

# A fatigue equivalence method applied to railway wheels submitted to in-service multi-input loads

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**ABSTRACT.** *This paper deals with a comprehensive method to determine Equivalent Fatigue Loads (EFL) from in-service load measurements, in the case of a structure submitted to multiple random fatigue loadings. This method, which aims at defining EFL, loads that generate similar damage to the in-service loads, can be useful for the definition of fatigue tests and simulations during the experimental and numerical validation of a new component. It can also be used in order to evaluate the severity of the in-service loads.*

*The aim of the work presented in this paper is to develop and apply the Equivalence Fatigue Method in the case of a structure submitted to a multi-input loading. Damages are evaluated using a life prediction method based on the Dang Van multiaxial fatigue criterion. EFL are computed for the entire structure with a damage equivalence optimization developed in this paper.*

*The method is used for rotating structures, applied to train wheel. Train wheels are critical safety railway components mainly subjected to random multiaxial fatigue induced by multi-input loading.*

## INTRODUCTION

Evaluating the damage induced to a structure by a random fatigue loading is difficult and even harder when there are multiple loadings involved in.

The first aim of this work is to propose a method to estimate damages generated by in-service measured loads. The method enables to find Equivalent Fatigue loadings (EFL) for the whole structure through Finite Element computation, the usage of the multiaxial Dang Van criterion and cumulative damage law for the evaluation of the damage induced by multiple loadings. Once the damage is calculated, the objective of this method is to find the best Equivalent Fatigue loadings (EFL) for a structure (choice of the complexity of EFL is really important). It can be used to perform a validation test on

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a bench and can also be used during the design phase to optimize the structure geometry for a specific use. Here, it is applied on a rotating structure: a train wheel.

In the automotive industry, the fatigue equivalence method is widely used [18, 3]. This method takes into account the variability of the loads in the calculation of the damage in the high cycle fatigue domain in order to evaluate EFL. Customer surveys on car usage and load measurements are used to identify the loading variability. The main features of this method are : (1) In-service loading measurements and customer surveys, (2) Analysis of the material fatigue strength, (3) Computation of damage from variable loading in HCF, (3) Equivalent fatigue loading research.

This method has been applied by Bignonnet [2] to railway wheel set axles. It has been extended to thermo-mechanical loading in low cycle fatigue [15]. Genet [8] has generalized the method using a multiaxial fatigue criterion (Morel [11]) for multiple but proportional loads. All these methods focus on critical points of the structure. The paper focuses on finding an accurate EFL for an entire structure.

The first part of the presented work focuses on the loading of the train wheel and the in-services data used in this study. The loading considered is variable and non proportional. The data are representative of the life of the structure. The second part develops the model used for the computation of damage. This model takes into account the complex multiaxial stress path using an extended Dang Van criterion. The total damage is computed with a fatigue equivalent stress built using damage accumulation. The third part focuses on the Equivalent Fatigue Loading research for the entire structure with genetic algorithm and local optimization. And finally, the last part shows the results obtained with this new fatigue equivalence method for a particular geometry.

## **1. THE RAILWAY WHEEL**

### ***1.1. Global fatigue loadings***

The fatigue loadings applied to train wheels [1] are the following: (1) Lateral load  $Y$ , (2) Vertical load  $Q$ , (3) Press fit  $F$  (considered as a load).

The variable loads  $Y$  and  $Q$  represent the resultant of the wheel-rail contact forces. The relative rotation of these loads induces a relevant fatigue cycle.  $F$  is the press fit load between the wheel and the axle (no precise statistical data available for this load).

### ***1.2. In-service data***

In this study, we use the in-service data obtained from a real train path. The large amount of in-service data enables us to make the assumption that they are representative of the wheel life.

8 instrumented wheels are used to acquire the data on a representative track and to obtain the in-service data. These wheels are equipped with several strain gauges in order to estimate in real time the loads  $Y$  and  $Q$ .

For each rotation of the wheel the maximum value of each load is extracted. The result of the extraction is given in a load matrix reported in figure 1. The figure represents the number of occurrences for each couple of load. This result represents the equivalent of

about 100 km running for a single wheel. These data are chosen to be representative of the whole life of the train wheel. The statistical characteristics of both variables  $Y$  and  $Q$  are therefore considered as known.

