A novel approach to predict the growth rate of short cracks under multiaxial loadings

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ABSTRACT. *The purpose of this paper is to present a method to predict the growth rate of short cracks using a non-propagation criterion proposed by Thieulot-Laure et al. [1] and modified by de Moura Pinho et al. [2] and material data such as the Paris' law and the fatigue threshold for long cracks. The criterion is based on linear elastic fracture mechanics including non-singular terms of the asymptotic developments i.e. the T-stresses. This criterion allows predicting the non-propagation fatigue threshold for short and long cracks. In this approach it is used to predict the crack growth rate of short and long cracks in the near threshold regime.*

INTRODUCTION

Predicting the residual fatigue life of structures that may contain flaws is a major concern in various industrial sectors in which safety is a key issue, such as the aircraft industry for instance. Impact marks or scratches on the surfaces of critical components, for instance, may evolve, when submitted to fatigue loadings, into short fatigue cracks and lead to the failure of the component. The prediction of the growth rate of short crack has long been a concern since the anomalous behaviour of short cracks was exhibited [6, 9]. The criterion developed by Thieulot-Laure *et al.* [1] and modified by de Moura Pinho *et al.* [2] allows predicting the fatigue threshold of short cracks as a function of the crack length, it reproduces, as well as, for instance, the El Haddad's criterion [7], the short crack to long crack transition in the Kitagawa and Takahashi diagram. However, unlike other criteria [7], this criterion can be used in multiaxial loading conditions, including non proportional loadings.

This criterion is based on the assumption that fatigue cracks propagate because free surfaces are created at the crack tip when it experiences plastic deformation. Hence, the criterion [1, 2] is basically a plastic yield criterion *f*, that derives from the von Mises criterion, but expressed in terms of linear elastic fracture mechanics quantites; stress intensity factors and T-stresses $K_I, K_{II}, K_{III}, T, \Gamma, T_z$. In each time step, if the plastic yield criterion *f* is negative, crack tip plasticity and hence fatigue crack propagation can be neglected. As soon as the yield criterion is fulfilled, plastic yield and hence fatigue crack growth can occur. In other words, the effective part of the fatigue cycle is that for which f is not negative. In this paper, the criterion was used to determine the threshold for fatigue crack growth in mode I conditions (K_I, T, T_z). It is then used to predict the fatigue crack growth rate for short or long cracks using the Paris' law of the material [4, 5].

CRITERION

Hypotheses

Emmanuelle Thieulot Laure [1] and Raùl de Moura Pinho [2] introduced a generalized von Mises yield criterion for the crack tip region. In this criterion, fatigue cracks are assumed to propagate only if cyclic plasticity occurs at crack tip. Thus, it is expressed as a threshold for plastic yield for a region of material located within a distance δ to the crack tip and per unit of length of the crack front. The radius δ is a length scale parameter which has to be identified from experiments. Like the El Haddad's equation [7], this criterion requires to know three material parameters:

- the actual flaw size or an equivalent flaw size, a_o,
- the non-propagation threshold for long cracks, ΔK_{th},
- a length scale parameter, δ.

This criterion [1, 2] allows reproducing the short cracks to long cracks transition as in the Kitagawa and Takahashi's diagram [6]. Moreover, once identified in mode I, it allows accounting very naturally for stress multiaxiality effects [3] by considering as many stress intensity factors (K_I, K_{II}, K_{III}) and T-stresses (T, Γ, T_z) as required for a given problem. Let consider, for instance, an infinite media subjected to a uniform diagonal stress tensor **S**.

$$\mathbf{S} = \begin{bmatrix} S_{xx} & 0 & 0 \\ 0 & S_{yy} & 0 \\ 0 & 0 & S_{zz} \end{bmatrix}_{(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})} \tag{1}$$

Let now consider that, a 2D crack with a length $2a$, lying in the plane normal to **y** and with its straight fronts directed by **z** is inserted in this infinite media. If we consider linear elastic conditions, the solution of the problem can be obtained by superposition (cf. Figure 1).

First, the problem can be solved analytically for equibiaxial and plane strain conditions $(S_{xx} = S_{yy}$ and $S_{zz} = 2\nu S_{yy})$. It would yield the Westergaard's solution and $K_I = S_{yy}\sqrt{\pi a}$.

