Localization rules for the extension of a HCF two-scale damage model to a Lemaitre LCF damage model

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ABSTRACT. *The aim of this work is to develop a lifetime prediction model based on continuum damage mechanics that can unify the Low Cycle Fatigue (LCF) and High Cycle Fatigue (HCF) domains. The concept of an inclusion embedded in a matrix is used to describe the fact that plasticity is no more observable when the load is below the yield stress. A review of the localization rules used in the literature for damage modeling is done and two new ones have been proposed. A numerical implementation allows us to calculate the responses of the inclusions for the different localization rules. Some calculations are presented under cyclic loadings.*

INTRODUCTION

Space engines are submitted to vibrations that lead to failure after a high number of cycles (HCF). During the starting phase of the engine, the components are submitted to high amplitude cycles of loading that are representative of the Low Cycle Fatigue domain (LCF). A good prediction of the components lifetime would then take into account both cases. A damage model based on the Lemaitre's damage law [1] gives a good response in the case of Low Cycle Fatigue when plasticity and damage may be observed at the scale of the structure (macro-scale). When the structure is submitted to low amplitude solicitations, for instance vibrations, the material behavior remains elastic at the macro-scale. However failure finally occurs due to micro-plasticity and micro-damage that leads to the initiation of a macro-crack. A two-scale damage approach has been developed in LMT Cachan in order to solve the problem of lifetime prediction in the HCF domain [2–4]. The aim of this work is to extend the validity of a multiaxial HCF model to the LCF domain in order to tackle the case of complex loadings such as experienced by space engines components.

INITIAL TWO-SCALE DAMAGE MODEL

Continuum damage mechanics is a powerful tool to predict crack initiation in a structure under multiaxial and random loadings. It handles as well monotonic and cyclic solicitations. Based on the concept of effective stress, Lemaitre [1] developed a thermodynamic framework that gathers elasticity, plasticity and damage equations. The incremental resolution of those equations gives access to the evolution of some internal variables that represent the material state. However, the model introduces a damage evolution driven by plasticity (and enhanced by the stress triaxiality). Thus lifetime prediction can be done under LCF conditions but is not straight forwardly possible under HCF conditions (when the structure remains elastic). A two-scale approach has been then developed to tackle this difficulty [2–4]. Plasticity is supposed to occurs at a lower scale (defects in the microstructure). The micro-plastic sites are gathered into a virtual inclusion with a lower yield stress taken equal to the fatigue limit. An elastic (possibly elastoplastic) calculation is performed at the structure scale from which the loading at critical points is taken. The loading is then applied to a Representative Volume Element (meso-scale) and a scale transition, based on the Eshelby inclusion problem, is used to calculate plasticity and damage in the inclusion. The time integration of the constitutive equations (elasticity and plasticity coupled with damage) at this micro-scale leads then to a predicted lifetime.

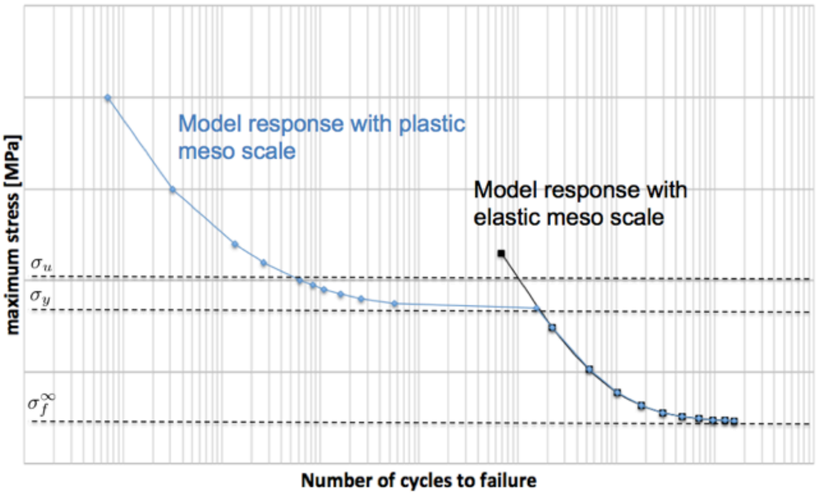

Figure 1: SN curve: response of the initial two-scale damage model from an elastic or an elastoplastic calculation at meso-scale.

