Deviatoric Formulation of the SWT Parameter

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ABSTRACT. *The Smith-Watson-Topper (SWT) parameter was originally suggested and is still widely used to account for a mean stress in fatigue life analysis. It is well recognized however, that the SWT parameter might be non-conservative for cyclic loads that involve relatively large compressive mean stresses. A new energy interpretation of the SWT parameter is proposed. This interpretation is formulated in terms of the sum of strain energy density, the complementary strain energy density supplemented by the strain energy density associated with a mean stress in the cycle. Then, a new deviatoric formulation of the* SWT_D *parameter is proposed. Capability of the deviatoric* SWT_D *parameter to correlate experimental data for 7075-T651Al and ASTM A723 steel under various positive and negative mean stresses is presented. At high negative mean stresses, the deviatoric* SWT_D *parameter demonstrates a fairly good correlation where the original SWT parameter is unable to correlate the data.*

INTRODUCTION

Engineering components subjected to cyclic loading often experience mean stresses. Figure 1 depicts the typical notation used in description of the fatigue hysteresis loop.

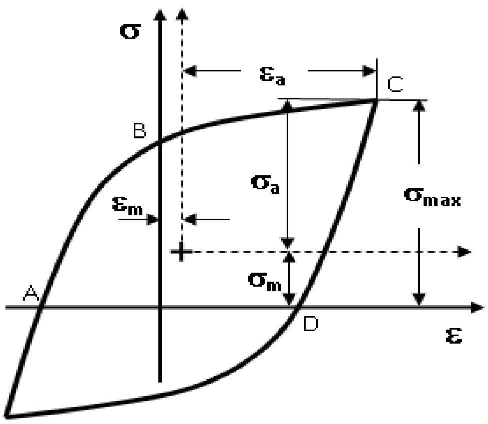


Figure 1 Typical notation used in description of the fatigue hysteresis loop.

It is well know that the positive mean stress is detrimental, whereas the negative mean stress is beneficial. In the past, various approaches have been proposed for estimating mean stress effect in terms of stress-life, strain-life, and stress-strain-life relationships. In this section a brief review of the most widely used models and some recent approaches will be summarized.

Stress-Life Models

In general, the stress-life formulations have been used at high-cycle fatigue (HCF). The first stress-life approaches are the classical works of Gerber and Goodman [1, 2]. It is well known that for positive mean stresses the Gerber and Goodman models can be too optimistic and too conservative, respectively. In addition, they are not usually applicable for negative mean stresses [1, 2].

In 1968, Morrow [3] proposed the following stress-life relation,

$$\sigma_a = (\sigma_f^{'} - \sigma_m)(2N_f)^b \tag{1}$$

where $\sigma_f^{'}$ is the fatigue strength coefficient and b is the fatigue strength exponent. The Morrow model is widely used for positive and negative mean stresses.

Another, stress-life model for mean stress effect has been proposed by Walker [4]

$$\sigma_{\max}^{(1-\gamma)}\sigma_a = f(2N_f) \tag{2}$$

where γ is a material fitting parameter. The Walker model requires fatigue data at different mean stresses or R-ratios (R= min. stress/max. stress) in order to calibrate γ.

Strain-Life Models

In terms of strain-life approach, Morrow [3] proposed the following equation,

$$\varepsilon_a = \frac{\sigma_f^{'} - \sigma_m}{E}(2N_f)^b + \varepsilon_f^{'}(2N_f)^c \tag{3}$$

where $\varepsilon_f^{'}$ is the fatigue ductility coefficient and c is the fatigue ductility exponent.

It can be noted, that Eq. (3) combines Eq. (1), in terms of the elastic strain amplitude, and the well known Coffin-Manson relation, in terms of the plastic strain amplitude [1, 2]. Equation (3), indicates that the mean stress correction is life dependent. The model predicts that the mean stress has a greater effect at HCF, where the elastic strain amplitude dominates. On the other hand, Eq. (3) predicts that the mean stress has a much smaller effect at low-cycle fatigue (LCF), where the plastic strain amplitude governs. This is in agreement with experimental observations, which indicate that the mean stress has a larger influence at HCF than at LCF [1, 2].

Manson and Halford [5] suggested that both; the elastic and plastic terms of the strain-life relationship should account for the mean stress effect, namely.

$$\varepsilon_a = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \varepsilon'_f \left(\frac{\sigma'_f - \sigma_m}{\sigma'_f} \right)^{c/b} (2N_f)^c \tag{4}$$

Usually, Eq. (4) overestimates the mean stress effects at LCF.

