MULTIAXIAL FATIGUE DESIGN OF CAST PARTS : DEFECT STRESS GRADIENT (DSG) APPROACH

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Abstract *Experimental multiaxial Kitagawa diagrams are produced for cast A356 T6 containing natural and artificial defects. Results are obtained for three different loadings: tension, torsion and combined tension-torsion, for a load ratio R = -1. The experimental critical defect size determined is 400 +/- 100 μm in A356 T6 under multiaxial loading. Below this value, the microstructure governs the fatigue limit mainly through the SDAS. We compared four theoretical approaches to simulate Kitagawa diagram for multiaxial loading: Murakami's equation, a defect is equivalent to a crack using Linear Elastic Fracture Mechanics, the Critical Distance Method (CDM) proposed by Susmel and Taylor and the Defect Stress Gradient (DSG) approach proposed here. It is shown that CDM and DSG methods give good results but need three fatigue data for the identification.*

Material and experimental results

The material employed in this study was Low-Pressure Die Cast (LPDC) strontium modified A356 (Al-7Si-0.3Mg). Tensile testing has resulted in a modulus of elasticity of 66 GPa, Poisson's ratio of 0.3, a yield strength of 164 MPa and an ultimate tensile strength of 317 MPa. While all specimens came from castings made with permanent steel dies, the majority of specimens came from a wedge-shaped casting, and a lesser number were cut directly from an automotive wheel. The wheel casting was actively cooled during solidification while the wedge casting was passively left to cool. As these two casting types provided a wide range of solidification conditions, so too did the specimens from a defect and microstructure standpoint. Therefore, the fatigue behaviour characterized in the current work is directly applicable to commercial castings. Experimental Kitagawa diagrams are reported here for the three different loadings. All Kitagawa diagrams are presented with the same scale in a bi-linear diagram presentation style. The defect size parameter used is the 'area' parameter proposed by Murakami [1]. Cast A356 T6 contains different types of defects also reported by [2-4]. Defects can be gas pores, shrinkages, oxides films or inter-metallic inclusions. Figure 1 presents the experimental Kitagawa diagram under pure tension and R = -1. In all samples, the initial defect size was easy to determine and the fracture plane was always perpendicular to the direction of the maximum principal stress. The first remarkable point from this curve is the relative high critical defect size: the tests T6, A1 and A2 have a very low impact on the fatigue limit (8 % reduction). The material is sensitive to defect when the size is bigger than 500 μm under tension. It is interesting to point out that this does not depend on the type of defect.

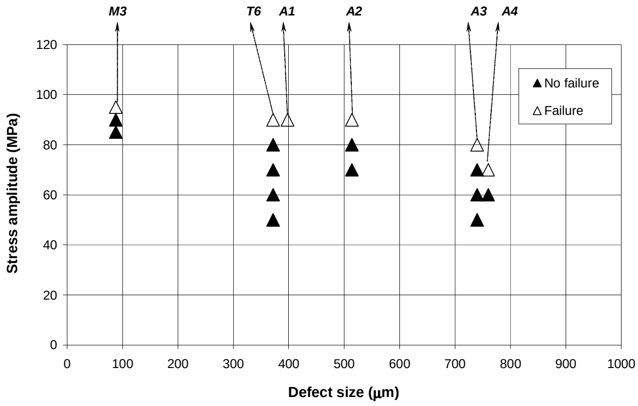

Figure 1: Kitagawa type diagram for A356-T6 under tension, R = -1.

Figure 2 presents the experimental Kitagawa diagram under pure torsion and R = -1. The experimental points presented on the curve below 100 μm are classified as either ‘no defect’ or ‘not identified defect’. The samples have been separated with different defect size between 0 and 100 μm but it is only in order to make the graph clear, it is not related to the defect size because there is no. The first remarkable conclusion on the Kitagawa diagram under torsion is the scatter. The fatigue limits vary from 55 to 95 MPa for samples with ‘no defect’ or ‘not identified defect’. This represents a huge scatter compared to the other results obtained for the other load cases.

