A Stress-based Method to Predict Multiaxial Fatigue Limits

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ABSTRACT. *A new criterion for multiaxial high-cycle fatigue limit for both smooth and notched specimens is proposed using two parameters: one governing crack initiation in Stage I and the other governing initial crack growth in Stage II. The axial fatigue limit for smooth specimen and the true fracture strength are used as the material properties. The prediction accuracies of conventional and the proposed criteria are compared using known data about fatigue limits under different loading conditions. The comparison is provided for the effects of phase difference, mean stress and the combination of phase difference and mean stress. The evaluation error of the proposed criterion is approximately 10%. The prediction accuracy of the proposed criterion is higher than that of the other criteria.*

INTRODUCTION

In this paper, a new approach that uses a combination of two parameters: governing crack initiation in Stage I and initial crack growth in Stage II is proposed as a multiaxial high-cycle fatigue criterion for both smooth and notched specimens [1]. In this criterion, the fatigue limit is regarded as the stress condition where a crack initiated in Stage I stops in initial crack growth. The prediction accuracies of the conventional and the proposed criteria are compared using the following data from previous studies: (1) fatigue limits under combined bending and torsion loadings for smooth specimens, (2) fatigue limits under biaxial loadings for smooth specimens, and (3) fatigue limits under combined bending and torsion loadings for notched specimens. The comparison is provided for the effects of phase difference, mean stress and the combination of phase difference and mean stress in that order. Bending and axial loadings are not distinguished because long crack growth behavior is not focused in this paper.

PROPOSED CRITERION

Parameters for evaluation of fatigue fracture

Nominal stress is used to evaluate the fatigue fracture of smooth surfaces. Whereas nominal stress multiplied by the notch factor is used to evaluate the fatigue fracture of notched surfaces.

Crack initiation parameters

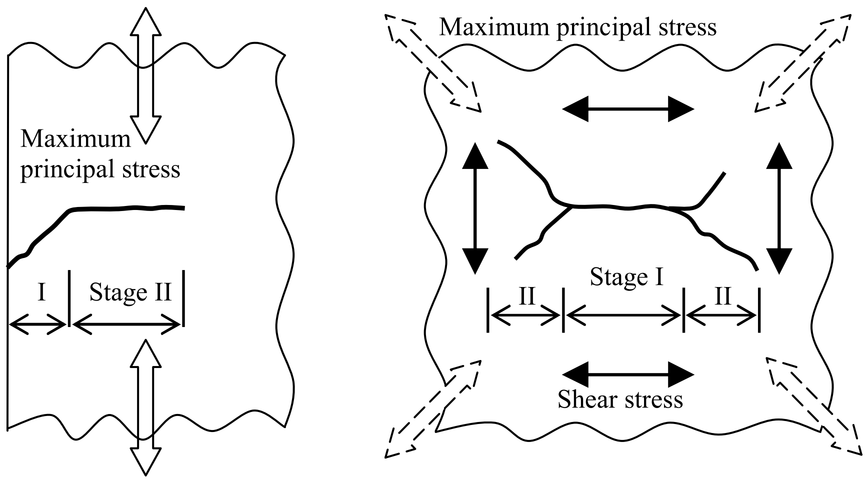
Nishitani et al. experimentally verified that fatigue crack initiation is solely dominated by the shear stress amplitude, independently of mean stress [2]. Therefore, the equivalent shear stress amplitude $\sqrt{J_{2,amp}}$ is used as the parameter for governing crack initiation in the proposed criterion. The definition of the equivalent shear stress amplitude follows the Li criterion [3]. The effect of phase difference can be considered. In the Li criterion, $\sqrt{J_{2,amp}}$ is expressed, using five stresses for stress conversion, as

$$\sqrt{J_{2,amp}} = \sqrt{S_1^2 + S_2^2 + S_3^2 + S_4^2 + S_5^2} \tag{1}$$

where $S_1 = \frac{\sqrt{3}}{2} s_{xx}$, $S_2 = \frac{1}{2}\left(s_{yy} - s_{zz}\right)$, $S_3 = s_{xy}$, $S_4 = s_{xz}$, $S_5 = s_{yz}$.