Stress-Strain-Life Models

Smith, Watson and Topper [6] advocated that the product of $\sigma_{\max}\varepsilon_a$ (the maximum tensile stress, $\sigma_{\max} = \sigma_a + \sigma_m$ and the strain amplitude, ε_a) controls the fatigue life.

$$\sigma_{\max}\,\varepsilon_a = f(2N_f) = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \varepsilon'_f\,\sigma'_f\,(2N_f)^{b+c} \tag{5}$$

The product of $\sigma_{\max}\varepsilon_a$ can be interpreted as the strain energy quantity. The SWT parameter is widely used and gives good estimation of mean stress effect in both HCF and LCF; however, for cyclic loads that involve relatively large compressive mean stress it might be non-conservative.

Based on experimental observations of partial unloading during cyclic creep of copper, Lorenzo and Laird [7] suggested replacing the total strain amplitude, ε_a, in Eq. (5) with the plastic strain amplitude, ε_{ap}.

$$\sigma_{\max}\,\varepsilon_{ap} = f(2N_f) = \varepsilon'_f\,\sigma'_f\,(2N_f)^{b+c} \tag{6}$$

The above Eq. (6) is rather limited to LCF where the plastic strain amplitude is significant.

Figure 2 shows correlation of experimental data of 7075-T651 Al [12] with: (a) Morrow, Eq. (3) and (b) SWT, Eq. (5) models. For the Morrow model, an elastic fully-reversed strain amplitude, $\varepsilon_{ar,e}$, was calculated as

$$\varepsilon_{ar,e} = \frac{\sigma_a / E}{1 - (\sigma_m / \sigma'_f)} \tag{7}$$

An examination of Fig. 2 indicates that the Morrow model is non-conservative for $\sigma_m>0$ whereas the SWT model for $\sigma_m<0$, respectively.

Some Recent Modifications of the SWT Parameter

In the past, a number of modifications of the SWT parameter have been proposed. For example, Dowling [8] has shown that that the SWT parameter given by Eq. (5) can be transform to the following strain-life relationship.

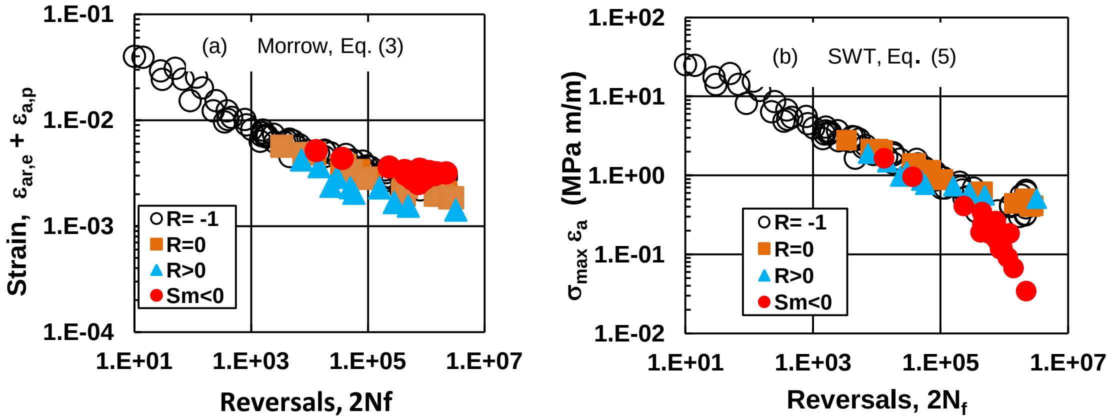


Figure 2 Correlation of experimental data of 7075-T651 Al [12] with: (a) Morrow and (b) SWT models.

$$\varepsilon_a = \frac{\sigma_f'}{E}\left[2N_f\left(\frac{1-R}{2}\right)^{1/2b}\right]^b + \varepsilon_f'\left[2N_f\left(\frac{1-R}{2}\right)^{1/2b}\right]^c \tag{8}$$

Another modification of the SWT parameter, in terms of plastic strain energy associated with tensile stresses, has been proposed by Chiou and Yip [9]

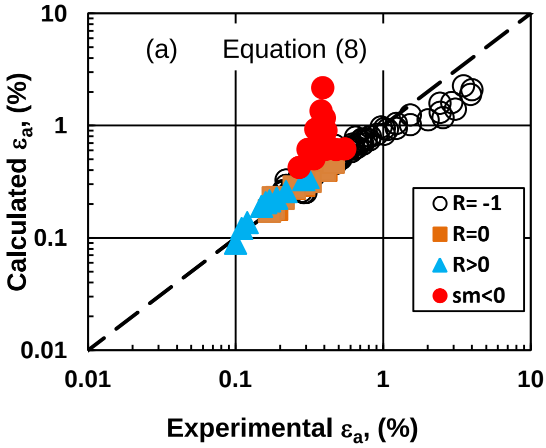
$$(\Delta W_P)_T = area\,(ABCDA) \tag{9}$$

where the area (ABCDA) is calculated according to Figure 1. The above Eq. (9), similarly as Eq. (6), is limited to LCF region where the plastic strain amplitude is significant. Recently, Ince and Glinka [10] made use of the SWT idea and modified the Morrow model, Eq. (3), in terms of the total equivalent strain amplitude.

