Effects of Small Defects and Cracks in Multiaxial Fatigue

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ABSTRACT. *In this study, the effects of small surface defect and small crack on the fatigue limit of metallic materials were investigated. As a theoretical framework, McEvily's fatigue crack growth equation was adopted to account for the Kitagawa effect, the elastoplastic bahavior in the vicinity of the tip of fatigue crack and the crack-closure development in a unified manner. Based on the comparisons between the theoretical predictions and the experimental data, the predictive capability of the present method was examined. Further, the influence of defect size and biaxial stress on the multiaxial fatigue behaviors were studied in a systematic manner. Moreover, in view of the* $\sqrt{area}$ *-parameter model, our effort was devoted to propose a simple yet reasonably accurate equation to predict the fatigue limit.*

INTRODUCTION

To improve the structural integrity against fatigue failure, thorough understanding of the role of small crack is required. In practice, most of fatigue cracks spend the vast majority of their lives as short cracks, the behavior of such a flaw is of significant when determining the fatigue lifetime. As is well known, the fatigue crack behaviour of short cracks differs in a non-conservative manner from that of long cracks. The fatigue limit for many steels is dictated by the non-propagating condition of a small crack [1]. Accordingly, the boundary between propagation and non-propagation state separates the safe from the potentially unsafe fatigue regimes. In general, the structural and machine components in service are often in a complex stress state due to biaxial or multiaxial loading. Moreover, defects, such as surface flaws and non-metallic inclusions, often act as the stress concentration site and they significantly reduce the fatigue strength.

The purpose of the present study is to examine the effect of biaxial stress on the fatigue limit of specimens containing small defects and cracks. To tackle the underlying problem, the modified liner-elastic fracture mechanics approach proposed by McEvily et al. [2] was adopted. Moreover, in view of the $\sqrt{area}$ -parameter model [1], our research effort was devoted to propose a simple yet reasonably accurate equation to predict the fatigue limit.

FATIGUE CRACK GROWTH EQUATION

According to McEvily et al. [2], the fatigue crack growth rate can be characterized by the following equation (i.e., McEvily equation):

$$\frac{da}{dN} = A\left(\Delta K_{\mathrm{eff}} - \Delta K_{\mathrm{effth}}\right)^2 \qquad (1)$$

where a is the crack length; N is the number of load cycles; A is a constant that depends on the material and the environment; ΔK_{eff} represents the effective range of Mode-I stress intensity factor (SIF) given by $K_{\mathrm{max}} - K_{\mathrm{op}}$, where K_{max} is the maximum value of SIF in a cycle and K_{op} is the SIF at the crack opening level; $\Delta K_{\mathrm{effth}}$ signifies the effective range of the SIF at the threshold level, a material constant. It is noted that Eq. (1) is dimensionally correct.

To tackle the biaxial fatigue with a stress ratio of $R = -1$, we rewrite Eq. (1) as :

$$\frac{da}{dN} = A \cdot M^2 \qquad (2)$$

where [3]:

$$\begin{aligned} M &= \Delta K_{\mathrm{eff}} - \Delta K_{\mathrm{effth}} \\ &= \left[\left(1-\frac{\sigma_{\mathrm{p}}}{\sigma_{\mathrm{n}}}\right)\sqrt{2\pi r_e F} + Y\sqrt{\pi a F}\right]\sigma_{\mathrm{n}} - \left(1-e^{-k(a-r_e)}\right)K_{\mathrm{opmax}} - \Delta K_{\mathrm{effth}} \end{aligned} \qquad (3)$$

In Eq. (3), r_{e} is termed an effective crack length that can account for the Kitagawa effect; F is the elasticplastic correction factor for crack length; Y is a geometric correction factor for SIF, σ_{n} and σ_{p} are the far-field applied stress normal and pararell to the crack plane, respectively. Further, k is a material constant that governs the rate of crack closure development; and K_{opmax} is a value of SIF when a macroscopic crack opens. The effective crack length, r_{e} is calculated based on the fatigue limit of smooth specimen, σ_{w0} at $R = -1$ as follows:

$$r_e = \frac{1}{\pi F_0(\sqrt{2}+1.12)^2}\left(\frac{\Delta K_{\mathrm{effth}}}{2\sigma_{\mathrm{w0}}}\right)^2, \qquad \text{where } F_0 = \frac{1}{2}\left(1+\sec\left(\frac{\pi\sigma_{\mathrm{w0}}}{2\sigma_{\mathrm{YS}}}\right)\right) \qquad (4)$$

where σ_{YS} signifies the yild strength for uniaxial tension.