From those two models, we aim at building a unified model able to predict the lifetime for the whole range of loading level from the fatigue limit to the ultimate stress level (monotonic failure). The transition domain between LCF and HCF (loading level just above the elastic limit) has to be treated carefully. The Eshelby-Kröner transition law currently used doesn't take enough into account the plastic evolution of the meso-scale. A review of the scale transition rules is presented next. A general framework is proposed to unify the literature rules for the spherical inclusion problem. Two proposals will be made to get flexibility on the behavior response of the inclusion.

LOCALIZATION RULES UNIFIED IN A GENERAL FRAME

In this work, the approach is based on the inclusion problem for which Eshelby gave an exact analytical solution under some restrictive hypothesis. The idea is to calculate the stress $\boldsymbol{\sigma}^{\mu}$ and strain $\boldsymbol{\varepsilon}^{\mu}$ fields to which an inclusion is submitted knowing the stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\varepsilon}$ fields applied to the matrix far from the inclusion, making the hypothesis of no interaction between different inclusions. The stress field in the inclusion is equal to the stress field seen by the matrix at infinity corrected with a mismatch stress field. The strain field is also corrected using a strain field called mismatch strain field. The upper script „F" that indicates the mismatch fields is used by reference to the „Free" strain introduced by Eshelby.

$$\boldsymbol{\sigma}^{\mu} = \boldsymbol{\sigma} - (\mathbb{I} - \mathbb{S}) : \boldsymbol{\sigma}^{F} \tag{1}$$

$$\boldsymbol{\epsilon}^{\mu} = \boldsymbol{\epsilon} + \mathbb{S} : \boldsymbol{\epsilon}^{F} \tag{2}$$

The tensor $\mathbb{I}$ is the fourth order identity tensor and $\mathbb{S}$ is a fourth order tensor introduced by Eshelby to describe the geometrical aspect of the inclusion. In the case of an elastic spheroidal inclusion, the expression of the Eshelby's tensor depends only on the Poisson ratio ν:

$$\mathbb{S} = \frac{\alpha}{3}\mathbf{1} \otimes \mathbf{1} + \beta\mathbb{K}$$
$$\alpha = \frac{1+\nu}{3(1-\nu)} \quad , \quad \beta = \frac{2(4-5\nu)}{15(1-\nu)} \tag{3}$$

The process of scale transition is next described by giving the relationship between $\boldsymbol{\sigma}^{F}$ and $\boldsymbol{\varepsilon}^{F}$. It should be noticed that as far as the free strain depends only on the plasticity of the matrix and the inclusion, all the localization rules presented next impose the equality of the hydrostatic part of the stress field in the matrix and in the inclusion. The localization process will then only modify the deviatoric parts of the strain and stress field in the inclusion. The problem becomes multiaxial in the inclusion even if the strain imposed to the matrix is uniaxial.

Voigt's bound

Voigt [5] made the assumption that the strain field in the inclusion was equal to the field in the matrix. In many analyses, this assumption is a lower bound in terms of effective stiffness. It can be seen as setting the Eshelby tensor $\mathbb{S}$ equal to zero: $\mathbb{S} = \mathbb{O}$.

Reuss bound

Reuss [6] made the assumption of a stress field in the inclusion equal to the one applied to the matrix. In that case, one speaks about an upper bound in terms of effective stiffness. It is equivalent to an Eshelby tensor taken equal to the fourth order identity tensor: $\mathbb{S} = \mathbb{I}$.

