Comparison of Frequency-Domain Methods for a Vibration-Fatigue-Life Estimation

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ABSTRACT. *Vibration fatigue analysis is a two part process. A structure model must be built, be it in a simulated or real environment, which enables the study of stress loads in material points. On the basis of those stress states, different methods are then used to asses fatigue life. Quite a few frequency domain methods exist, that can operate on obtained power spectral densities, to give the fatigue life estimate. The results differ among methods and thorough analysis must be made, before such methods can be applied with confidence. Comparison studies have already been made, but they compare different sets of methods on different data. Based on specifics of structural dynamics responses, a new approach to comparison has been devised. Different groups of spectra are aimed at evaluating the sensitivity of the frequency domain methods on different changes in spectra, such as changing the distance between modes or raising and lowering the level of background noise. The accuracy is also tested on some typical accelerated-vibration test spectra, often used in the automotive industry. To come as close as possible to real-world problems, stress data is measured in the form of electro-dynamic shaker vibration. Fatigue life estimates are compared based on the value of fatigue life given by more conventional approach in the time-domain. Conclusions show, that methods by Tovo and Benasciutti, Zhao and Baker and by Dirlik give good results.*

INTRODUCTION

For the purpose of vibration fatigue analysis, knowledge of structural dynamics and fatigue must be combined. This paper researches the available methods that can be used to evaluate the fatigue-life of a structure (or to find a critical point), once the stress response in the nodes on the structure is known. A number of frequency domain methods for fatigue life analysis have been collected and compared side-by-side, based on some reference estimate, that was calculated by a more conventional approach in the time domain (using the Palmgren-Miner [1, 2] hypothesis and the Rainflow count [3]). The methods considered for comparison are: narrow-band approximation [4], empirical $\alpha_{0.75}$ [5], Wirsching-Light [6], Gao-Moan [7], Tovo-Benasciutti (2 versions) [8, 9], Zhao-Baker (2 versions) [10] and Petrucci-Zuccarello [11].

Similar studies have already been made, but the choice of compared methods is different, and so are the analyzed random responses. Aforementioned studies (*e.g.* by Gao and Moan [7] and by Tovo and Benasciutti [9]) find the Dirlik method to be most accurate and this is indeed one of the more popular methods available [12]. However, it is relatively old, being devised in 1985 [13] and quite a few methods have been developed since.

Before relying on any of the frequency domain methods which have been developed based on numerous numerically simulated spectra, the accuracy of results must be evaluated specifically on the spectra that are of interest. One should aim to chose the best possible (*i.e.* most accurate) method for the specific application. In the course of this research, the studied spectra are those exhibited by material nodes in a structure, responding to some random vibration of the Gaussian distribution.

Different groups of carefully devised power-spectral-densities were analyzed using the collected methods, a total of 27 spectra. Spectral shapes were determined based on typical structural responses of structures and on accelerated-vibration-tests spectra, that are often used in the automotive industry.

The data was produced by means of experiment, to resemble as close as possible a real-world scenario. The rainflow counting method, developed by Matsuishi and Endo [3] was used to calculate the reference time-domain fatigue-life estimate. To take into account the effect of material (*i.e.* the $S-N$ curve parameters), pointed out by existing studies [14, 9], three different sets of parameters, found in literature [11] were used.

Throughout this study, the basics of vibration fatigue are presented in the following section. The experiment is described next. Concluding remarks are given in the end.

VIBRATION FATIGUE

When approaching fatigue-life evaluation of a structure, the stress distribution is first calculated or measured. Modal analysis is a powerful tool that can aid in deducing the response spectra in material nodes. The basics of this approach are presented in the following section, but for a detailed explanation, the reader is referred to [15].

Once the responses are known, they are characterized based on spectral moments. More detailed explanation can be found in books by Shin and Hammond [16] or by Newland [17]. Only very brief introduction is made in the following sections.

Structural Dynamics

Modal analysis can already be applied in FEM environment [18, 19, 20] and supplemented later with the experimental modal analysis. Structures are modeled as linear, multi-degrees-of-freedom systems with second-order differential law [15]:

$$[\mathbf{M}]\,\{\ddot{\mathbf{x}}\} + [\mathbf{C}]\,\{\dot{\mathbf{x}}\} + [\mathbf{K}]\,\{\mathbf{x}\} = \{\mathbf{f}\} \tag{1}$$

where $[\mathbf{M}]$ is the mass matrix, $[\mathbf{C}]$ is the damping matrix, $[\mathbf{K}]$ is the stiffness matrix, $\{\mathbf{x}\}$ is the vector of degree- of-freedom and $\{\mathbf{f}\}$ is the excitation force vector. The frequency-response-functions (FRF) connecting a material point on the structure to another point are deduced. There are different FRFs, that can be calculated, but for fatigue-life evaluation, the FRF from acceleration a (vibration load) to material stress s is of special interest. For many nodes it takes the matrix form:

$$\mathbf{s} = \mathbf{H_{as}}\,\mathbf{a} \tag{2}$$

where $\mathbf{s}$ is the stress tensor in the frequency domain, $\mathbf{a}$ is the forced vibration acceleration profile and $\mathbf{H_{as}}$ is the FRF matrix. Once the vibration load $\mathbf{a}$ and the transfer functions are known, stresses $\mathbf{s}$ can be calculated and evaluated for fatigue.

