Lifetime of semi-ductile materials on the critical plane

K. Walat and T. Łagoda

Department of Mechanics and Machine Design, Opole University of Technology
ul. Mikołajczyka 5, 45-271 Opole, Poland
[1] k.walat@po.opole.pl
[2] t.lagoda@po.opole.pl

ABSTRACT. *This paper presents the results of estimation of fatigue strength depending on the variable orientation of the critical plane for proportional and non-proportional bending and torsion with regard to specimens made of aluminium alloy 2017A. The algorithm applied for the estimation of the fatigue strength was based on the Carpinteri-Spagnoli proposal and its subsequent modifications in accordance with the ideas developed by the authors. The objective in this paper is to search for a model which offers the best possible results of estimating fatigue strength of materials with properties which are intermediate between elastic-brittle and elastic-plastic such as aluminium alloys and a further insight into them with regard to their fatigue strength.*

INTRODUCTION

The multiaxial fatigue criteria applied nowadays are predominantly based on the determination of equivalent stress in the critical plane [2,6,7]. The orientation of the critical plane denotes the orientation of the surrounding of a material point in the space; however, it cannot be identified with the plane of the fatigue failure. The orientation of the critical plane and the location of fatigue failure plane are relative to the type of material which was used. The materials are often found in extremely varied situations in the elastic-brittle and elastic-plastic states as well as indicate intermediate properties between these states. Intermediate properties between elastic-brittle state and elastic-plastic states are demonstrated by aluminium alloys [1]. This paper focuses on the variable relations between the orientation of the critical plane with regard to fatigue strength for proportional and non-proportional bending and torsion. The determination of the critical plane was undertaken in accordance with a proposition made by Carpinteri and Spagnoli [2]

$$\beta = \frac{3}{2}\left[1-\left(\frac{\tau_{af}}{\sigma_{af}}\right)^2\right]45^\circ , \qquad (1)$$

where angle β is denoted in relation to the direction given by the maximum in the normal direction, σ_{af} is fatigue boundary for swing bending, τ_{af} is the fatigue boundary for bilateral bending.

Finally, the angle which describes the orientation of the critical plane (normal direction) is determined by the angle β in relation to the plane given by the maximum of normal stresses in the space.
The way in which relation (1) was derived is not provided in [2]. It was adopted arbitrarily. In connection with this, this paper makes several original proposals which in extreme conditions are pertinent both to both elastic-brittle and elastic-plastic materials. As a result, this gives the angle 0° for the ratio of bending and torsion fatigue boundary $(\sigma_{af} / \tau_{af})$ equal to 1, as for the case of materials which display the properties of elastic-brittle, and 45° for the ratio of $\sqrt{3}$, which is the case for elastic-plastic materials. The boundary conditions are fulfilled for instance in the following relations:

$$\beta = \frac{9}{8}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)^4\right]45° , \quad \beta = \frac{3\sqrt{3}}{3\sqrt{3}-1}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)^3\right]45° , \tag{2,3}$$

$$\beta = \frac{3\sqrt{3}}{3\sqrt{3}-1}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)\right]45° , \quad \beta = \frac{3\sqrt{3}}{(\sqrt{3}-1)^2}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)\right]^2 45° . \tag{4,5}$$

Fig.1 presents a graphical interpretation of formulae (1) – (5). One can note that depending on the adopted model, the resulting angle can change to considerable degree. For instance, for the ratio of fatigue boundaries of $\sigma_{af} / \tau_{af} = 1.4$, the difference can be equal to as much as 16°, which can significantly affect the fatigue strength calculated by applying selected criteria of multiaxial fatigue [1, 3, 4]. In connection with this, it is necessary to derive an adequate model on the basis of experiments, one which can be used to relate the resulting angle to the ratio of the fatigue boundaries.

The estimation of the fatigue strength in the critical plane is quite complex due to the necessity of selecting one criterion of multiaxial fatigue among all the criteria based on the concept of a critical plane.