In essence, the following three modifications are considered in Eq. (3).

- A modification to overcome the inability of the standard approach to predict fatigue crack growth behavior in the regime of short cracks.
- A modification to account for the elastic-plastic behavior.
- A modification to deal with the development of crack closure in the wake of a newly-formed crack.

In this study, rather than performing a three-dimensinal stress analysis, we conducted a two-dimensinal stress analysis to determine Y in Eq. (3) for the sake of simplicity. Concerning the elasticplastic behavior or calculation of F, we adopted a modified Dugale strip yield model.

Consideration of Elasticplastic Behavior

By taking advantage of the Dugdale model [4], the modified crack length [5] under uniaxial tension can be calculated as:

$$a_{\mathrm{mod}} = aF\,, \qquad \text{where} \quad F = \frac{1}{2}\left(1+\sec\frac{\pi\sigma^{\infty}}{2\sigma_{\mathrm{YS}}}\right) \tag{5}$$

In Eq. (5), σ^{∞} represents the far-field applied stress. On the other hand, under the biaxial loading condition, the stress component parallel to the crack plane (i.e., non-singular stress) influences the plastic behavior near the crack tip [6]. Therefore, we employed the following yield strength that is based on the von Mises yield condition [6]:

$$\sigma_{\mathrm{YS}bi} = \frac{\sigma_x + \left[\sigma_x^2 - 4(\sigma_x^2 - \sigma_{\mathrm{YS}}^2)\right]^{1/2}}{2} \tag{6}$$

To make use of Eq. (6), we calculated the local stress at the crack tip for σ_x based on the elastic stress analysis. It is noted that concerning a crack emanating from a hole under uniaxial loading condition, the value of $\sigma_{\mathrm{YS}bi}$ approaches σ_{Y} as the crack propagates (i.e., long crack). Calculation of the elastic-plastic correction factor, F in Eq. (3) is not a trivial task for a crack emanating from a stress concentration site (e.g., hole). Therefore, we adopted a numerical method to calculate F and the procedure is explained elsewhere [7].

EXPERIMENTS

The used materials were an annealed 0.37 % carbon steel (JIS S35C) and a quenched/tempered Cr-Mo steel (JIS SCM435). Table 1 lists the mechanical properties of each material.

Table 1. The mechanical properties.

Material	Yield strength (MPa)	Tensile strength (MPa)	Elongation (%)	Vickers hardness (*HV*)	σ_{w0} (MPa)
S35C	328	586	48	164	230
SCM435	858	947	57	306	490

The smooth specimens were a round-bar with the minimum diameter of 8.5 and 10 mm. After heat treatment, a 30μm thickness of surface layer was removed by electro-polishing, and thereafter pre-cracks or slits was introduced into the surface to simulate a defect, as illustrated by Figure 1. A single cylindrical hole is referred to as a 1-hole defect and a defect connected with two or three adjacent holes is referred to as a 2-hole or 3-hole defect. To pre-crack the specimens, fatigue cracks were introduced at a crack starter of 1-hole or 2-hole defects, and they were grown to total surface lengths, $2a_0$, of 150μm, 160μm, 200μm, 300μm, 400μm, 600μm, and 1200μm, respectively. Another type of small defect was a hole with slits made by the focused ion beam (FIB) technique at the both ends. The major axis of cracks and slits was perpendicular to the direction of the maximum principal normal stress. All specimens were annealed at 873K in vacuum to remove the residual stresses. The $\sqrt{area}$ -parameter model [1] was used to evaluate the size of initial defect into $\sqrt{area}$.

Uniaxial fatigue tests were carried out by using a rotating bending machine with an operating frequency of 50-60 Hz. The torsional fatigue tests were performed by using a servo-hydraulic axial/torsional testing machine with an operating frequency of 30-45 Hz. All fatigue tests were conducted by applying the sinusoidal loading with a stress ratio of R = –1. Table 2 summarizes the material constants adopted for the present analysis. Based on Eq. (3), the threshold condition can be predicted by seeking the condition of M = 0 (i.e., da/dN = 0).