Then, the T-stress $T = S_{xx} - S_{yy}$ can be superimposed to this first solution, to correct it considering now a biaxial and plane strain loading case.

Finally, T_z-stress is added to correct the solution from the fact that the media is not necessarily loaded in plane strain conditions and its value is $T_z = S_{zz} - 2\nu S_{yy} - \nu T$.

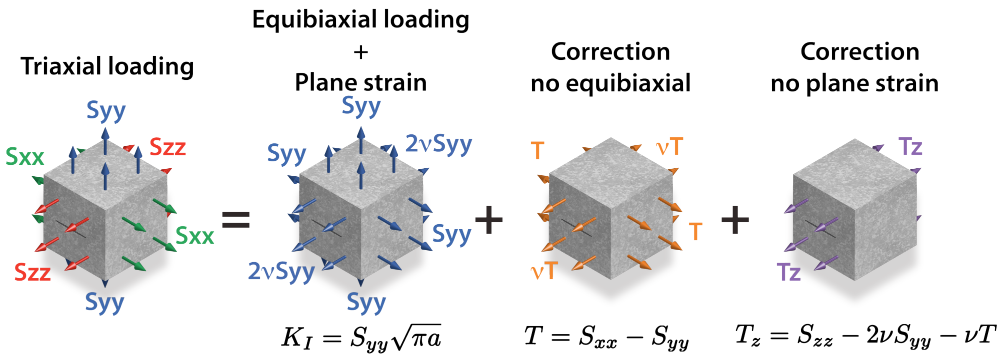

Figure 1. Illustration of the significance of T and T_z stresses

If the stress tensor $\boldsymbol{S}$ was not diagonal, a mode II stress intensity factor would yield from $\boldsymbol{S}_{xy}$, a mode III stress intensity factor from $\boldsymbol{S}_{zy}$ and $\boldsymbol{\Gamma}$ the third T-stress would stem from $\boldsymbol{S}_{xz}$. The number of required fracture mechanics quantities being equal to the number of independent degrees of freedom of the stress tensor in the uncracked component. When 3D cracks in 3D components are considered, the T-stresses can be determined from FE calculations, or can be found in the literature for simple cracks and specimens geometries.

Considering not only the stress intensity factors but also the T-stresses allows therefore accounting for all the components of the stress multiaxiality, but it also introduces a short crack effect. Indeed, the T-stress components are function of the stress field in the uncracked media (cf. Figure 1), while the stress intensity factors are also function of the size of the crack. In the Westergaard's solution, the terms associated with the T-stresses are constant while those associated with the stress intensity factors vary with $(\sqrt{a/r})$. As a result, considering the T-stresses in a fracture mechanics criterion will naturally induce a crack size effect in the criterion. For long cracks, the effect of the T-stresses is usually neglected. But, if short cracks are considered, the effect of the T-stresses cannot be neglected anymore.

Expression of the criterion

We apply the same approach to get the yield criterion for the crack tip region, as that used to express the von Mises criterion for a uniformly loaded volume of material. First, the distortional part of the elastic energy is determined as a function of the quantities used to represent the loadings. Second, a critical value of this elastic energy is determined as a function of an experimental data. And third, the criterion is obtained by assuming that the yield condition is obtained in any multiaxial loading condition for the same critical value of the distortional part of the elastic energy.

To do so, the distortional part of the elastic energy within the crack tip region [1, 2, 8] is calculated using the LEFM stress, strain and displacement fields at crack tip. Since, it is aimed at using this criterion for small cracks, non-singular terms (the T-stresses), are also considered. For instance, for a 2D generalized plane strain problem in mode I, the Westergaard's asymptotic development [3] of the displacement field **u** including the T-stresses, is as follows:

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2.\pi}}\cos\frac{\theta}{2}(\kappa - \cos\theta) + +\frac{T}{8\mu}(\kappa+1)r\cos\theta$$
$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2.\pi}}\sin\frac{\theta}{2}(\kappa - \cos\theta) - \frac{T}{8\mu}(3-\kappa)r\sin\theta \qquad (2)$$
$$u_z = \frac{T_z}{E}z$$

where $2\mu = \frac{E}{1+\nu}$ and $\kappa = 3 - 4\nu$.