The general form of an expression for the history of equivalent stress in the critical plane can be presented as [3]

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\sigma_{\eta}(t) , \tag{6}$$

where B and K are used for the selection of a specific form of the expression (6) depending on the way in which the orientation of the critical plane is defined, $\tau_{\eta s}(t)$ is the history of the static stress in a given plane, $\sigma_{\eta}(t)$ is the course of the normal stress in a plane given by the respective formulae:

$$\sigma_n(t) = \cos^2 \alpha\sigma_{xx}(t) + \sin 2\alpha\tau_{xy}(t) , \tag{7}$$

$$\tau_{\eta s}(t) = -\frac{1}{2}\sin 2\alpha\sigma_{xx}(t) + \cos 2\alpha\tau_{xy}(t). \tag{8}$$

The earlier analyses were conducted in [8] with the use of the method of maximum variance of the normal stress and maximum variance of the shear stress.
The method of the maximum variance of the normal stress can be expressed by the formula

$$\mu_{\sigma,\sigma} = \frac{1}{T_0}\int_0^{T_0} \sigma_\eta(t)\sigma_\eta(t)dt \quad , \tag{9}$$

where T_0 is the observation time.
For the case of cyclic loadings, the observation time is equal to a single period T_0=T. Concurrently, $\sigma_\eta(t)$ is the history of the normal stress oriented under the angle of α in relation to the stress $\sigma_{xx}(t)$ for combined bending and torsion.
The method of the maximum variance can be expressed by the formula

$$\mu_{\tau,\tau} = \frac{1}{T_0}\int_0^{T_0} \tau_{\eta s}(t)\tau_{\eta s}(t)dt, \tag{10}$$

while $\tau_{\eta s}(t)$ is the history of the shear stress under the angle α in relation to stress $\sigma_{xx}(t)$ for combined bending and torsion.
The search for the maximum variance of the shear and normal stresses in accordance with the respective formulae (9) and (10) provides the values of angles α_η and $\alpha_{\eta s}$, which determine the directions in which normal and shear stresses are adopted to be the critical plane.
By analyzing the relations expressed in (1) - (5), the formulae for B and K coefficients are derived for the ratios of Mohr's circle diameters at the level of fatigue boundary for pure bending and pure torsion in the form

$$k = \frac{\sigma_{af}}{2\tau_{af}} . \tag{11}$$

As a result of testing in the circumstances of swing bending and bilateral torsion, one can derive the weighted coefficients to be used in formula (6) in the expression for the equivalent stress:

$$B = \frac{k - \dfrac{\sin(90° + 2\beta)}{\cos^2\beta}}{\dfrac{\sin 2\beta \sin(90° + 2B)}{\cos^2\beta} + \cos(90° + 2\beta)} \quad , \tag{12}$$

$$K = \frac{2 + B\sin 2\beta}{2\cos^2\beta} \quad , \tag{13}$$

$$\alpha = \alpha_\eta + \beta \, , \tag{14}$$

where α_η is the angle which determines the position of the critical plane under normal stress (formula (9)), and angle β is given by one of the formula from (1) – (5).

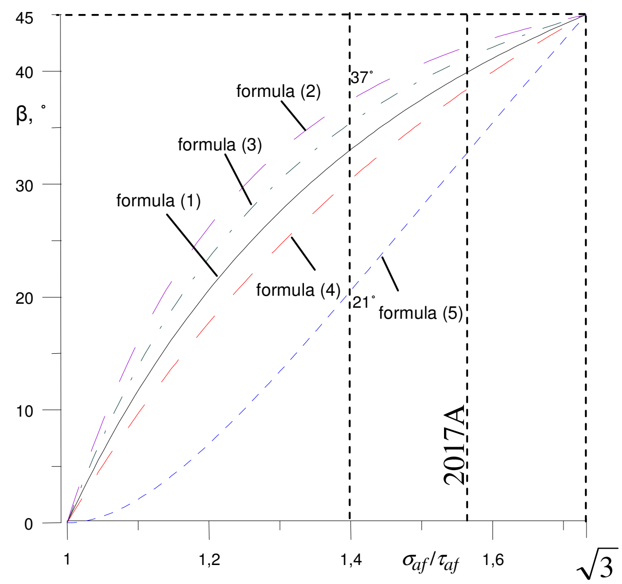


Fig.1. Angle in relation to the maximum normal stress depending on the ratio of bending and torsional fatigue boundaries