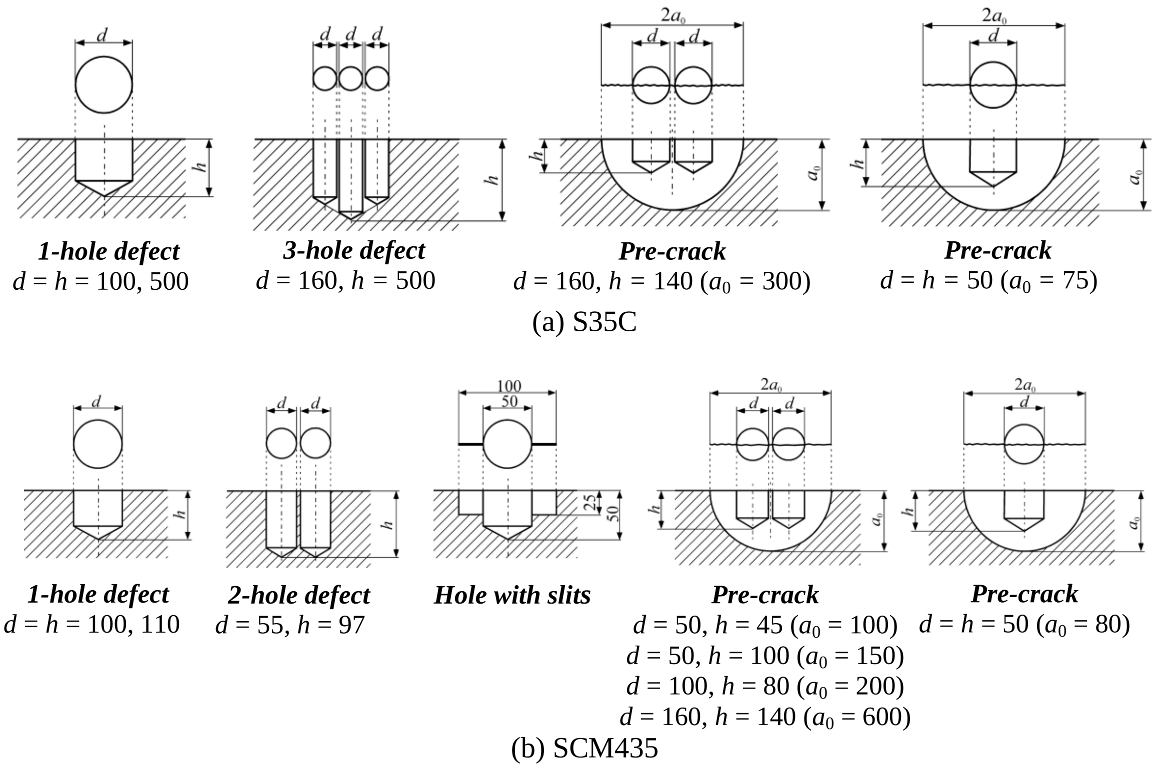


Figure 1. Shapes and dimensions of artificial defects (in μm)

Table 2. The material constants used in the McEvily equation.

Material	σ_{YS} (MPa)	k (m^{-1})	K_{opmax} ($MPa{\cdot}m^{1/2}$)	ΔK_{effth} ($MPa{\cdot}m^{1/2}$)	σ_{w0} (MPa)
S35C	328	6000	3.5	3.0	230
SCM435	858	25000	3.5	3.0	490

RESULTS AND DISCUSSIONS

By conducting the comparisons between the predictions and the experimental data, the predictive capability of the present method was examined. Further, by taking advantage of the $\sqrt{area}$ -parameter model, a simple yet reasonably accurate equation to predict the fatigue limit was proposed.