Eshelby's law and Kröner's approach

Eshelby [7] gave the demonstration of the solution in the case of an elastic inclusion embedded in an elastic matrix and submitted to a free strain. In that case the strain field in the inclusion is homogeneous. Kröner [8] used this result and assumed that the free strain is due to the difference between the plastic states of the inclusion and the matrix. This approach shows its limits when the plasticity of the matrix tends to be high and evolves much, as it is the case in the LCF domain. Using the former general framework, Eshelby-Kröner is obtained with the assumption of an elastic mismatch behavior ($\mathbb{E}$ is the fourth order Hook's tensor):

$$\boldsymbol{\sigma}^{F} = \mathbb{E} : \boldsymbol{\epsilon}^{F} \tag{4}$$

Hill's approach

Hill [9] takes into account the matrix plasticity by replacing the elastic modulus $\mathbb{E}$ by the tangent modulus $\mathbb{L}$ of the matrix (assumed symmetric):

$$\dot{\boldsymbol{\sigma}} = \mathbb{L} : \dot{\boldsymbol{\epsilon}} \tag{5}$$

$$\begin{cases} \text{elasticity at RVE scale:} & \mathbb{L} = \mathbb{E} \quad (h = \infty) \\ \text{plasticity at RVE scale:} & \mathbb{L} = \mathbb{E} - \frac{4G^2}{h+3G} \boldsymbol{n} \otimes \boldsymbol{n} \end{cases} \tag{6}$$

With G the shear modulus, h the plastic modulus and $\boldsymbol{n}$ the normal tensor to the yield surface:

$$\dot{\boldsymbol{\sigma}} : \boldsymbol{n} = h\dot{p} \tag{7}$$

The localization process has then to be written in terms of strain and stress rate instead of finite quantities. To set

$$\dot{\boldsymbol{\sigma}}^F = \mathbb{L} : \dot{\boldsymbol{\epsilon}}^F \tag{8}$$

recovers Hill's localization rule. It can be noticed that in the case of elastic unloading at RVE meso, one recovers an elastic behavior and doing so, the same response as Kröner's rule. It is equivalent as taking the hardening modulus h as infinite. Using the true tangent modulus may be problematic as it is non isotropic but directed by the normal to the yield surface. In homogenization processes, many authors [10], [11] have obtained too stiff responses compared to the reality. They have then proposed isotropized forms instead of tensorial form.

Berveiller and Zaoui's approach

Berveiller and Zaoui [12] used Hill's approach under the Hencky-Mises condition and they considered a monotonic loading. They then integrated the constitutive equation laws written to introduce finite quantities:

$$\boldsymbol{\sigma}^F = 2G \frac{\frac{\sigma_{eq}}{\epsilon^p_{eq}}}{3G + \frac{\sigma_{eq}}{\epsilon^p_{eq}}} \boldsymbol{\epsilon}^F$$

$$\sigma_{eq} = \sqrt{\frac{3}{2} (\boldsymbol{\sigma}^D - \boldsymbol{X}) : (\boldsymbol{\sigma}^D - \boldsymbol{X})} \qquad \epsilon^p_{eq} = \sqrt{\frac{2}{3} \boldsymbol{\epsilon}^p : \boldsymbol{\epsilon}^p} \tag{9}$$

This rule applies to a monotonic loading. After a first plastification of the matrix, the second plasticization after an elastic unloading will show discontinuities. This approach cannot be used to describe properly cyclic loadings.

Gonzalez and Llorca's approach

Gonzalez and Llorca [10] started from Hill's approach and used an isotropized tangent modulus by projecting the localization rule on the normal to the yield surface. In the particular case of a radial the normal $\boldsymbol{n}$ remains constant. The projection of the localization rule becomes a scalar relationship:

$$\dot{\boldsymbol{\sigma}}^F = 2G \frac{h}{3G + h} \dot{\boldsymbol{\epsilon}}^F \tag{10}$$

With h the plastic modulus at the RVE scale that can either be expressed as a function of the chosen hardening law parameters or more generally the rate:

$$h = \frac{\dot{\boldsymbol{\sigma}} : \boldsymbol{n}}{\dot{p}} \tag{11}$$

González-Llorca's formulation is rewritten with rate quantities and will give continuous responses even when applied to cyclic loadings.