Initial crack growth parameter

A crack in Stage II propagates perpendicularly to the principal stress direction irrespective of the crack propagation direction in Stage I as shown in Figure 1. In Stage II, mean stress affects not only the crack propagation threshold but also the crack propagation rate. On the other hand, Tanaka et al. confirmed that in a multiaxial stress field, the crack grows in the direction of the maximum of the effective stress intensity factor range and that crack growth behavior is governed by the effective stress intensity factor range [4]. However, they point out that crack closure does not form to any sufficient degree in a small crack region, suggesting that initial crack growth is governed by the maximum of the stress intensity factor range. The crack growth region in the proposed criterion is the initial growth region after crack initiation, in which the crack length is short. For this reason, the crack non-propagation behavior in the proposed criterion is considered to be governed by the stress intensity factor range.



(a) Pure reversed axial (b) Pure reversed torsion

Figure 1. Schematics of crack initiation and growth

The proposed parameter S_{max} that governs initial crack growth is the stress of the maximum of the stress intensity factor range in the fatigue limit. Among the normal cyclic stresses on the critical planes, the normal cyclic stress with the maximum stress range is selected. The maximum stress during the selected stress cycle is defined as S_{max}. The schematic is shown in Figure 2. In the figure, the area where the crack has generated is critical. The cyclic stresses σ_1 to σ_4 are the stresses that occur perpendicularly to the critical plane. σ_1 is the normal cyclic stress with the maximum stress range. The maximum value σ_{1max} of this normal cyclic stress σ_1 is defined as S_{max}.

For evaluation of notch surfaces, stress correction is conducted when S_{max} at a notch root exceeds the yield stress on account of stress concentration. The Neuber's law is used for the stress correction [5]. Koe's equation [6] that is a cyclic stress–strain curve is used in the Neuber's law. The Koe's equation is derived from statistical processing of a large amount of experimental data. Eq. 2 and 3 represent the Neuber's law and the Koe's equation, respectively.

$$K_\varepsilon K_\sigma = K_t^2 \tag{2}$$

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{1.73\sigma_B}\right)^{\frac{1}{0.16}} \tag{3}$$

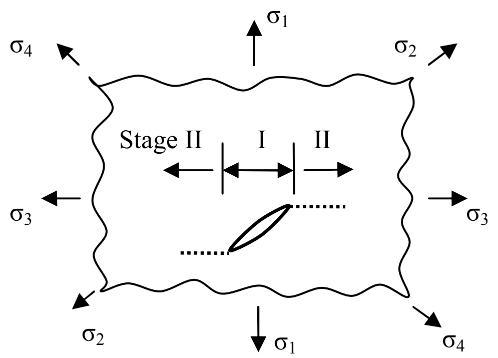


Figure 2. Schematic of crack growth

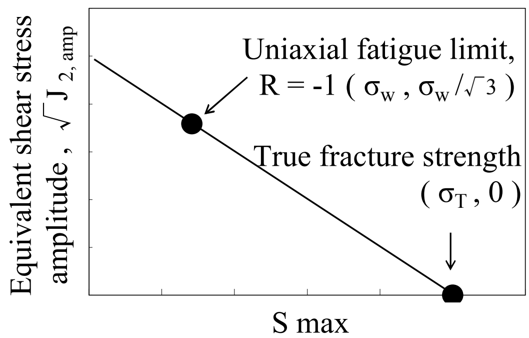


Figure 3. Proposed criterion

Proposed criterion equation

Conventional multiaxial high-cycle fatigue criteria use the bending and torsion fatigue limits as material properties. However, the torsion fatigue limit is equivalent to the axial fatigue limit in the proposed criterion. In the proposed crack initiation parameter $\sqrt{J_{2,amp}}$ and initial crack growth parameter S_{max}, the parameters of the fully reversed torsion fatigue limit τ_w are considered to be equal to the parameters of the axial fatigue limit ($\sigma_{max}=\tau_w$, $\sigma_{amp}=\tau_w\times\sqrt{3}$). In the proposed criterion, the torsion fatigue limit is not the specific material property, rather the material property is the value that indicate the effect of the mean stress of the fatigue limit.

The Goodman law is the criterion that evaluates the effect of the mean stress of the fatigue limit. Nishihara et al. proposed a modified Goodman law in 1938 [7]. The modified Goodman law linearly connects the fully reversed bending fatigue limit σ_w to the true fracture strength σ_T. The prediction accuracy of the fatigue limit with mean stress is improved. Thus, the material properties in the proposed criterion for both the smooth specimen and the notched specimen use the fully reversed axial fatigue limit σ_w of the smooth specimen and the true fracture strength σ_T. The equation for the proposed criterion is given by

$$\sqrt{J_{2,amp}} + \alpha S_{max} = \beta \tag{4}$$

$$\text{where} \quad \alpha = \frac{\sigma_w}{\sqrt{3}} \frac{1}{\sigma_T - \sigma_w}, \quad \beta = \frac{\sigma_w}{\sqrt{3}} \frac{\sigma_T}{\sigma_T - \sigma_w}.$$