### 1.3. Material properties

The material used for the wheel is ER7 (a material close to the C45 according to AFNOR designation). The monotonic material parameters are given in the standard [1].

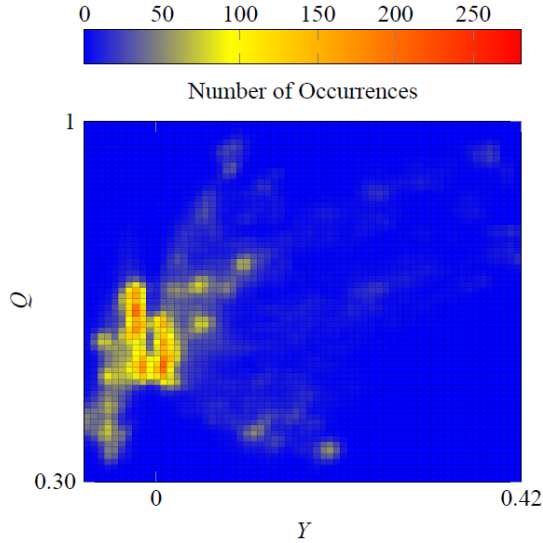


Figure 1. Loads matrix, distribution of loads  $Y$  and  $Q$  extracted over about 100 km (normalized values)

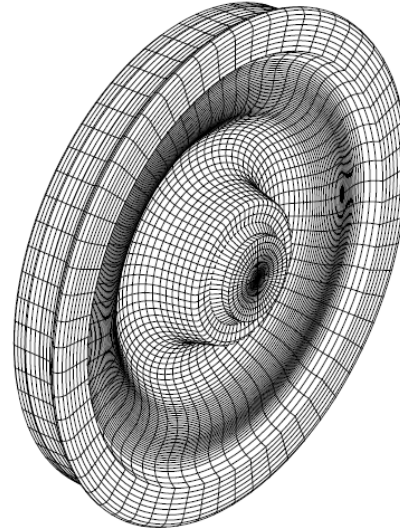


Figure 2. 3D mesh of the train wheel with axle

## 2. DAMAGE EVALUATION: FINITE ELEMENT COMPUTATIONS AND FATIGUE CRITERION

In order to evaluate damage by means of a fatigue criterion, elastic Finite Element computations are performed on the wheel under the previous loading conditions.

As we are concerned with a rotating structure, constant loads rotating over the structure are applied in order to reproduce the evolution of the stresses. The mesh of the wheel is shown in figure 2. Advantage is taken from the geometrical symmetries of the structure to reduce the number of computations: 1 elastic computation is sufficient to simulate a 180 instants stress path for one rotation to the time-space equivalence.

As the structure remains elastic, a superposition method is used to generate any stress path for a wheel rotation under a combination of loads  $Y$ ,  $Q$  and  $F$ . The stress evolution at a point  $M$ , at the instant  $t$ , due to the loading  $Y$ ,  $Q$  and  $F$  can be written as follows:

$$\sigma(M, Y, Q, F, t) = Y \cdot \sigma_Y(M, t) + Q \cdot \sigma_Q(M, t) + F \cdot \sigma_F(M) \quad (1)$$

$\sigma_y(M,t)$  and  $\sigma_o(M,t)$  are the stress path (rotation of the load) for respectively  $Y=1$  kN and  $Q=1$  kN.  $\sigma_F(M)$  is the stress for  $F=1$  mm interaction between the wheel and the axle for a given point of the structure. In our case the rim and so the contact zone are not studied because it is subjected to fretting fatigue. The Hub is also not considered because mostly subjected to constant loading. In the studied part of the wheel, it is verified that the maximum stresses are below the yield limit of the structure. This is consistent with a long life fatigue behavior.

## 2.2. Fatigue criterion

To evaluate the damage induced by a given combination of loads, a fatigue equivalent stress  $\tau_{DV}$  based on the Dang Van multiaxial high cycle fatigue criterion [6] is used. The infinite fatigue life domain is defined as:

$$\max_t (\tau(t) + a \cdot p(t)) \leq b \text{ and } \tau_{DV} = \max_t (\tau(t) + a \cdot p(t)) \quad (2)$$

$\tau$  is the mesoscopic shear stress and  $p$  the hydrostatic stress.  $a$  and  $b$  are two material parameters. The fatigue stress  $\tau_{DV}$  deduced from this criterion is defined in Eq. 2.