Proposed extension

The formulation given by Gonzalez and Llorca [10] can be extended to make the inclusion response more flexible (softer of stiffer) depending on the choice of the mismatch parameters (Ω, Ω_0 and ω):

$$\dot{\boldsymbol{\sigma}}^F = 2G\Omega_0 \left(\frac{h}{3G\Omega + h} \right)^{\omega} \dot{\boldsymbol{\epsilon}}^F \tag{12}$$

The quantity h is still the plastic modulus calculated from the RVE plastic state. The sensitivity to parameters (Ω, Ω_0 and ω) is studied next to show the ability to give a modular response. For Berveiller and Zaoui's rule, one can also propose an integrated version of this rule in terms of stress and strain fields but by keeping the parameterization by the hardening modulus:

$$\boldsymbol{\sigma}^F = 2G\Omega_0 \left(\frac{h}{h + 3G\Omega} \right)^{\omega} \boldsymbol{\epsilon}^F \tag{13}$$

For the same reasons as for Berveiller and Zaoui's rule, this proposal cannot be used for cyclic loading because of discontinuities at plasticity/elasticity transition.

NUMERICAL IMPLEMENTATION

The different localization rules have been implemented as a procedure of DAMAGE LMT-Cachan post-processor. The input loading is here a uniaxial strain at RVE meso-scale. The behavior RVE scale the macroscopic behavior of the material, with a power law kinematic hardening [13]. The behavior of the inclusion is taken elastoplastic linear kinematic hardening with a softer plastic modulus $h^\mu = C^\mu$. The yield stresses are taken equal at both meso- and micro-scales. The plastic modulus C^μ of the inclusion is taken 100 times lower than C at meso-scale. One first calculates the elastoplastic response of the RVE. Then the elastoplastic state of the inclusion is calculated regarding to the different localization rules. Both monotonic and cyclic responses are presented.

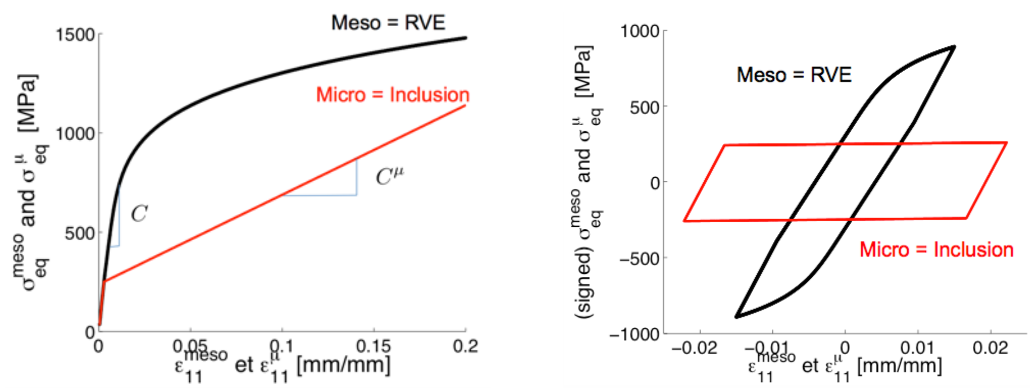

Figure 2. Behavior at meso- and micro-scales under monotonic and cyclic loading

The classical localization rules have been first implemented. When plotting the stress of the inclusion with respect to the strain in the inclusion, one recovers of course the chosen behavior (linear kinematic hardening). The difference is then the final plastic state obtained for a given maximum load applied to the meso scale.