Here, $\sqrt{J_{2,amp}}$ is the equivalent shear stress amplitude in the Li criterion. S_{max} is the proposed parameter. The material properties are the fully reversed axial fatigue limit σ_w of the smooth specimen and the true fracture strength σ_T. Figure 3 shows the proposed evaluation diagram. The parameters α and β of the proposed equation are derived from the fully reversed uniaxial fatigue limit (S_{max}=σ_w , $\sqrt{J_{2,amp}}$ =$\sigma_w/\sqrt{3}$) and the true fracture strength (S_{max}=σ_T , $\sqrt{J_{2,amp}}$ =0).

VALIDIATION OF PROPOSED CRITERION

The fatigue limit data for steels which are drawn from previous studies are used for validation. Three types of data are used: (1) fatigue limits under combined bending stress σ and torsion stress τ for smooth specimens [8-11], (2) fatigue limits under biaxial stresses σ_1 and σ_2 for smooth specimens [10,12], and (3) fatigue limits under combined bending stress σ and torsion stress τ for notched specimens [13,14]. The accuracy of the Li criterion is comparable to that of the Papadopoulos criterion [15, 16] and the Mamiya and Araújo criterion [17]. The Li criterion is a criterion that is improved to evaluate simply the effect of the phase difference based on the Crosland criterion in the stress invariant approach [18]. Therefore, the Li criterion and the Crosland criterion are used for comparison of the proposed criterion.

The error index is given by Eq. 5. The numerator of Eq. 5 is the relative difference between the left-hand side, for which the experimental value is substituted, and the right-hand side in each estimate equation. The denominator is the right-hand side of the estimation equation.

$$I = \frac{\text{left hand side} - \text{right hand side}}{\text{right hand side}} \times 100 \tag{5}$$

Combined bending stress σ and torsion stress τ for smooth specimens

Figure 4 shows the relationship between the mean absolute error and the phase difference for smooth specimens from previous data [8-11] with mean stress zero and phase differences. The mean absolute error in the Crossland criterion increases as the phase difference increases. The effect of phase difference is negligible and the maximum error is less than 10% in the Li criterion and the proposed criterion.

Figure 5 shows the relationship between the absolute error and the stress ratio for smooth specimens without phase difference and with mean stress. Diamond symbols indicate data with mean bending stress, and square symbols indicate data with mean torsion stress. In the Li criterion, the error appears to be large when R = 0, however, the errors of data when R = -1 are small. The maximum error of the Li criterion is 15%, whereas that of the proposed criterion is less than 10%.

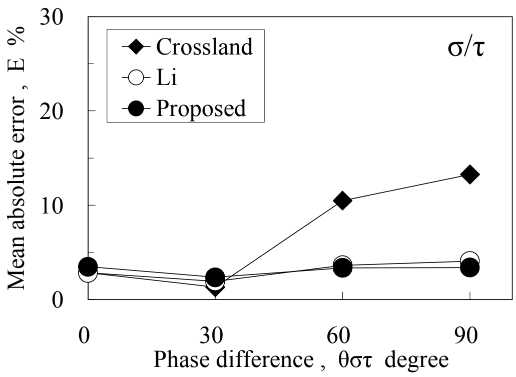


Figure 4. Relationship between E and $\theta_{\sigma\tau}$

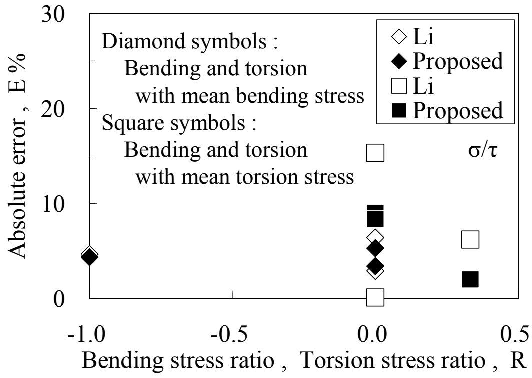


Figure 5. Relationship between E and R

Figure 6 shows the fatigue test results for smooth specimens under combined bending and torsion loadings. The vertical axis indicates the non-dimensional equivalent shear stress amplitude following each criterion. In the figure, the solid line indicates the non-dimensional equivalent shear stress amplitude under each criterion, and the dashed lines indicate a range of ±10% from the solid line. Circle symbols in the figure indicate data with mean stress and phase difference, and square symbols indicate the data in other cases. The figure shows that there are no differences in accuracy between the data with mean stress and phase difference and the data in other cases. The experimental data are within the error of approximately ±10% in the Li and the proposed criteria. Under combined bending and torsion loadings, there are no significant differences between the Li and proposed criteria.