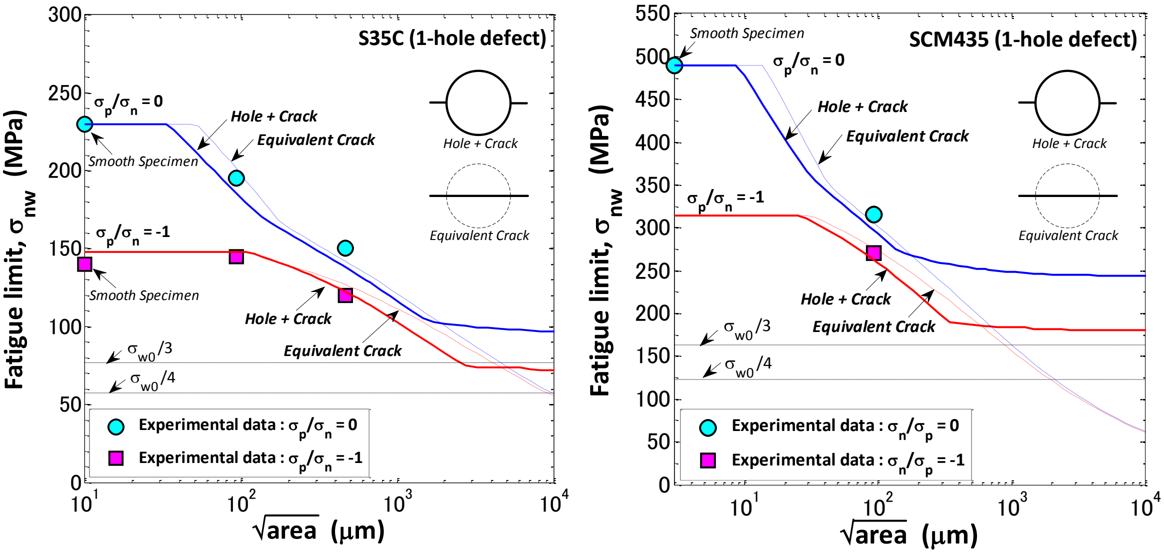
Comparisons with the Experimental Data

For illustration, the fatigue limit for 1-hole defect was predicted and compared to the experimental data. Since the two-dimensional stress analysis was conducted with Eq. (3), the $\sqrt{area}$ of 1-hole defect for prediction was evaluated as [1]:

$$\sqrt{area} = \sqrt{10} \times \frac{d}{2} \tag{7}$$

where d is the diameter of hole. Moreover, the fatigue limit of smooth specimen under biaxial loading can be estimated based on Eqs. (3) and (4):

$$\sigma_{nw0} = \left(\sqrt{2} + 1.12\right)\left[\left(1 - \frac{\sigma_p}{\sigma_n}\right)\sqrt{2} + 1.12\right]^{-1} \times \sigma_{w0} \tag{8}$$



(a) S35C (b) SCM435

Figure 2. Comparisons with the experimental data.

As shown by Figure 2, the predictions are in reasonable agreement with the experimental data. It is noted that when $\sqrt{area}$ is small (i.e., the size of defect is small), two different predictions ("Hole + Crack" and "Equivalent Crack") render similar results. Namely, small defect can be regarded as a crack. With respect to the effect of biaxial stress, though the stress biaxility of σ_p/σ_n = -1 significantly reduces the fatigue limit, the degradation responses (i.e., the slope of the curve) associated with $\sqrt{area}$ is seems to be independent of the stress biaxility.

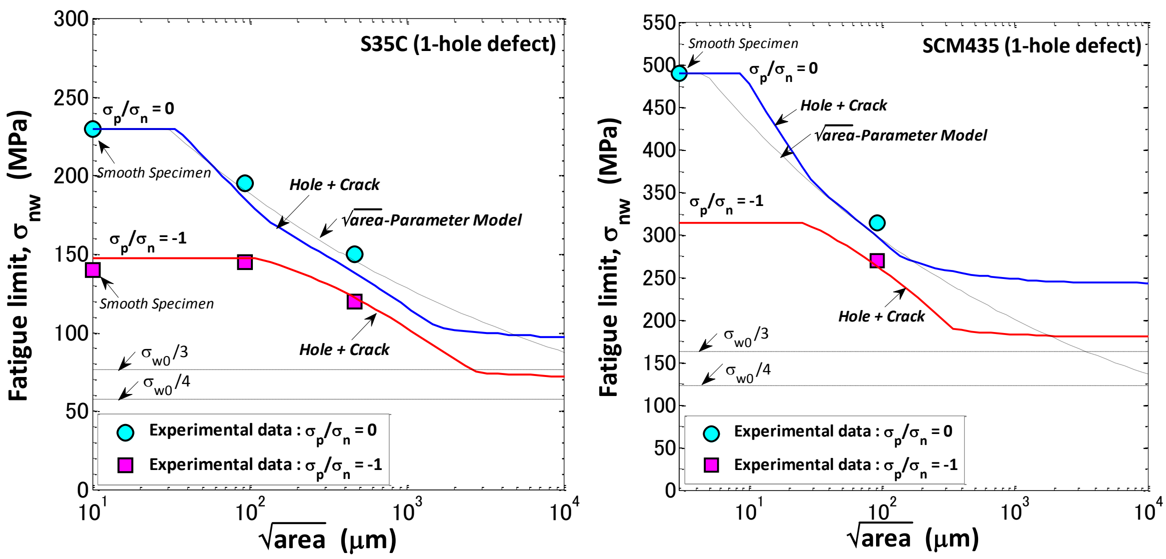
Comparisons with the $\sqrt{area}$ -parameter Model