Other fatigue criteria as proposed by Papadopoulos [12], Sines [14] and Crossland [5] can be used.

The used criterion is recommended by standard [4] in the railway industry and applied by authors on railway wheels [13].

## 2.3. Fatigue life model

To evaluate the fatigue life, an extension of the Dang Van criterion in the finite life domain is adopted. It consists in the use of a power law (Basquin like model, as introduced by Papadopoulos [12]) relating the number of cycle  $N$  to failure to the fatigue stress  $\tau_{DV}$  by:

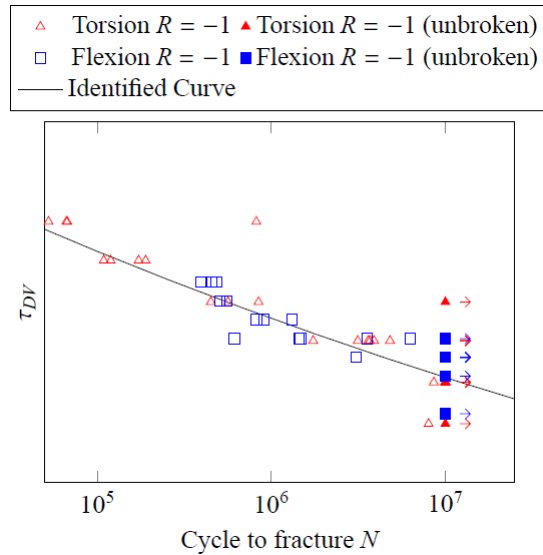


Figure 3. Fatigue life model for ER7 steel

$$N = N_e \cdot \left( \frac{b}{\tau_{DV}} \right)^m \quad (3)$$

The calibration of the material parameters is performed as follows:  $a$  is obtained from two different fatigue limits [6];  $m$  and  $b$  are deduced from a Wöhler curve based on the fatigue stress  $\tau_{DV}$  (figure 3);  $N_e$  is a parameter of the method, more precisely defined in section 3.1. The fatigue properties are determined from [4], and due to the confidentiality of the data, the values are not given.  $\sigma_{mat}$ , the standard deviation of

fatigue strength identified on a  $\tau_{CL}$  based Wöhler curve, in  $R = -1$  torsion only, is also identified.

Once all these fatigue parameters are determined the values of the fatigue stress are computed and the lifetime can be evaluated. The fatigue stress  $\tau_{DV}$  takes into account the material properties without the variability of the variable  $a$ . This choice is made in order to simplify the method. Other fatigue life models as proposed in [10] can also be used.

#### 2.4. Damage accumulation

Palmgren-Miner linear damage accumulation model [8, 10] is used to compute the damage in the structure for its whole life. Using Eq. (3), a cycle  $i$  (single wheel rotation) with a damage variable  $\tau_{DV,i}$ , induces a damage  $D_i$  given by:

$$D_i = \frac{1}{N_i} = \frac{1}{N_e} \left( \frac{\tau_{DV,i}}{b} \right)^m \Rightarrow D = \sum_i D_i = \sum_i \frac{1}{N_i} = \frac{1}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right)^m \quad (4)$$

$$d = N_e \cdot \frac{1}{N_e} \left( \frac{\tau_{eq}}{b} \right)^m = \left( \frac{\tau_{eq}}{b} \right)^m \quad (5)$$

$d$  is the damage induced by the equivalent loading. Because the in-service data do not represent the whole life of the wheel, the damage accumulation from in-service data has to be multiplied by a scalar  $k$ . This extrapolation to virtual life is also used in the literature (see [2]). We introduce the equivalent fatigue stress thanks to  $\tau_{eq}$ :

$$\left( \frac{\tau_{eq}}{b} \right)^m = \frac{k}{N_e} \cdot \sum_i \left( \frac{\tau_{DV,i}}{b} \right)^m \Rightarrow \tau_{eq} = \left( \frac{k}{N_e} \cdot \sum_i (\tau_{DV,i})^m \right)^{1/m} \quad (5)$$

$\tau_{eq}$ , obtained for every point of the structure is a measurement of the local damage.