Biaxial stresses σ_1 and σ_2 for smooth specimens

Figure 7 shows the relationship between the absolute error and the stress ratio under biaxial loadings. The data without phase difference and with mean stress are shown in the figure [10, 12]. The error index appears to be large at R = 0 as in the case of the combined bending and torsion loadings in Figure 5. The maximum error of the proposed criterion under biaxial loadings is 12%, whereas that of the Li criterion is as high as 30%.

Figure 8 shows the results of the fatigue test with biaxial loadings. The format of Figure 8 follows that of Figure 6. Circle symbols indicate the data with mean stress and phase difference, and square symbols indicate the data in other cases. The figure shows that there are no differences in accuracy between the data with mean stress and phase difference and the data in other cases. In the Li criterion, the solid line has a high value with more than 10% deviation from the experimental data. The Li criterion is a non-conservative evaluation under biaxial loadings. However, the error of the proposed criterion is within ±10% similar to the case of combined bending and torsion loadings.

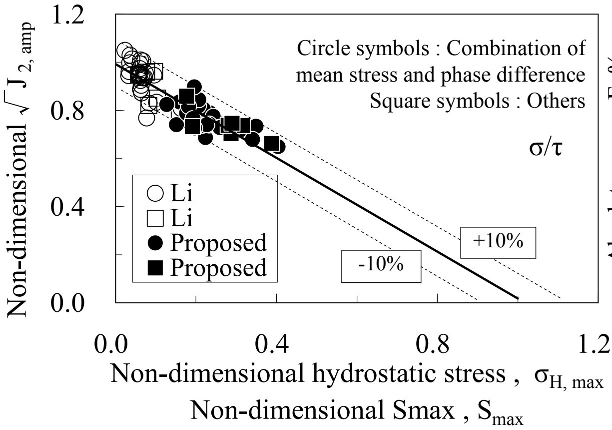


Figure 6. Comparison of predicted results with experimental results under σ and τ

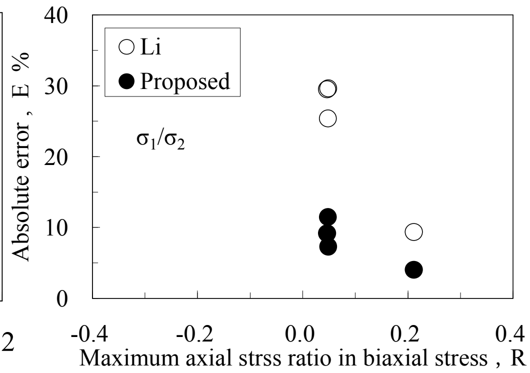


Figure 7. Relationship between E and R

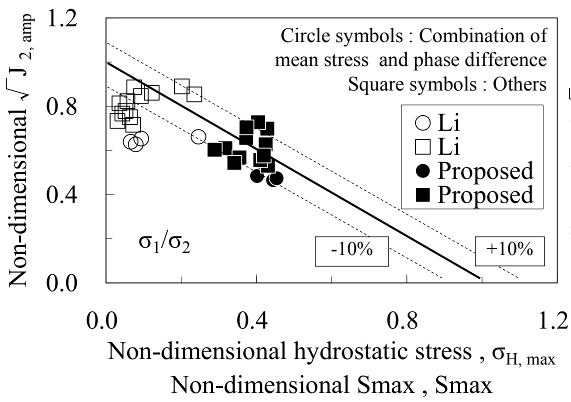


Figure 8. Comparison of predicted results with experimental results under σ_1 and σ_2

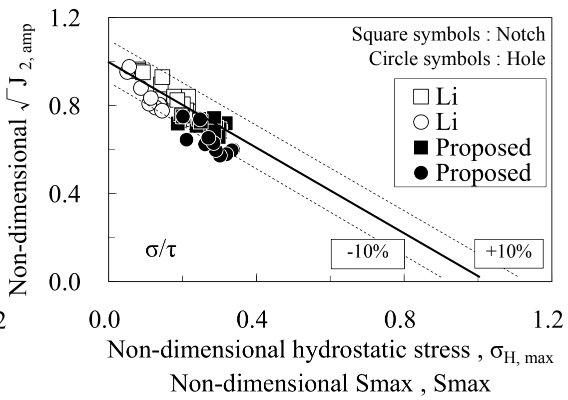


Figure 9. Comparison of predicted results with experimental results under σ and τ