Figure 3 shows the comparisons of predictions by the McEvily equation and the $\sqrt{area}$ -parameter model. The $\sqrt{area}$ -parameter model renders the fatigue limit by the following equation [1]:

$$\sigma_w = \frac{F_{loc}(HV+120)}{(\sqrt{area})^{1/6}}, \qquad \text{where } F_{loc} = 1.43 \text{ (for surface defect)} \tag{9}$$

As shown, regarding S35C (Figure 3(a)), two predictions render similar results for a wide range of $\sqrt{area}$. On the other hand, the similitude of two predictions in SCM435 can be observed for smaller $\sqrt{area}$ in comparison with S35C.

In practice, the $\sqrt{area}$ -parameter model does not require a fatigue test to make a prediction, and it has been successfully applied for a number of uniaxial fatigue problems. Given the successful predictions demonstrated in Figures 2 and 3, it might be possible to extend the $\sqrt{area}$ -parameter model to the biaxial fatigue problem with the aid of the McEvily equation. Accordingly, in what follows, our effort was devoted to propose a simple yet reasonably accurate equation to predict the fatigue limit.



(a) S35C (b) SCM435

Figure 3. Comparisons with the $\sqrt{area}$ -parameter model.

Extension of the $\sqrt{area}$-parameter Model for Biaxial Fatigue Problem

Based on the previous studies [8], we assumed that the fatigue limit can be characterized by the following equation:

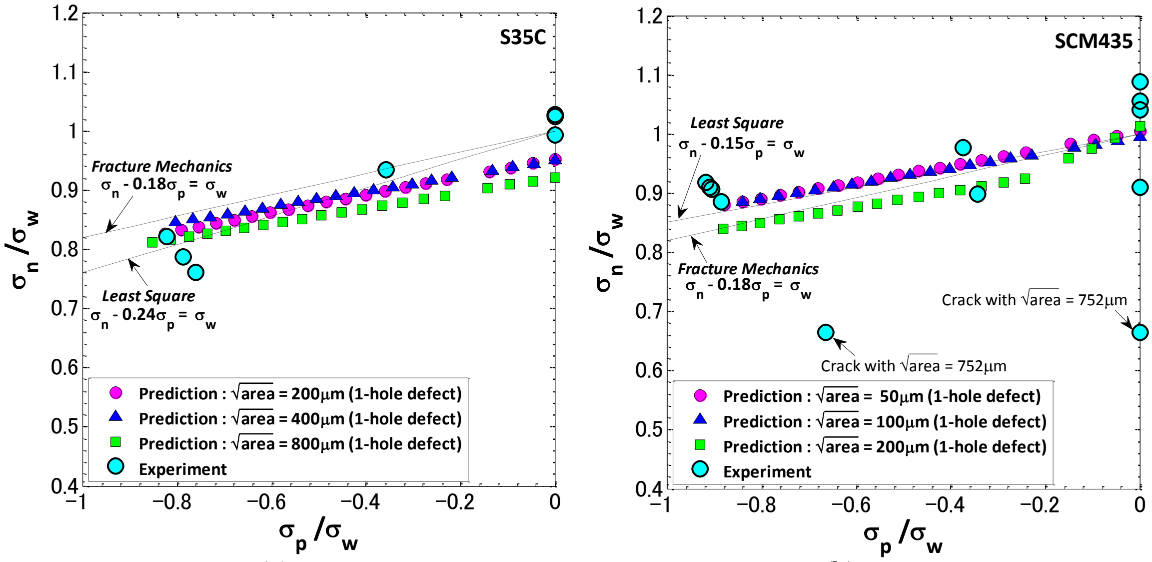
$$\sigma_{\mathrm{n}} + \kappa\sigma_{\mathrm{p}} = \sigma_{\mathrm{w}} \tag{10}$$

where σ_w is calculated by Eq. (9), κ is a parameter that can accounts for the stress biaxility. Based on the fracture mechanics point of view, κ = -0.18 can be rendered [8][9]. In this study, we estimated the parameter κ by making use of the McEvily equation.