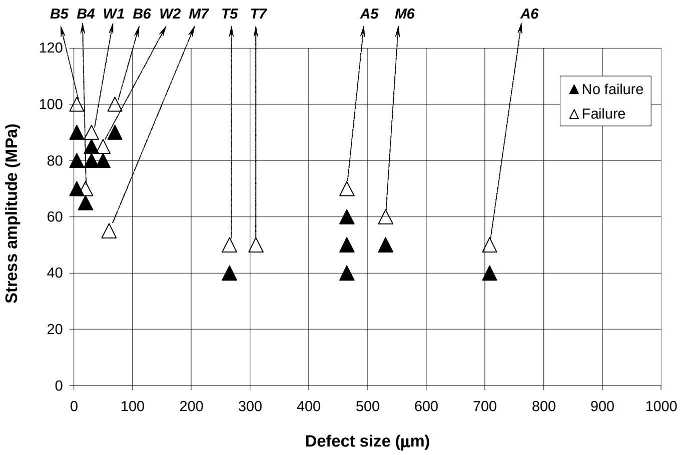

Figure 2: Kitagawa type diagram for A356-T6 under torsion, R = -1.

Figure 3 presents the experimental Kitagawa diagram under combined tension-torsion and R = -1. In this curve, no artificial defects are involved, only natural ones.

Macroscopic fractures surfaces are similar to tension ones: flat surface in the plane perpendicular to the direction of the maximum principal stress with a clear identification of the initiation area. The Kitagawa curve shows a very small influence of 500 μm defect. In fact for defects below this size, it seems that there is no influence of the defect on the fatigue limit.

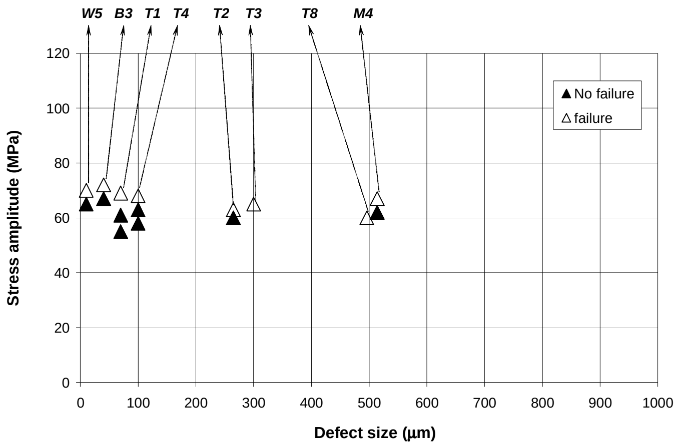

Figure 3: Kitagawa type diagram for A356-T6 under tension-torsion, R = -1.

From the previous fatigue results obtained on A356 T6 containing artificial and natural defects, we can conclude that the critical defect size that does not affect the fatigue limit is relatively large. It ranges from 300 to 500 μm depending on the loading type. It is also interesting to observe that when the defect is described by the 'area' parameter, an artificial defect behaves similarly to a shrinkage or an oxide film against fatigue limit. It is another demonstration that this parameter is really powerful to describe the morphology of a given defect. From the pure torsion results, it is not so easy to find the initial defect size due to the complex topology of the fracture surface and the multiple initiation sites. This last point should be studied into more details especially because we were not able to find any fatigue results under torsion on A356 T6 with experimental characterization of defect size in the literature.

Comparison of Multiaxial fatigue criteria

LEFM describes crack propagation threshold with the amplitude of the stress intensity factor ΔK, a function of crack length a_c and stress amplitude $\Delta\sigma_c$. The defect size is given trough the 'area' parameter that is transformed into a semi circular crack here. The relation between the fatigue limit and the defect size is given by:

$$\Delta\sigma_C = \frac{\Delta K_{th,eff}}{Y\sqrt{\sqrt{2\pi}\sqrt{area}}}$$

Where Y is the crack shape factor and $\Delta K_{th,eff}$ is the effective stress intensity factor threshold. With a spherical defect: $Y = \frac{2}{\pi}$. $\Delta K_{th,eff} = 1.5 MPa\sqrt{m}$ for A356-T6 alloy; this value

was not experimentally determined in our material but from a large compilation of published data from [5-10]. Murakami proposes to represent the defect as a surface entity and introduces the $\sqrt{area}$ parameter to describe defect size. He justifies this choice using fracture mechanics concepts. Observing non-propagating cracks in a small stressed zone around de defect, he considers that endurance threshold corresponds to crack growth threshold. He shows that the maximum stress intensity factor $K_{I\,max}$ is linearly related to the $\sqrt{area}$ parameter for different crack geometry and then links the endurance threshold and this size parameter. He shows that for a given Vickers hardness, fatigue crack growth threshold depends mainly on $\sqrt{area}$ parameter. Murakami has proposed an empirical equation based on the defect size ($\sqrt{area}$) and the material hardness to predict the fatigue limit of materials containing small defects.
Murakami proposed the following empirical relations for tension loading:

$$\sigma_W = \frac{A(H_v + 120)}{\left(\sqrt{area}\right)^{1/6}} \left[\frac{1-R}{2}\right]^{\alpha}$$