EXPERIMENTAL TESTING

The experiments used in this research applied specimens made of aluminium alloy 2017A. The tested one belongs to the group of zinc-free aluminium alloys applied for plastic working. The characteristic properties of aluminium alloys include good material strength, high tensile strength, adequate fatigue strength, low density and high corrosion resistance. The principal components of this alloy include copper which gives the strength and hardness and manganese addition in order to improve corrosion resistance. Aluminium alloy 2017A is commonly applied in various branches of the economy and transport. It is often used on heavy-duty machine constructions, military equipment and it finds applications in the aerospace, ship-building, automotive industries as well as in aesthetic casings of photographic equipment, electronic equipment, household appliances, components of power tools, small hand tools and decoration and finishes for building engineering applications.

The tests were conducted on diabolo-shaped specimens, whose geometry enabled the identification of the spots with the highest stress levels, as illustrated in Fig.2.

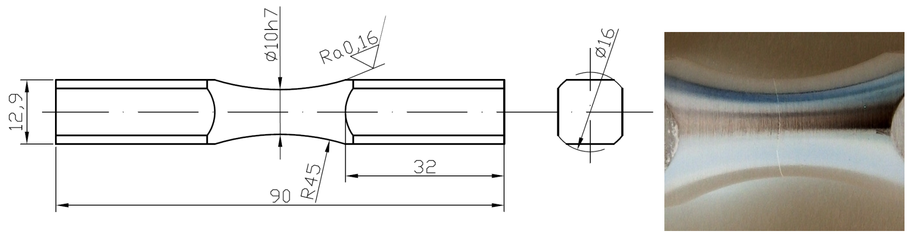


Fig. 2.a) Geometry of specimens applied in the testing, b) Specimen with an apparent fatigue crack

The specimens were subjected to pure swing bending and pure bilateral torsion as well as proportional (λ=0.25; λ=0.5; λ=1) and non-proportional combinations of bending and torsional moments (λ=0.25 φ=90°; λ=0.5 φ=60°; λ=0.5 φ=90°; λ=1 φ=90°). The properties of aluminium alloys are less commonly known; therefore, they were undertaken as the subject of the current testing. The static and cyclic characteristics of the examined material are summarized in Tables 1 and 2, respectively.

Table 1. Static properties of aluminium alloy 2017A

R_e, MPa	R_m, MPa	A_5, %	E, GPa	ν
395	545	21	72.06	0.32

Table 2. Cyclic properties of aluminium alloy 2017A

K`, MPa	n`	$\sigma`_f$, MPa	$\varepsilon`_f$	b	c
489	0.032	642	1.890	-0.065	-1.008

The characteristic gained from the testing (in accordance with ASTM norm [3]) for swing bending can be expressed as:

$$\log N_f = 21.87 - 7.03 \log \sigma_a, \tag{15}$$

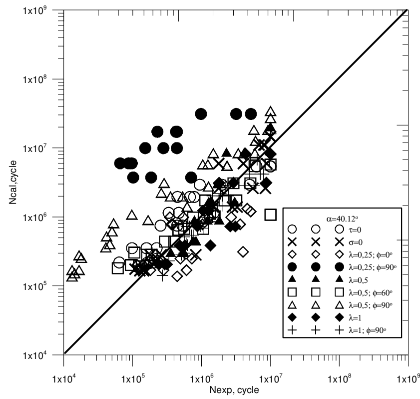
and for bilateral torsion:

$$\log N_f = 19.94 - 6.87 \log \tau_a. \tag{16}$$

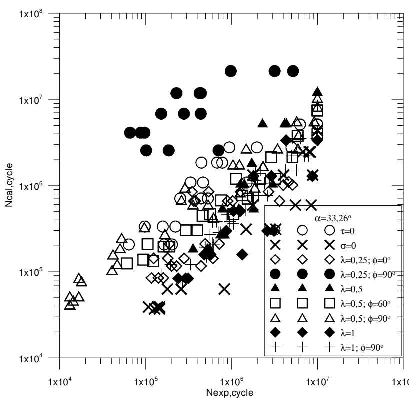
The results offer a conclusion that for the case of materials with semi-ductile material properties these methods do not lead to satisfactory levels of conformity between the calculated fatigue strength and experimental results.

The comparison of the calculated fatigue strength with results from experiments is presented in Fig.3 for various definitions of angle β in accordance with relations (1) – (5).

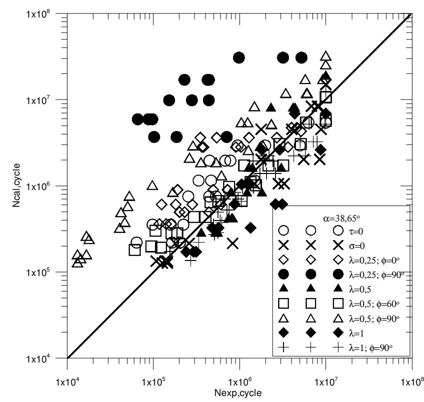
a)



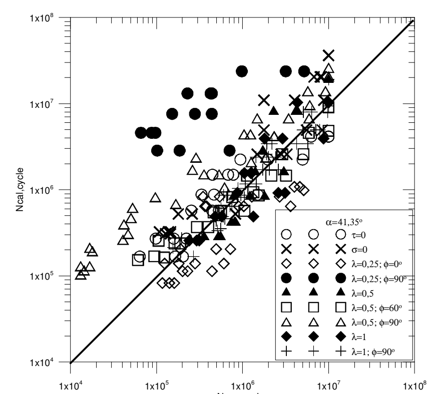
b)



c)



d)



e)

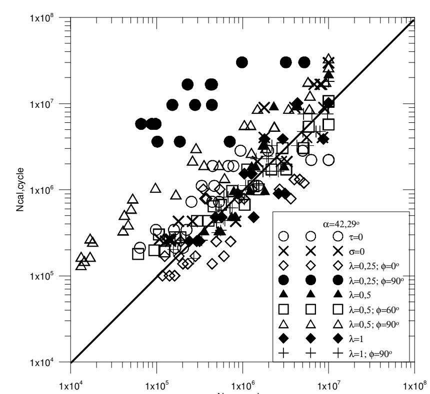


Fig.3. Comparison between calculated and experimental fatigue strength of aluminium alloy 2017A for various orientations of the critical plane a) β=40.12° - formula (1) in accordance with Carpinteri-Spagnoli proposal, b) β =33.26° - formula (2), c) β =38.65°- formula (3), d) β =41.35°- formula (4), e) β =42.29° - formula (5)

The comparison of the calculated strength with the results gained from experiments leads to the conclusion that for non-proportional bending and torsion λ=0.25 φ=90° it was not possible to gain results at a satisfactory level– as all the calculations yield considerably exceeded values.

ANALYSIS OF THE RESULTS

The method of the statistical interpretation of the results of fatigue tests by means of a root mean square error was applied for the analysis of the results. This is a universal method applied for the comparison of fatigue strength gained via various methods (fatigue criteria).