The strain tensor $\boldsymbol{\varepsilon}$ is then derived from the displacement field **u** and the stress field $\boldsymbol{\sigma}$ is obtained by the Hooke's law. The distortional elastic energy density $w(r,\theta)$ in each point (r,θ) can be expressed as follows:

$$w(r,\theta) = \frac{1}{2}Tr(\boldsymbol{\sigma}'.\boldsymbol{\varepsilon}') \qquad (3)$$

where $\boldsymbol{\sigma}'$ and $\boldsymbol{\varepsilon}'$ are the deviatoric parts of the stress and strain tensors. The distortional elastic energy density is then integrated over a domain within a distance $\boldsymbol{\delta}$ to the crack tip, to get the distortional energy per unit of length of the crack front $U(K_I,T,T_z)$:

$$U(K_I,T,T_z) = \int_{r=0}^{r=\delta}\int_{\theta=-\pi}^{\theta=\pi} w(r,\theta).rd\theta.dr \qquad (4)$$

The yield criterion is obtained by assuming that the yield condition is obtained in any multiaxial loading condition (K_I,T,T_z) for the same critical value of the distortional part of the elastic energy U_C.

$$U(K_I,T,T_z) = U_C \qquad (5)$$

The value of U_C can be determined from the threshold stress intensity factor amplitude measured for long cracks (i.e. T and Tz are neglected) so that:

$$U_C = U(K_I = K_{IY}, T = 0, T_Z = 0) \qquad (6)$$

with
$$K_{IY} = \Delta K_{Ith\,(R=-1)} \qquad (7)$$

After calculation and some simplifications, the equation (4) becomes:

$$f=\left(\frac{K_I}{K_{IY}}\right)^2+\left(\frac{T}{T_Y}\right)^2+\left(\frac{T_Z}{T_{ZY}}\right)^2+f_1\frac{K_I}{K_{IY}}\frac{T}{T_Y}+f_2\frac{K_I}{K_{IY}}\frac{T_Z}{T_{ZY}}+f_3\frac{T}{T_Y}\frac{T_Z}{T_{ZY}}-1=0 \quad (8)$$

The material parameters are the yield threshold K_{IY}, the distance δ and the Poisson's ratio ν. The length scale parameter $\boldsymbol{\delta}$ allows adjusting the short to long crack transition. The other coefficients are reported in Table 1.

Table 1. Values of the coefficients in equation (8) calculated for ν=0.29.

T_Y	T_{ZY}	f_1	f_2	f_3
$T_Y=\dfrac{0.430K_{IY}}{\sqrt{\delta}}$	$T_{ZY}=\dfrac{0.318K_{IY}}{\sqrt{\delta}}$	-0.419	-0.614	-0.390

The same approach can be used to get the expression of ***f*** when the LEFM quantities are all considered, i.e. the three stress intensity factors (K_I, K_{II}, K_{III}) and the three T-stresses (T, Γ, T_z). No additional material parameter is necessary.

SHORT CRACK GROWTH RATE PREDICTION FOR THE INCO 718 DA ALLOY

The previous non-propagation criterion is now used to model the growth rate of small cracks. To illustrate the methodology the material data of the superalloy INCO 718 DA are considered and mode I, constant amplitude fatigue, conditions are considered.

During each fatigue cycle, different phases may appear. During a loading step, f is first negative (elasticity) then becomes positive above the yield threshold (plasticity occurs). Then, at unloading, ***f*** is positive but ***df*** is negative.

During cycling in constant amplitude fatigue at $R \leq 0$, the effective part of the loading cycle is thus defined by $\boldsymbol{f_{max}}$, which corresponds to the integration of ***df*** over the fatigue cycle, considering only the time steps during which plasticity is promoted i.e. the time steps during which both ***f*** and ***df*** are positive. The Paris' law for long cracks of the material is then used to determine a relation between $\boldsymbol{f_{max}}$ and the crack growth rate per cycle da/dN. This relation is then used to predict the growth rate per cycle for short cracks.