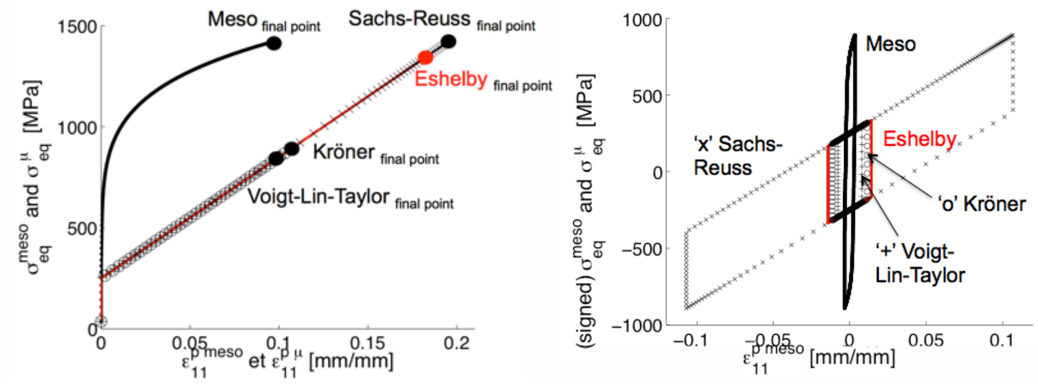

Figure 3. Behavior of the inclusion regarding 4 classical localization rules

The response of the inclusion can be plotted as the equivalent stress in the inclusion with respect to the strain at the meso scale (applied loading).

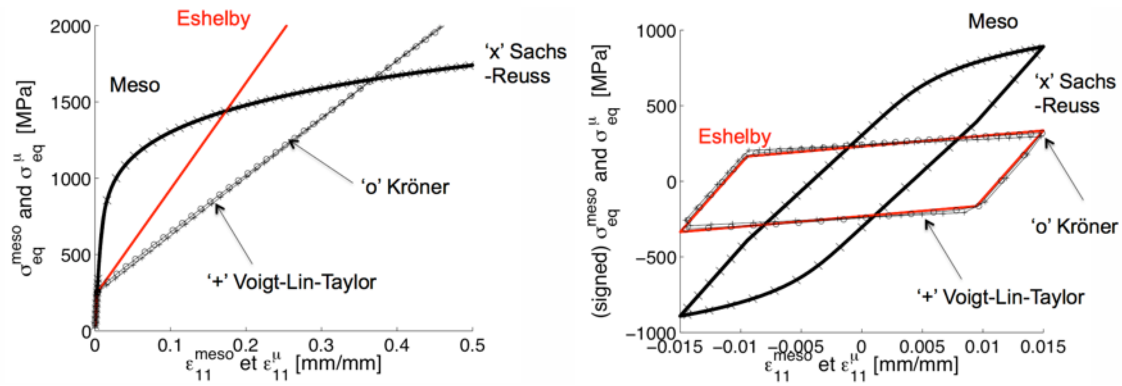

Figure 4. Response of the 4 classical localization rules when a strain loading is applied at the meso-scale

The response given by the Reuss' approach fits the meso behavior as it translates the equality between the stresses at the two scales. One can observe that the Kröner's, Eshelby's and Voigt's responses under cyclic loadings are tightly the same. The three localization rules don't take enough into account the plasticity of the meso scale. Because of this consideration, the Hill's approach was developed and the response presented below is the one used by Gonzalez and Llorca under the restriction of a radial loading. The response of the parameterized proposal is also given in blue. Here is just presented the sensitivity to the power coefficient ω.