With $A = 1.43$ for surface defects and $A = 1.56$ for internal defects and $\alpha = 0.226 + H_v * 10^{-4}$.
For torsion loading and surface defects, the fatigue limit is given by the following equation:

$$\tau_W = \frac{0.93(H_v + 120)}{F(b/a)\left(\sqrt{area}\right)^{1/6}} \left[\frac{1-R}{2}\right]^{\alpha}$$

With $F(b/a) = 0.0957 + 2.11(b/a) - 2.26(b/a)^2 + 1.09(b/a)^3 - 0.196(b/a)^4$ and $b/a = 1$ for spherical defects. For combined tension-torsion loading, the fatigue limit is given by the following relation:

$$\sigma_1 + k\sigma_2 = \frac{A(H_v + 120)}{\left(\sqrt{area}\right)^{1/6}} \left[\frac{1-R}{2}\right]^{\alpha}$$

Where σ_1 is the maximum principal stress, σ_2 is the minimum principal stress and $k = -0.18$ for cracks emanating from a round defect. There are two parameters that have to be identified for Murakami's relations: the macroscopic Vickers hardness (H_v = 80 MPa for A356 T6) and the b/a parameter (b/a = 1 for spherical defect). Murakami's equation is very simple to identify but the limitation is mainly due to the description of the stress state that is not able to take into account for a general multiaxial loading. The 'critical distance' approach and the 'gradient' one needs a multiaxial fatigue criterion to be applied. In this paper, we decided to use the equivalent stress proposed recently by Vu [11] that aims to describe the multiaxial behavior for complex loading using invariant approach. This equivalent stress needs only analytical computations but we could use another criterion so that the following results comparing the approaches are not depending on the criterion.

$$\sigma_{eqVu} = \sqrt{\gamma_1 J'_2(t)^2 + \gamma_2 J_{2,mean}^2(t)^2 + \gamma_3 I_f(I_{1,a}, I_{1,m})} \leq \beta$$

See reference [11] for more details. The values of $\gamma_1, \gamma_2, \gamma_3$, β and α depend on the strength of the metal and are identified using the parameters t_{-1}, f_{-1} and R_m. For the

A356 T6 alloy, $\gamma_3 = \frac{t_{-1}^{2} - \frac{f_{-1}^{2}}{3}}{f_{-1}} = 41.1MPa$, $\beta = t_{-1} = 80MPa$ and $R_m = 317MPa$ so $\gamma_1 = 0.65$, $\gamma_2 = 0.8636$ and $\alpha = 1$.

The second input data needed by both 'critical distance' and 'gradient' approaches is the stress distribution around the defect. In order to compute local stresses around the defect represented here by a spherical void, the analytical theory of Eshelby have been used. This CDM is based on the approach proposed by Susmel and Taylor. [12] This approach describes the influence of the defect through the measurement of the stresses to compute the equivalent fatigue stress at a given distance from the defect. We need therefore the evaluation of the stress field around the defect as explained before. The maximum equivalent stress is calculated at the critical distance L/2 from the tip of the notch. The fatigue limit σ_C is established as follow:

$$\underset{\sigma=\sigma_C}{Max}\left(\sigma_{eqVu}\left(\frac{L}{2}\right)\right) = \beta$$

With σ the nominal applied stress.

The parameter we have to identify to use this criterion is the parameter L/2. This distance is the distance from the notch tip to the point where $\sigma_{eqVu} = \beta$. We need therefore an experimental fatigue limit for a given defect size to make the identification of L/2. The following case has been used $\sigma_{\infty ref} = 85MPa$ for $\sqrt{area}_{ref} = 400\mu m$. It has been found that L/2 = 79µm. Nadot, Gadouini (DSG criterion) [13, 14] proposed to compute an equivalent stress that includes the effect of the defect through the description of the stress gradient around the defect. The criterion is written as follow:

$$\sigma_{eq}^{*} = \sigma_{eqVu,Max} - b_g \frac{\sigma_{eqVu,Max} - \sigma_{eqVu,\infty}}{\sqrt{area}} \leq \beta$$