This makes it possible to identify a single method which is the most suitable, i.e. one which offers the highest conformity between the fatigue strength gained from computations and the results of the experiment. The value of the root mean square error is derived in the form:

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} \log^2 \frac{N_{expi}}{N_{cali}}}{n}}\ , \tag{15}$$

where n denotes the total number of measurements, N_{exp} is the experimental strength, N_{cal} is the calculated strength. Finally, the mean square error T_{RMS} of the scatter is derived in the form:

$$T_{RMS} = 10^{E_{RMS}}\ . \tag{16}$$

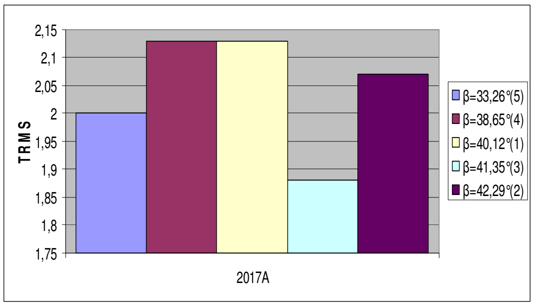
Fig. 4a presents the root mean square error gained for all values of the fatigue strength in accordance with the five analyzed models, and in Fig. 4b – the error with the exception of one combination, i.e. λ=0.25 φ=90°. On the basis of the analysis of the results one can conclude that the model proposed by Carpinteri-Spagnoli offers high conformity of the calculated fatigue strength and experimental results. However, the analysis of the result of the error indicates that the higher conformity is gained for the orientation of the critical plane under the angle 41.35°, i.e. one which follows from the proposition in formula (4). From the analysis of the error it is also clear that with the exception of the results gained for non-proportional loading λ=0.25 φ=90°, the most accurate results are provided by the analysis with the application of the model with the critical plane oriented under the angle of 33.26° - formula (5).

CONCLUSIONS

1. The Carpinteri-Spagnoli proposal with its subsequent modifications is applicable for the estimation of the position of the critical plane during the formulation of the criteria of multiaxial fatigue.

2. Aluminium alloy 2017A displays intermediate states between elastic-brittle and elastic-plastic, but is closer to elastic-plastic material properties.
3. It is necessary to undertake a further analysis involving other materials, including aluminium alloys, which have intermediate properties between elastic-brittle and elastic-plastic.

a) b)



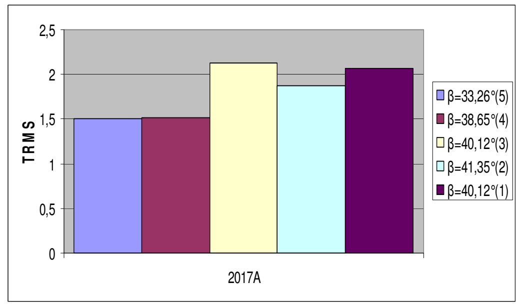


Fig.4. a) Mean square discrepancies for testing specimens made of 2017A alloy in accordance with each analyzed model, b) mean square scatter for testing specimens made of 2017A alloy with the exception of the results for λ=0.25 φ=90°

REFERENCES

1. Walat K., Łagoda T., (2010) *Procedia Engineering* **2**, 1211-1218.
2. Carpinteri A., Macha E., Brighenti R., Spagnoli A., (1999) *Int. J.Fatigue* **21**, 89-96
3. Łagoda T., Ogonowski P., (2005) *Mat.-wiss.u. Werkstofftech* **36**, 429-437
4. Skibicki D.,Pejkowski Ł, (2012) *J. of theoretical and applied mechanics* **50,** 1073-1086
5. Walat K., Kurek M., Ogonowski P., Łagoda T., (2012), *Int. J.Fatigue*, **37**, 100-111
6. Carpinteri A., Spagnoli A., Vantatori S., *Fatigue life estimation under multiaxial random loading using a critical plane-base criterion*, Second International Conference on Material Performance under Variable Amplitude Loading, Proceedings, Vol.I, Darmstadt, 2009, Berlin, pp.474-486
7. Kueppers M., Sonsino C.M.(2003) *Fatigue Fract. Engng. Mater. Struct.,* **26**, 507-513
8. Walat K., (2011) *Fatigue strength of machine parts in the critical plane given by extreme of stress covariance,* SiM Vol.294, Multiaxial random fatigue of machine parts and constructions. Section XV, Opole University of Technology [in Polish]

"The project financed from the funds of the National Centre of Science – decision number 2011/01/B/ST8/06850"