### 3. EQUIVALENCE METHOD

#### 3.1. Equivalent loading

The value of the fatigue stress  $\tau_{DV}$  for any values of the loads parameters can be calculated. In the specific case of the train wheel, the parameters are the values of the rotating loads  $Q$  and  $Y$ . Loading parameter vector is the following ( $N_c$  is the number of cycles in the in-service data), as the equivalent loading is chosen for  $N_e$  cycles:

$$\lambda = \begin{Bmatrix} Q_{,1} & \cdots & Q_{,i} & \cdots & Y_{,N_c} \\ Y_{,1} & \cdots & Y_{,i} & \cdots & Y_{,N_c} \end{Bmatrix}, \lambda_i = \begin{Bmatrix} Q_{,i} \\ Y_{,i} \end{Bmatrix}, \lambda^e = \begin{Bmatrix} Q_{,1}^e & \cdots & Q_{,i}^e & \cdots & Y_{,N_c}^e \\ Y_{,1}^e & \cdots & Y_{,i}^e & \cdots & Y_{,N_c}^e \end{Bmatrix}, \lambda_i^e = \begin{Bmatrix} Q_{,i}^e \\ Y_{,i}^e \end{Bmatrix} \quad (6)$$

As shown before (Eq. 4), for one cycle (and at any point  $M$  of the structure),  $\tau_{DV}(\lambda, M)$  can be computed. Furthermore, we can compute for every cycle:  $D_i(\{\lambda_i\}, M)$ . And finally, the damage from the life of the structure and from the EFL are:

$$D(M) = \sum_i D_i(\lambda_i, M), D^e(M) = \sum_i D_i^e(\lambda_i^e, M) \quad (7)$$

Strict fatigue equivalence is obtained when, for all the points  $M$  of the structure,  $D^e(M) = D(M)$ .

### 3.2. Finding best equivalent loading

Obviously, the equivalent load should be as simple as possible, so the equality of the damages will not be obtained for all points.

So choices have to be made for the equivalent load. For a given variable load, the best equivalent  $\lambda_e$  is the following:

$$\lambda^e = \underset{(\lambda^e)}{\operatorname{argmin}} \sqrt{\sum_M \left[ \frac{D(M)}{\sum_i D_i^e(\lambda_i^e, M)} - 1 \right]^2} \quad (8)$$

In order to obtain a good evaluation of the damage from the Equivalent Fatigue Loading, we can try more complex EFL.

For instance the first EFL can be a single value of the loads parameters, one block repeated  $N_e$ . In order to have a better equivalence it is important to try more blocks.

The function minimized in Eq.8 is the mean error over the structure between the damage induced by the loading during the life of the structure and the damage induced by the Equivalent Fatigue Loading. Other indicators are used as the maximum damage and the minimum damage over the structure.

In order to compute  $\lambda^e$  with equation 8 an optimization scheme is needed. In this paper both a genetic algorithm and a local algorithm are used to realize the optimization.

## 4. APPLICATION AND RESULTS

### 4.1. Severity on the structure

First, the severity of the service loading is evaluated. We will use the  $\tau_{eq}$  indicator for every point. Figure 4 shows the values of the indicator on a 2D mesh of the web of the structure.

This indicator is used to find the hot point of the structure because it takes into account the variability of the loading and the damage induced by every cycle.

This indicator is also studied for the in-service loading corresponding to the 3 loading configurations defined as: straight track,  $Q_i > 0$  and  $Y_i \approx 0$  ; curved track,  $Q_i > 0$  and  $Y_i > 0$ ; switch point,  $Q_i > 0$  and  $Y_i < 0$ .

#### 4.2. Equivalent loading for a loading configuration: one block

First, the equivalent loading chosen with a single block of  $Q^e$  and  $Y^e$  as loading parameters. This equivalent is a single block repeated  $N_e$  times. This is the simpler EFL in our case. We will identify for each configuration an associated EFL.

Then the in-service loads identified in every loading configuration are extracted and used to find an EFL in this configuration. So, three EFLs are computed with these constraints: straight track,  $Q_1^e > 0$  and  $Y_1^e = 0$  ; curved track,  $Q_2^e > 0$  and  $Y_2^e > 0$  ; switch point,  $Q_3^e > 0$  and  $Y_3^e < 0$ .