$\sigma_{eqVu,Max}$ is the maximum equivalent stress calculated at the tip of the defect and $\sigma_{eqVu,\infty x}$ is the nominal equivalent stress calculated far from the defect. To use this criterion, the parameter b_g has to be identified. It has the dimension of a length. This parameter allows accounting for the defect geometry and is calculated using fatigue limit of material containing a known artificial defect. Identification is performed on the reference loading case ($\sigma_{\infty ref} = 85MPa$ and $\sqrt{area}_{ref} = 400\mu m$):

$$b_g = \sqrt{area}_{ref}\left(\frac{\sigma_{eqVu,Max} - \sigma_{eqVu,\infty}}{\sigma_{eqVu,Max} - \beta}\right)$$

The calculation of the stress field around the defect is performed using the analytical computation explained before. Under tension, critical distance and gradient approaches are very good for the values but this is not surprising because one experimental point is used for the identification. The trend is also well described. Murakami's equation leads to non conservative result but the trend is well described. LEFM gives conservative results with a good trend. Under torsion, all approaches are relatively good for values and trend; expect Murakami that is again non conservative and critical distance that tend to assess a small allowable defect size. Under combined loading, all approaches are non conservative except Murakami. Again the critical defect size is largely underestimated by the critical distance theory.

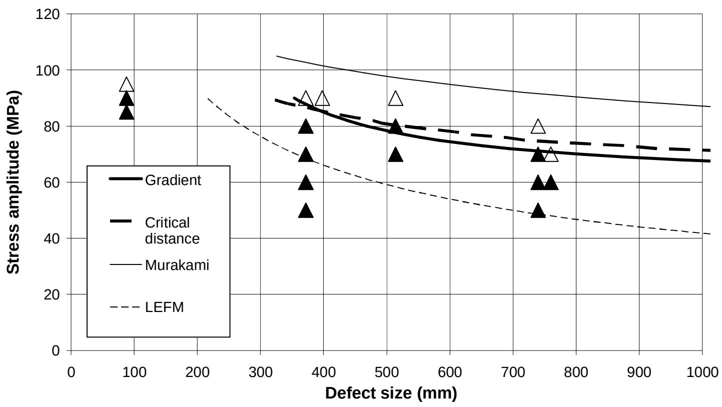

Figure 4: comparison between experimental results and simulations (tension, R -1).

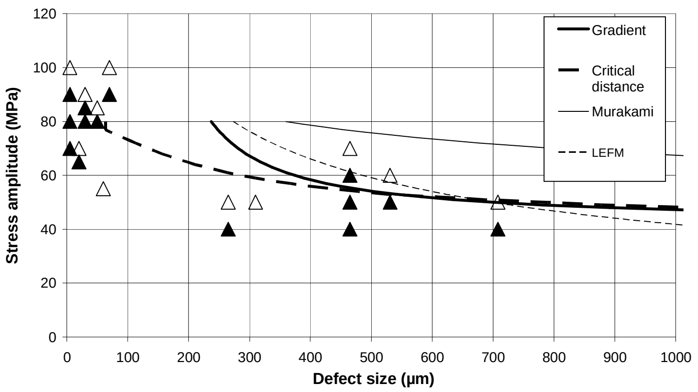

Figure 5: comparison between experimental results and simulations (torsion, R -1).

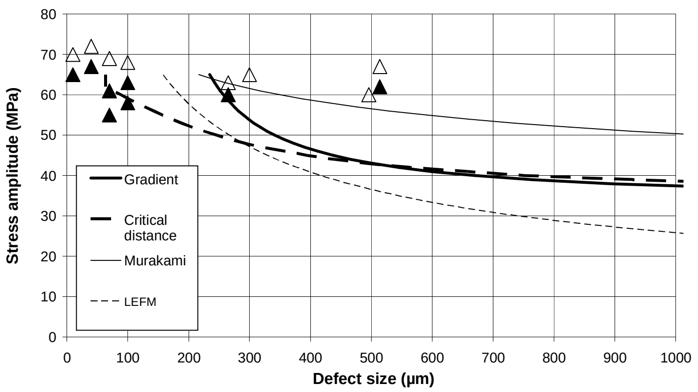

Figure 6: comparison between experimental results and simulations (tens-tor, R -1).