Closure effect and other history effects are not considered there, to account for them, an internal stress state (analogous to a kinematics hardening) and its evolution rule should be introduced for each LEFM quantity [10].

Modification of the Paris-Erdogan's law

The Paris-Erdogan law [5] for the INCO 718 DA superalloy was obtained from [4] and yields, where da/dN is in m/cycle and ΔK in MPa.m$^{1/2}$,

$$\frac{da}{dN} = 1.64 \times 10^{-11} \Delta K^{2.91} \tag{9}$$

Thereafter, f is calculated assuming that the experimental results were obtained for a long crack ($T = T_z = 0$) and $K_{IY} = 2$ MPa$\sqrt{\text{m}}$:

$$f_{max} = \left(\frac{\Delta K}{K_{IY}}\right)^2 - 1 \tag{10}$$

and the experimental crack growth rate per cycle da/dN was then replotted versus f_{max} (cf. Fig. 2).

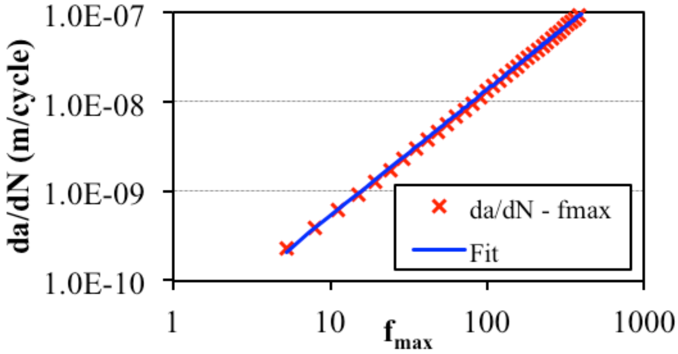

Figure 2. Evolution of da/dN as a function of f_{max}

This curve can be fitted by a power law to obtain the eq. (11):

$$\frac{da}{dN} = \alpha \left[\left(\frac{\Delta K}{K_{IY}}\right)^2 - 1\right]^{\gamma} \tag{11}$$

with $\alpha = 2.02 \times 10^{-11}$ and $\gamma = 1.42$.

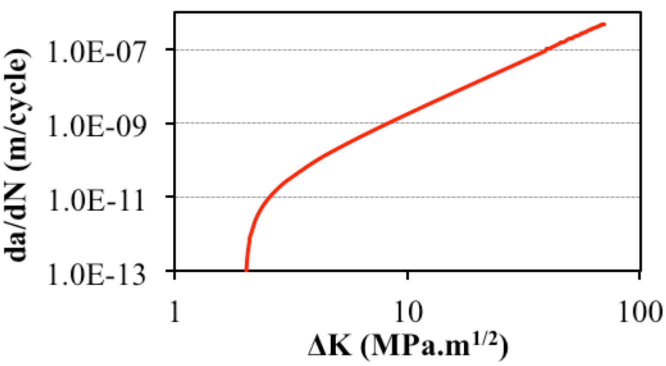

Figure 3. Evolution of crack growth rate da/dN versus ΔK

Identification of the parameter T_y

In the following, it is assumed that the short crack to long crack transition appears for a crack length of 300 µm in uniaxial loading conditions. This assumption is used to determine the parameter T_y of the non propagation threshold ***f=0*** (equation 8). Indeed, T_y is adjusted to get for a=300 µm a threshold stress intensity factor ΔK_{th} of 90% of the long crack threshold. To do so we condidered a through thickness crack with length 2a in plane strain:

$$K_I = S_{yy}\sqrt{\pi a} \text{ and } T = S_{xx} - S_{yy} = -\beta S_{yy} \text{ and } T_z\text{=0} \qquad (12)$$

Where the biaxiality ratio $\beta = \left(\frac{S_{yy}-S_{xx}}{S_{yy}}\right)$. Hence, the threshold can be obtained from equation (8) as follows:

$$f = \left(\frac{\Delta K_{th}}{K_{IY}}\right)^2 + \left(\frac{\beta \Delta K_{th}}{T_Y\sqrt{\pi a}}\right)^2 - f_1\beta\frac{\Delta K_{th}{}^2}{K_{IY}\,T_Y\sqrt{\pi a}} - 1 = 0 \qquad (13)$$

Finally, is identified for $\beta = 1$ at $T_y = 204.74$ MPa and δ =17.6 µm is obtained from the expressions in Table 1.The evolution of ΔK_{th} vs. crack length can be plotted for various biaxiality ratio β (cf. Fig.4).