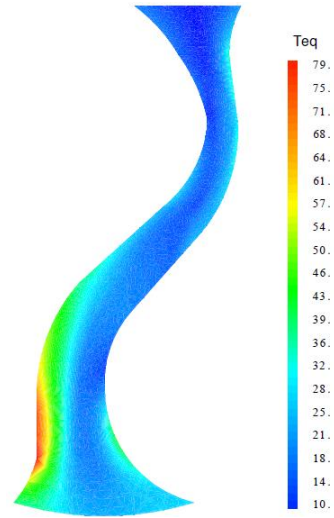


Figure 4. Values of  $\tau_{eq}$  (MPa) on the structure

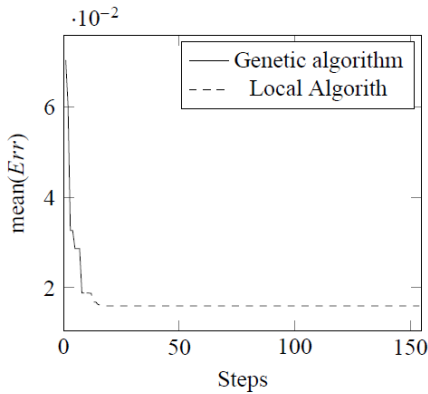


Figure 5. Mean values of the difference of damage over the structure

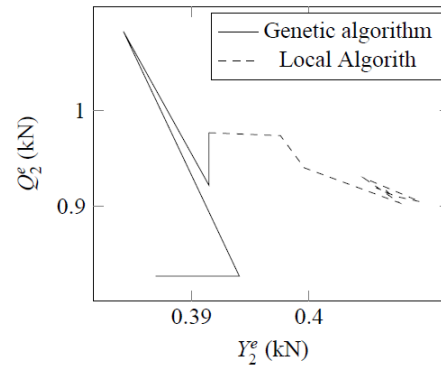


Figure 6. Values of the loads during the optimization

The second configuration (curved track) is studied. The possible values of loads for the algorithm are given in Eq. 9:

$$\begin{array}{lll}
Q_2^e \in [0, 1.6] & Q_2^e = 0.97 & \max_M(Err) = 0.1238 \\
Y_2^e \in [0, 0.8] & Y_2^e = 0.39 & \text{mean}(Err) = 0.016 \\
& N_e = 10^7 & \min_M(Err) = 1.34 \cdot 10^{-15}
\end{array} \quad (9)$$

The values of the mean error over the steps of optimization are shown on figure 5. The values of the loading parameters during the optimization are illustrated in figure 6.

#### 4.3. Equivalent loading for a loading configuration: several blocks

Because one block can not be relevant for each configuration, the next step to find a good EFL is to try multiple blocks EFL. The aim is to know how many blocks are needed in each configuration in order to have an accurate EFL. We propose for the following type of EFL (example for two blocks with  $f \in [0, 1]$ ).

$$\text{For } i \in [1, N_e \cdot f], \lambda_i^e = \begin{Bmatrix} Q_2^e \\ Y_2^e \end{Bmatrix} \text{ and for } i \in [f \cdot N_e + (1-f) \cdot N_e], \lambda_i^e = \begin{Bmatrix} Q_2^{e'} \\ Y_2^{e'} \end{Bmatrix} \quad (10)$$

In this part the configuration of life 2 (curved track) is studied.

The obtained equivalence indicators are really good, for exemple,  $\text{mean}(Err) = 0.0065$ .

Choosing more blocks will indeed give more accurate EFL but the results will be more difficult to use (in a test bench for instance).

#### 4.3. Equivalent loading for the life of the structure

In order to find an Equivalent Fatigue Loading for the entire life of the structure, the choice has to be made with several blocks in different configurations. It is also important to use more blocks in order to find an accurate EFL.

## CONCLUSION

This paper gives a method for designing reliable industrial structures subjected to multi-input and multiaxial variable fatigue loads. This paper focuses on rotating structures but the method could be extended to non-rotating structures. (1) The method requires large in-service data in order to take into account the variability of the loads during the whole life of the structure; (2) A simple analysis of the fatigue strengths is required (Classical fatigue tests must be performed and numerical models of the structure are needed) to compute damage (using in this paper an extended Dang Van Criterion), (3) A fatigue equivalence method is used to compute equivalent fatigue stress taking into account the variable loads. This variable is a local measurement of the damage; (4) A mathematical method to find Equivalent fatigue Loading is described. This method uses a genetic algorithm in order to compute an EFL for the complete structure.



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