Figure 7 gives another viewpoint on this comparison by plotting the error between simulation and each experimental result for the four approaches and all load cases. Negative values are related to non conservative assessment and positive to conservative ones. We can also average the absolute error given by each approach; this leads to the following result LEFM = 19 %, Murakami = 20 %, critical distance = 11 % and gradient = 9 %. From this general average comparison we can conclude that the description of the defect through the elastic stress field (gradient or critical distance approach), gives better results that LEFM or Murakami. But if we include in the comparison the 'identification cost', then Murakami's equation remains very good because you can give the fatigue limit of the material for different defect sizes, three load cases using only the macroscopic hardness of the material. LEFM gives also in average interesting results related to the fact that only one experimental parameter is used: the effective threshold stress intensity factor for long cracks. Both critical distance method proposed by Susmel and Taylor and gradient one proposed by Nadot are good in average but you have to describe the elastic stress field around the inclusion and get three experimental fatigue limits including one with a defect. It is important to note that elastic computation of stresses is relevant in the case of A356-T6 because fatigue limit is half the yield stress so that there is a very little amount of plasticity at the tip of the defect.

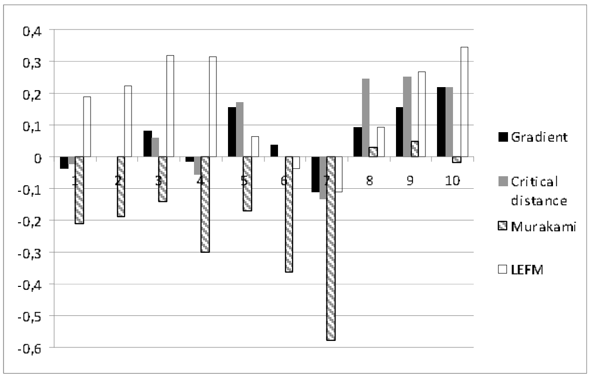

Figure 7: Quantitative comparison between simulations and experimental results for the four approaches tested.

CONCLUSIONS

- In cast A356-T6 submitted to multiaxial fatigue loading, fatigue cracks can initiate either on casting defects or inside the microstructure. Both scales are in competition for the localization of cyclic plastic deformation that induces the initiation of the crack that leads to failure.
- When the crack initiates on a defect, it can be of different types: oxide, pore, or shrinkage.
- The critical defect size that does not affect the fatigue limit is 400 µm +/- 100 µm in A356 T6. This result is obtained for both artificial and natural defect and tension, torsion and combined loading for a load ratio R = -1.

- Multiaxial Kitagawa type diagram are simulated using four different approaches: Murakami, LEFM, ‘critical distance’ and ‘gradient’. Results shows that Murakami’s equation gives mainly non conservative results for A356-T6 with an average error of 20 % error. LEFM is mainly conservative with an average error of 19 %. Critical distance and gradient are both equivalent with mainly conservative results and average error of respectively 11 and 9 %.

References

[1] Murakami Y. Metal fatigue: effects of small defects and non-metallic inclusions. Elsevier; 2002.
[2] Gao YX, Yi JZ, Lee PD, Lindley TC. Acta Mater 2004;52:19.
[3] Brochu M, Verreman Y, Ajersch F, Bouchard D. Int J Fatigue 2010;32:8.
[4] McDowell DL, Gall K, Horstemeyer MF, Fan J. Eng Fract Mech 2003;70:1.
[5] Buffière JY, Savelli S, Jouneau PH, Maire E, Fougères R. Mater Sci Eng A 2001;316:1–2.
[6] Couper MJ, Nesson AE, Griffiths JR. Fatigue Fract Eng Mater Struct 1990;13:3.
[7] Atzori B, Filippi S, Lazzarin P, Berto F. Fatigue Fract Eng Mater Struct 2004;28:1–2.
[8] Dabayeh AA, Xu RX, Du BP, Topper TH. Int J Fatigue 1996;18:2.
[9] Zhu X, Yi JZ, Jones JW, Allison JE. Metall Mater Trans A 2007;38:5.
[10] Kumar A, Torbet CJ, Pollock TM, Jones JW. Acta Mater 2010;58:6.
[11] Vu QH, Halm D, Nadot Y. Int J Fatigue 2010;32:7.
[12] Susmel L, Taylor D. Eng Fract Mech 2008;75:15.
[13] Nadot Y, Billaudeau T. Eng Fract Mech 2006;73:1.
[14] Gadouini H, Nadot Y. Int J Fatigue 2008;30:9.