination of mixed mode (ϕ = 18°) and overload (P_{maxol} = 17993 N, R_{ol} = 2.5, a_{ol} = 20.40 mm). External loading, prior to overload, was a constant amplitude (P_{max} = 7197 N, R = 0.1). Geometry parameters of the specimen are: a_0 = 17.75 mm, b = 52 mm, t = 6.5 mm [20]. The considered specimen is made of 2024 T3 Al Alloy and the following parameters are assumed: σ_{ut}=469 MPa, σ_{ys} = 324 MPa, E = 73100 MPa, ν = 0.33, C = 1.61 x 10^{-10}.

Based on the fatigue performance data, equivalent stress intensity factor ranges and stress intensity factor ranges for both modes can be computed by applying Eq.2 together with Eqs.8-9 and Eq.12. Calculated results for the equivalent stress intensity factor range and stress intensity factor range (Mode I) as a function of crack lengths are presented in Fig.3.a.

Furthermore, by using Eqs.4-9 together with Eq.2 and Eq.12, the crack length is computed as a function of the number of loading cycles up to failure. All calculated results are shown in Fig.3.b. Additionally, the estimated number of loading cycles is compared (see Fig.3.b) to available crack growth data [20]. The comparison between the different results shows good agreement.

a



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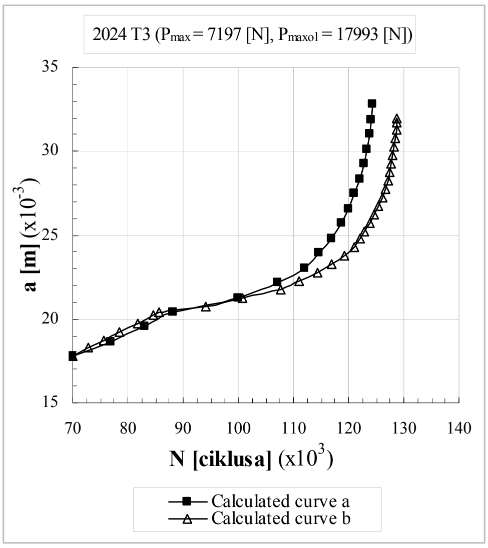


Figure 3. a) Stress intensity factor versus crack length (a - K_{eq}-a, b – K_I-a);
b) Crack length versus number of loading cycles (calculated curve b from Ref. [20]).

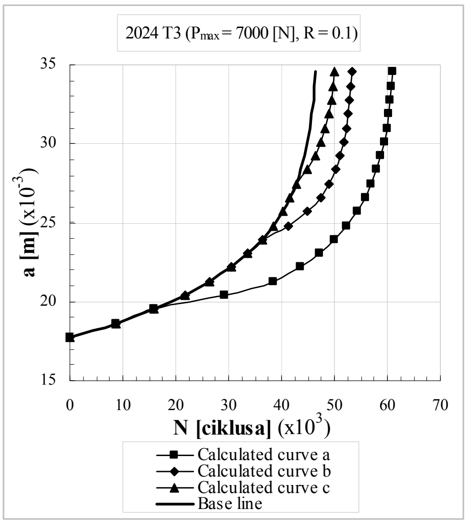
Additionally, the same geometry of the SEN specimen and material are considered in order to analyze the effect of the value of the crack length at overload as well as the level of maximum force of constant amplitude loading on fatigue life up to failure.

When analyzing the effect of the value of the crack length at overload on fatigue life, three different lengths are tackled (a_{ol} = 19.52 mm, a_{ol} = 23.93 mm, a_{ol} = 27.47 mm). After the length a_{ol} the SEN specimen is subjected to the loading combination mixed mode (ϕ =18°) and overload (P_{maxol} = 17500 N, R_{ol} = 2.5). The level of a constant amplitude loading before the overload is assumed to be P_{max} = 7000 N (R = 0.1).

Moreover, in order to investigate the effect of maximum force of constant amplitude loading on the number of loading cycles up to failure, three levels are considered (P_{max}=7000 N, P_{max} = 8400 N, P_{max} = 9800 N). The stress ratio for a constant amplitude loading was always R = 0.1 as well as the level of maximum force at overload was the same (P_{maxol} = 17500 N) in all cases of maximum force.