$$\varepsilon_{eq,a} = \frac{\sigma_{\max}}{\sigma_f'}\varepsilon_{ae} + \varepsilon_{ap} = \frac{\sigma_f'}{E}(2N_f)^{2b} + \varepsilon_f'(2N_f)^c \tag{10}$$

In Eq. (10), the quantity $(\sigma_{\max}/\sigma_f')\varepsilon_{ae}$ is called the equivalent elastic strain amplitude. It was concluded [10] that Eq. (10) provides moderate improvement with respect to the Morrow model for several materials investigated. It can be noted that both Eqs. (8) and (10) can be regarded as a modified version of the SWT parameter written in terms of the strain-life relation.

Figure 3 shows correlations of the right-hand-sides of Eqs. (8) and (10) with experimental data of 7075-T651 Al [11]. It is seen from Fig. 3 that Eqs. (8) and (10) exhibit non-conservative predictions, in particular for relatively large compressive mean stresses.



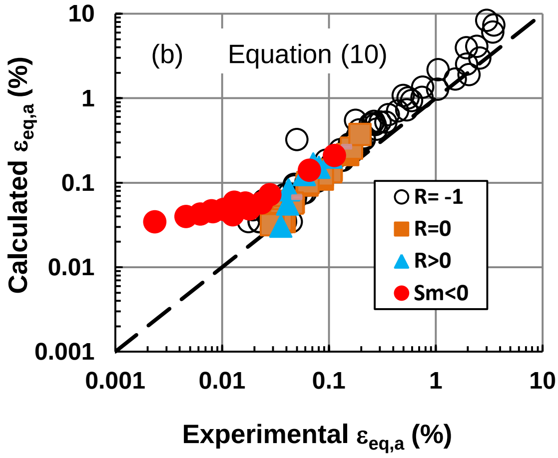


Figure 3 Comparison of the right-hand-side of (a) Eq. (8) and (b) Eq. (10) with experimental data of 7075-T651 Al [12].

STRAIN ENERGY INTERPRETATION AND A NEW DEVIATORIC FORMULATION OF THE SWT PARAMETER

Strain Energy Interpretation of the SWT Parameter

Figure 4 depicts a fully-reversed hysteresis loop together with the cyclic stress-strain curve. The product of the stress and strain amplitudes, $\sigma_a\varepsilon_a$, represents the amplitude of the total strain energy density, which is the sum of the strain energy density, W_ε, and the complementary strain energy density, W_σ.

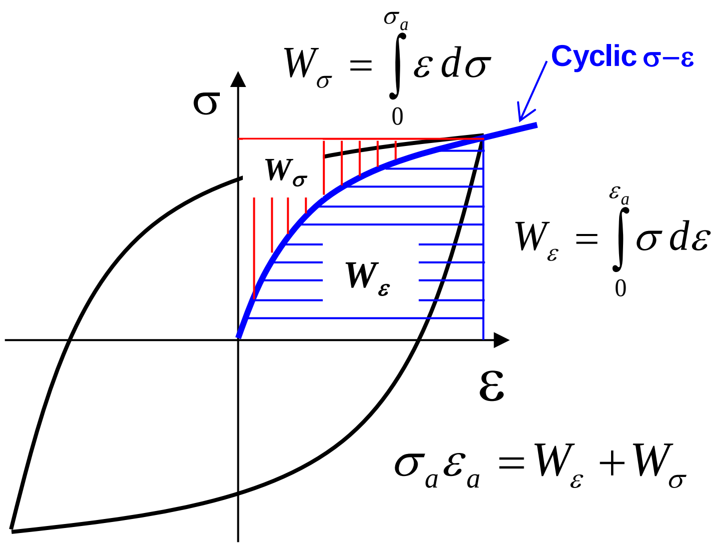


Figure 4 Graphical representation of the strain energy density, W_ε, and the complementary strain energy density, W_σ, for fully-reversed hysteresis loop.

Figure 5 shows two hysteresis loops; one with positive, $\sigma_m>0$, and another with negative, $\sigma_m<0$, mean stress, respectively. The product of $\sigma_{max}\varepsilon_a$ is also illustrated in Fig. 5. The $\sigma_{max}\varepsilon_a$ product corresponds to the SWT parameter and is equal to the amplitude of the total strain energy density, $\sigma_a\varepsilon_a$, supplemented by the strain energy density associated with mean stress in the cycle, $\sigma_m\varepsilon_a$.