Fatigue-Life in Frequency-Domain

In fatigue analysis, the Palmgren-Miner [1, 2] hypothesis is used to sum the damages of respective cycles, that have been identified beforehand using the Rainflow [3] counting method. The accumulated damage can be written as a product of time T and damage intensity $\overline{D}$:

$$D = T\,\overline{D} = T\left[\nu_p\,C^{-1}\int_0^\infty s^k\,p_a(s)\,\mathrm{d}s\right] \tag{3}$$

where C and k are material parameters, that define the $S-N$ curve. ν_p gives the expected frequency of peaks and the peak amplitudes are given with probability density function $p_a(s)$. These values, as it turns out, can be estimated using the moments of the PSD response spectra [17]:

$$m_i = \int_0^\infty f^i\,G_{XX}(f)\,\mathrm{d}f. \tag{4}$$

$$\nu_p = \sqrt{\frac{m_4}{m_2}} \tag{5}$$

where $G_{XX}(f)$ is a one sided PSD of the stress response, and its moments correspond to the variance of the random process σ_X^2 and, for the higher moments, the variances of the derivatives of the random process $\sigma_{\dot{X}}^2$.

$$\sigma_X^2 = m_0 \qquad\qquad \sigma_{\dot{X}}^2 = m_2 \tag{6}$$

The probability density function $p_a(s)$ is calculated differently, depending on the frequency-domain method used, but in most cases it is derived empirically, because of the complexity of the Rainflow counting rules that are applied in the time-domain.

A group of different frequency domain methods was compared side-by-side in this study, namely: the narrow-band approximation [4], Wirsching-Light method (WL) [6], the $\alpha_{0.75}$ method (AL) [5], the Gao-Moan method (GM) [7], the Dirlik method (DK) [13], both Zhao-Baker methods (ZB1 and ZB2) [10], the Tovo-Benasciutti methods (TB1 and

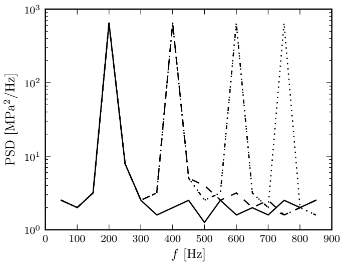

Figure 1: The multi-mode (MM) group of spectra.

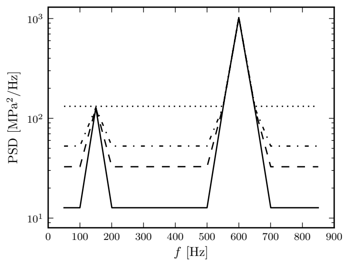

Figure 2: The background noise (BN) group of spectra.

TB2) [8, 9] and the Petrucci-Zuccarello method (PZ) [11]. The formulae for the considered methods are available in literature. All listed methods are aimed at analysis of Gaussian processes.

EXPERIMENT

Real data was obtained using an experimental setup, by which the stress response spectra were simulated directly on the electro-dynamic shaker. The collected data was then processed using different frequency-domain methods and also with more standard time-domain approach, for which it was assumed it is correct.

Response Spectra

The spectra, considered for the comparison of the accuracy of frequency-domain methods were devised based on experience in structural dynamics analysis. Specifically the spectra that characterize response in different material nodes were the focus of our research. Producing each of the devised response spectra using a vibrating structure would be very hard to achieve and since the response was already predicted, it was simulated on the shaker directly, without any structure used.

Five different groups of response spectra were conceived, namely the multi-mode (MM), background noise (BN), spectral width (SW), close modes (CM) and automotive (AM) group. The spectra were varied by some parameter in each group, such as the number of modes, level of background noise (see Figures 1 and 2) and distance between modes. This way, the effect of different structural dynamics phenomena on the fatigue-life estimate accuracy was studied. The automotive group contained profiles, that are often used in the automotive industry for accelerated-vibration tests, to validate the fatigue strength of a part or component. The frequency range of each spectra was 10 to 1000 Hz.

Experimental set-up

The experimental set-up consisted of an electro-dynamic shaker, shaker controller, conditioning amplifier and data acquisition hardware. The spectra were programmed into

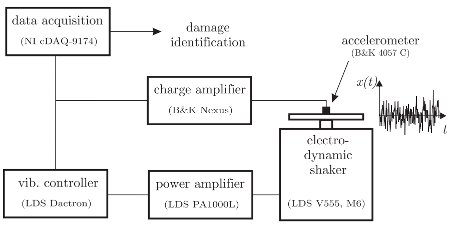

Figure 3: Schematics of the experiment.