Impact of different crack length at overload and maximum force of constant amplitude loading on the number of loading cycles up to failure are presented in Fig.4.

a



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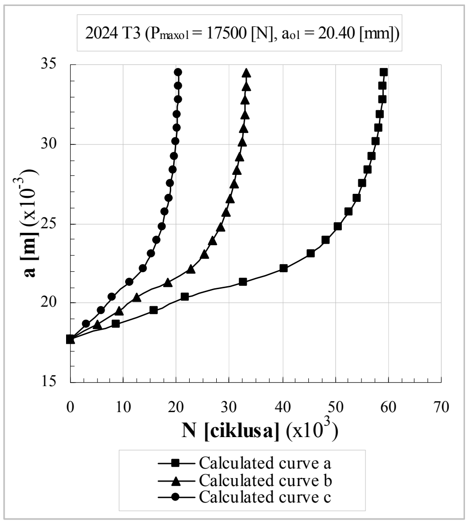
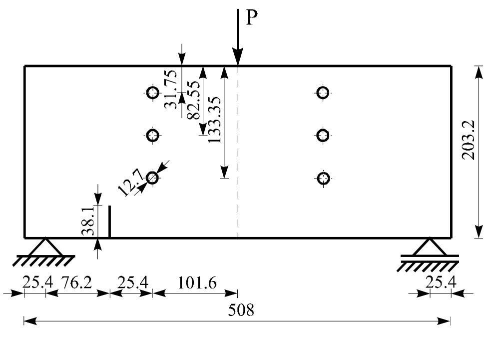


Figure 4. a) Effect of the crack length at overload on number of cycles up to failure (a - a_{ol} = 19.52 mm, b - a_{ol} = 23.93 mm, c - a_{ol} = 27.47 mm);
b) Crack length versus number of loading cycles for different levels of maximum force (a - P_{max}= 7000 N, b - P_{max}= 8400 N, c - P_{max}= 9800 N).

Modeling of mixed mode crack growth path

Numerical simulation of crack path under mixed mode loading is tackled in this example. For integrity of engineering structures, if there is already crack near holes, it is very important to investigate the position of the initial crack as well as its length so that crack will bypass holes during propagation. The considered plate has six holes (see Fig.5.a) and the initial crack with length 38.1 mm [21]. The crack is located at the distance of 127 mm from the middle of the plate. Due to the crack configuration (the position of crack and the crack length) the crack growth path bypasses holes. For the crack path simulation singular Q-P finite elements together with the MTS criterion [6] are employed (see Fig.5.b). The evaluated crack path is in a good agreement with experimental results [21]. Additionally, the longer distance of the crack from the middle of the plate and the smaller the crack length lead the crack to the hole [19].



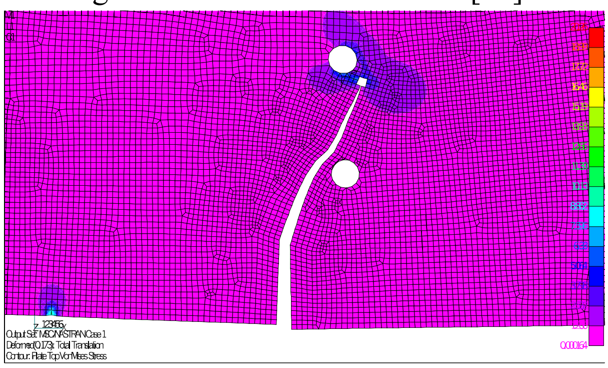


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Figure 5. a) Geometry of the plate with six holes subjected to cyclic loading;
b) Numerical simulation of the mixed mode crack growth path employing FEM.

CONCLUSIONS

An engineering procedure for crack growth analysis of mixed mode configurations is proposed. The computational procedure considers the stress analysis and the residual fatigue life evaluation. The stress analysis of mixed mode configurations is performed employing analytical and/or numerical approaches. For the simulation of stress field around the crack tip the software package MSC/NASTRAN and quarter-point (Q-P) singular finite elements are applied. Additionally, crack growth path under mixed modes is modelled using finite elements.

The comparison between the calculated and the experimental results indicates that proposed computational model adequately describes the nature of the crack growth process under mixed mode loading either with or without overload. Furthermore, the evaluated mixed mode crack growth path by applying finite elements is in a good correlation with experimental results [21].

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