Combined bending stress σ and torsion stress τ for notched specimens

Figure 9 shows the fatigue test results for notched specimens with combined bending and torsion loadings without mean stress and phase difference [13, 14]. The format of Figure 9 follows that of Figure 6. Square symbols indicate the data of circular notched specimens, and circle symbols indicate the data of specimens with circular hole. The figure shows that there is no difference in the accuracies of the data for the notch and for the circular hole between both criteria. The majority of the experimental data without mean stress for notced specimens for both criteria falls within the ±10 % range.

Figure 10 shows the relationship between the absolute error and the maximum Mises stress for notched specimens with combined bending and torsion loadings with mean stress and without phase difference in S65A [13]. The maximum Mises stress of the horizontal axis is the maximum stress among the cyclic mises stresses with combined bending and torsion loadings. Nominal stresses multiplied by the stress concentration factors for the bending and torsion loadings are used to calculate the Mises stress. In the proposed criterion, nominal stress multiplied by the notch factor is used for evaluating the fatigue limit. When S_{max} exceeds the yield stress, the stress correction is conducted by the Neuber's law. Circlule symbols indicate data with combined cyclic bending and cyclic torsion loadings. Square symbols indicate data with combined constant bending and cyclic torsion loadings. Diamond symbols indicate data with combined cyclic bending and constant torsion loadings. The figure shows the error of the proposed criterion is smaller than that of the Li criterion. However, the error of the proposed criterion of the notched specimens appears to be larger than that of the proposed criterion of the smooth specimens with mean stress. The error of the Li criterion for the case of combined cyclic bending and cyclic torsion loadings is larger than that for the case of the others. The maximum error of the Li criterion for the case of the others is within 20% similar to the case of combined bending and torsion loadings for the smooth specimen. Therefore, the experimental data of the case of combined cyclic bending and cyclic torsion loadings appears to be larger than that of the others.

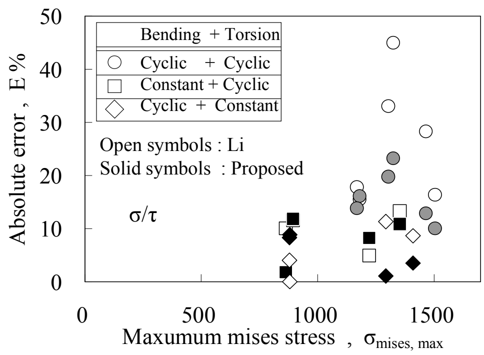


Figure 10. Relationship between E and $\sigma_{mises,max}$

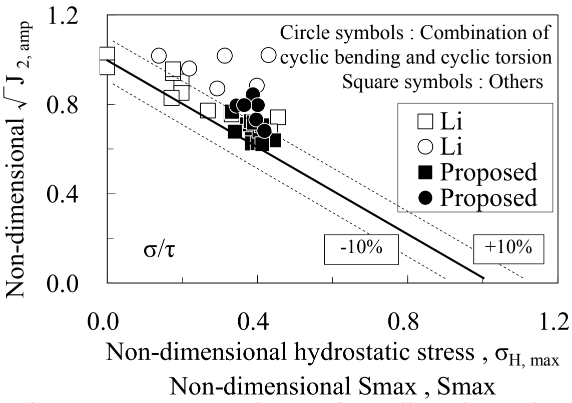


Figure 11. Comparison of predicted results with experimental results under σ and τ

Figure 11 shows the fatigue test results for notched specimens with combined bending and torsion loadings with mean stress and without phase difference in S65A [13]. The format of Figure 11 follows that of Figure 6. Circle symbols indicate the data with combined cyclic bending and cyclic torsion loadings, and square symbols indicate the data in other cases. The figure shows that the accuracy of the proposed criterion is higher than that of the Li criterion. The error of the proposed criterion is within ±10% except for combined cyclic bending and cyclic torsion loadings.

CONCLUSION

A new fatigue criterion for evaluating fatigue both smooth and notched specimens under multiaxial high-cycle stress is proposed. The proposed criterion uses a parameter for governing crack initiation in Stage I and a parameter for governing initial crack growth in Stage II. The material properties required for evaluation are the axial fatigue limit for the smooth specimen and the true facture strength. The evaluation error of the proposed criterion is approximately 10%. The prediction accuracy of the proposed criterion is higher than that of the other criteria.

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