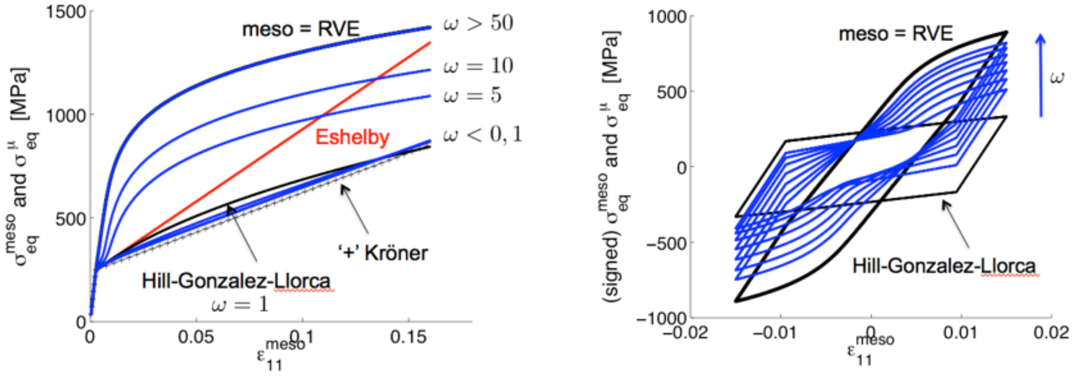

Figure 5. Response of localization rules written in terms of stress and strain rates
(in blue: sensitivity to the mismatch parameter ω)

Localization rules can be used as integrated quantities (as Berveiller and Zaoui's rule). Below is shown the sensitivity analysis to the power law coefficient ω. As expected the response under cyclic conditions is not continuous.

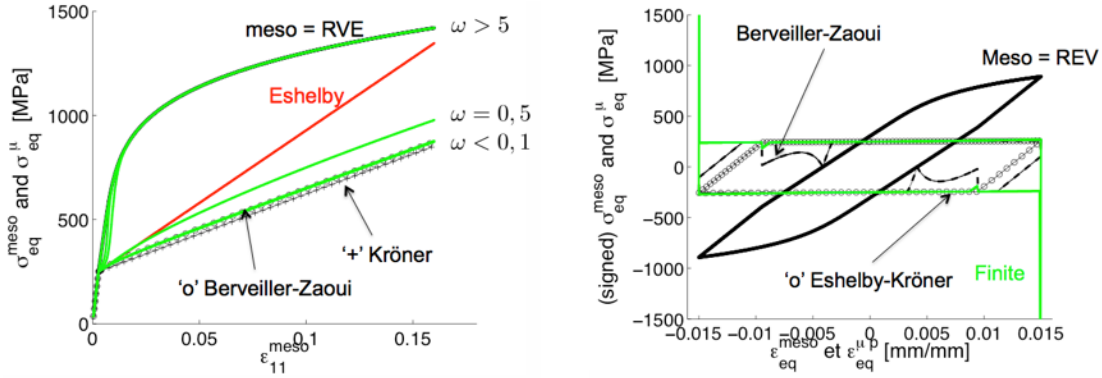

Figure 6. Response of localization rules written in terms of finite quantities
(in green: sensitivity to ω)

CONCLUSION AND PERSPECTIVES

Using the concept of mismatch behavior between matrix and inclusion allows unifying the localization rules in a same framework. In order to take into account the matrix yielding, Hill proposed to use the instantaneous tangent modulus instead of the elastic modulus in the localization rule. A numerical implementation of the corresponding isotropized rule but also to a modular family of localization rules has been done successfully. Both monotonic and cyclic loadings have been tested. The continuous response of the model under cyclic loading is guaranteed by the introduction of the plastic modulus in the so called $\dot{\boldsymbol{\sigma}}^F$ versus $\dot{\boldsymbol{\epsilon}}^F$ mismatch behavior and the fact that the localization rule is written in terms of rates. The proposed rules allow getting either softer or stiffer response of the inclusion for the same mesoscopic loading.

Acknowledgement: Research supported by CNES (Centre National d'Etudes Spatiales) and Snecma Space Engine Division SAFRAN Group

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