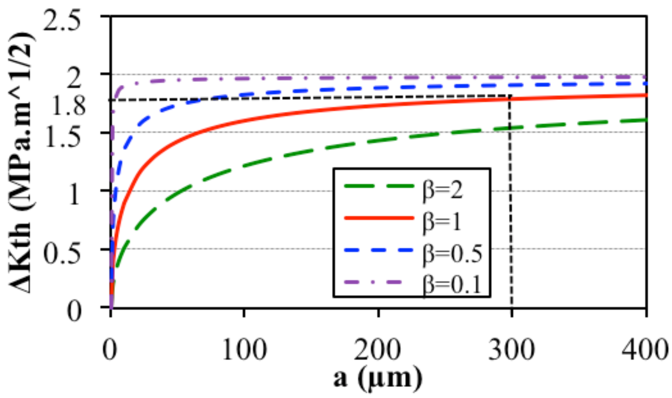

Figure 4. Evolution of the threshold stress intensity factor ΔK_{th} versus crack lenght a

Short crack growth curves

Let consider a semi-circular short crack with a radius $(a_0 + a)$ initiated (with a radius a_0) at the surface of a component loaded by a uniaxial stress S_{yy}. The expressions of K_I, T and T_z are now [2]:

$$K_I = 0.6375\,S_{yy}\sqrt{\pi(a_0 + a)}, T = -0.5564\,S_{yy}, T_z = -0.6882\,S_{yy} \qquad (14)$$

In this case the equation (11) becomes:

$$\frac{da}{dN} = \alpha\left[\left(\frac{K_I}{K_{IY}}\right)^2 + \left(\frac{T}{T_Y}\right)^2 + \left(\frac{T_Z}{T_{ZY}}\right)^2 + f_1\frac{K_I}{K_{IY}}\frac{T}{T_Y} + f_2\frac{K_I}{K_{IY}}\frac{T_Z}{T_{ZY}} + f_3\frac{T}{T_Y}\frac{T_Z}{T_{ZY}} - 1\right]^\gamma \qquad (15)$$

This equation can be used to plot several da/dN – ΔK curves for differents values of the initial crack length using eqs. (14) and (15). The curves plotted in Fig. 5 indicate clearly that the model is able to reproduce the short crack effect [11].

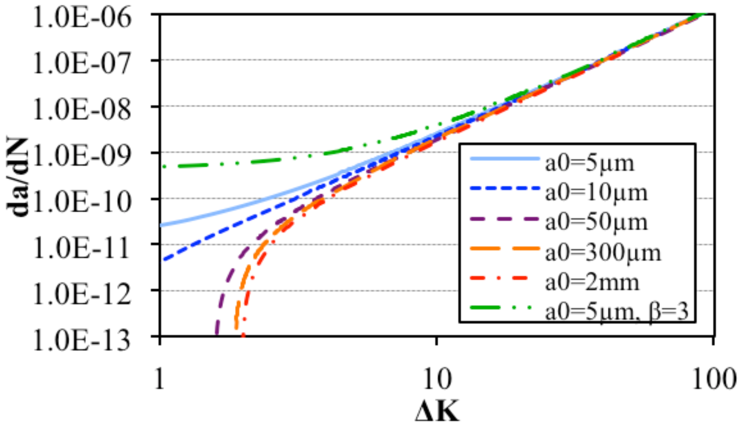

Figure 5. Evolution of da/dN versus ΔK for different a_0 for a semi-circular short crack

CONCLUSION

A plasticity index f was defined to characterize the loading conditions in the crack tip region in multiaxial conditions. This index includes the T-stresses and can hence be used either for short and long cracks. The Paris'law was modified in order to determine the crack growth rate per cycle in constant amplitude fatigue as a function of this plasticity index, and this modified law reproduces well the short crack effect in fatigue.

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