Figure 4 shows the relationships between σ_n and σ_p at fatigue limit. The least square fit for the predictions with the McEvily equation renders that κ = -0.24 for S35C and κ = -0.15 for SCM435, which are close to κ = -0.18. It is noted that when the defect is crack with a relatively large $\sqrt{area}$ (=752μm), the experimental data tend to deviate from Eq. (10) as shown by Figure 4(b).

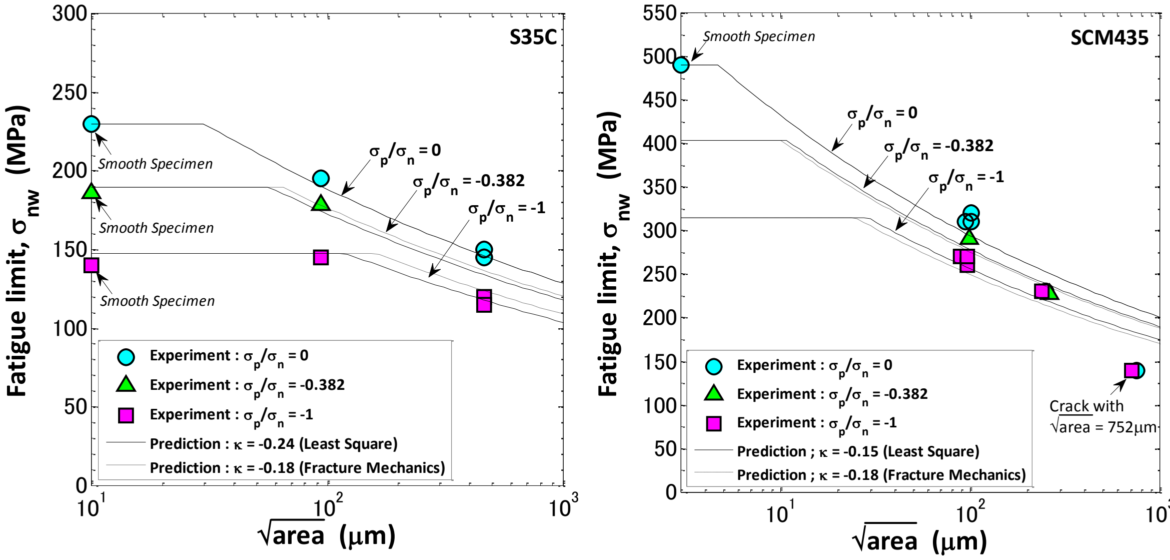
Given the applicability of Eq. (10) demonstrated in Figure 4, the following equation is proposed to estimate the fatigue limit based on Eqs. (9) and (10):

$$\sigma_{\mathrm{nw}} = \left(1+\kappa\frac{\sigma_{\mathrm{p}}}{\sigma_{\mathrm{n}}}\right)^{-1} \times \frac{1.43(HV+120)}{(\sqrt{area})^{1/6}} \tag{11}$$



(a) S35C (b) SCM435

Figure 4. The relationships between σ_n and σ_p.



(a) S35C (b) SCM435

Figure 5. Comparisons between the predictions and the experimental data.

Figure 5 shows the predictions with Eq. (11). As demonstrated, when $\kappa \cong -0.18$, the predictions are in good agreement with the experimental data. It is noted that the present prediction is straightforward and does not require a complex computer code. Concerning the crack with large $\sqrt{area}$ in Figure 5(b), Eq. (11) overestimate the experimental data. On the other hand, McEvily equation is in moderate agreement with the experimental data (cf. Figure 2(b) and Figure 5(b)) though it is computationally expensive.

CONCUDING REMARKS

In this study, the effects of small surface defects and small cracks on the fatigue limit of metallic materials were investigated. Based on the McEvily equation and the $\sqrt{area}$ -parameter model, a simple yet reasonably accurate equation was proposed to predict the fatigue limit.

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