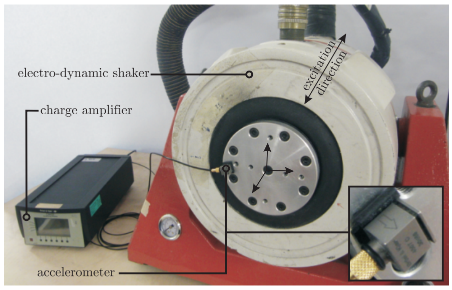

Figure 4: Electro-dynamic shaker with the conditioning amplifier and the accelerometer.

shaker controller and an accelerometer, mounted on the vibrating plate measured 5 min time-histories of vibration, that were processed later on. The fatigue-life estimate was found to converge well for a 5 min sample.

Results

The calculations were made for different materials, namely steel, aluminum and spring steel. The relative error T_{err} of each frequency-domain method T^{XX} was calculated in reference to the time-domain approach estimate T^{RFC}:

$$T_{\text{err}} = \frac{T^{\text{XX}} - T_{\text{RFC}}}{T_{\text{RFC}}} \tag{7}$$

After close inspection of the results it was clear, that methods by Tovo and Benasciutti, Dirlik and by Zhao and Baker give results well above average. These methods are compared side-by-side in Figures 5, 6 and 7. For each comparison a different pair of material parameters was used: steel with $k = 3.324$, aluminum with $k = 7.300$ and spring steel with $k = 11.730$ [11].

CONCLUSIONS

Different frequency-domain methods for fatigue-life evaluation have been compared side-by-side in the course of this research. Real data was used, obtained by experiment and carefully constructed response spectra were used in comparison.

The comparison was made based on the reference estimate, provided by the time-domain approach. Although the Palmgren-Miner hypothesis is not exact in the sense that it disregards the importance of the sequence and frequency with which the loading cycles occur, it can still be used as a good measure of accuracy of frequency-domain methods based on its wide-spread use. The main point is actually the ability to recover cycle amplitudes directly in the frequency-domain.

Data, gathered by the application of frequency-domain methods shows, that a narrow group of methods is especially suitable for fatigue-life evaluation. The Tovo-Benasciutti,

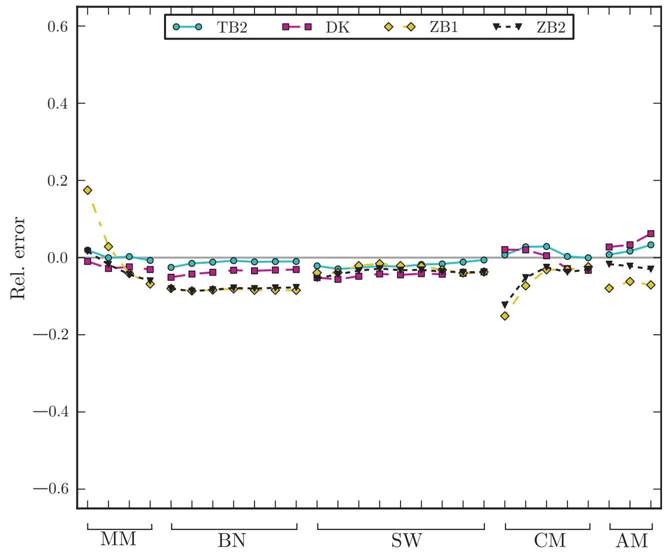

Figure 5: Comparison of relative errors for $k = 3.324$.

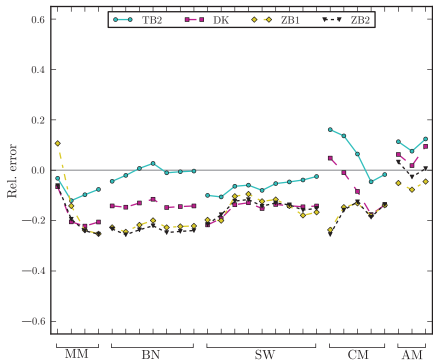

Figure 6: Comparison of relative errors for $k = 7.300$.

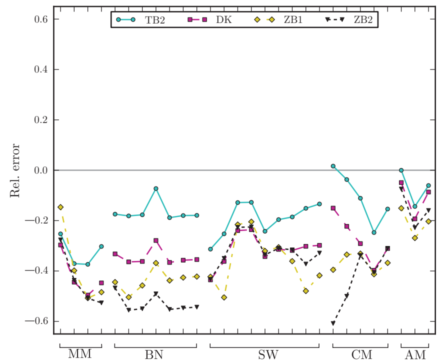

Figure 7: Comparison of relative errors for $k = 11.73$.

Dirlik and Zhao-Baker methods should be the preferred choice as they proved to be most accurate on the spectra used in this research.

The most accurate method for the typical automotive-industry accelerated test profiles was the improved Zhao-Baker method, consistently providing a conservative fatigue-damage estimate. Overall, the improved Tovo-Benasciutti and Dirlik methods were most accurate. Results are a bit different from those presented by Benasciutti and Tovo [5, 9], where Dirlik was deemed most accurate.

The observed effect of the $S-N$ curve parameter k was expected. Studies by Buoyssy *et al.* [14] and by Benasciutti and Tovo [5] came to similar conclusions. For high values of parameter k, the errors and inconsistencies in results grow bigger.

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