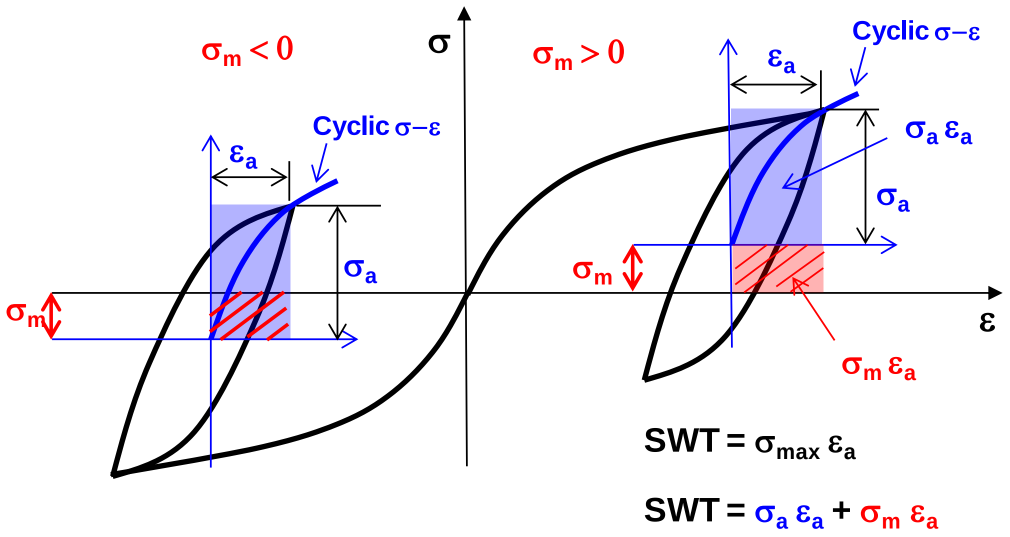


Figure 5 Energy interpretation of the SWT parameter.

The Proposed Deviatoric Interpretation of the SWT Parameter

For uniaxial loading the maximum stress, σ_{max}, and the minimum stress, σ_{min}, can be represented as

$$\begin{bmatrix} \sigma_{\max} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} S_{\max 1} & 0 & 0 \\ 0 & S_{\max 2} & 0 \\ 0 & 0 & S_{\max 3} \end{bmatrix} + \begin{bmatrix} \sigma_{\max}/3 & 0 & 0 \\ 0 & \sigma_{\max}/3 & 0 \\ 0 & 0 & \sigma_{\max}/3 \end{bmatrix} \tag{14}$$

and

$$\begin{bmatrix} \sigma_{\min} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} S_{\min 1} & 0 & 0 \\ 0 & S_{\min 2} & 0 \\ 0 & 0 & S_{\min 3} \end{bmatrix} + \begin{bmatrix} \sigma_{\min}/3 & 0 & 0 \\ 0 & \sigma_{\min}/3 & 0 \\ 0 & 0 & \sigma_{\min}/3 \end{bmatrix} \tag{15}$$

where S_{max1}, S_{max2}, S_{max3} and S_{min1}, S_{min2}, S_{min3} are the principal deviatoric stresses corresponding to maximum and minimum cyclic stresses, whereas $\sigma_{max}/3$ and $\sigma_{min}/3$ are the analogous hydrostatic stresses.

Using the following relations: $\varepsilon_{kk} = \varepsilon_a - 2\nu_{eff}\varepsilon_a$ and $\nu_{eff} = 0.5 - \dfrac{0.5-\nu}{E\Delta\varepsilon_{eq}}\Delta\sigma_{eq}$ where ν

and ν_{eff} are elastic and effective Poisson's ratios, one can write similiary for strains.

$$\begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & -\nu_{eff}\varepsilon_a & 0 \\ 0 & 0 & -\nu_{eff}\varepsilon_a \end{bmatrix} = \begin{bmatrix} e_{a1} & 0 & 0 \\ 0 & e_{a2} & 0 \\ 0 & 0 & e_{a3} \end{bmatrix} + \begin{bmatrix} \varepsilon_{kk}/3 & 0 & 0 \\ 0 & \varepsilon_{kk}/3 & 0 \\ 0 & 0 & \varepsilon_{kk}/3 \end{bmatrix} \tag{16}$$

The deviatoric SWT_D parameter is defined as,

$$SWT_D = MAX \begin{bmatrix} S_{\max 1}|e_{a1}|, & S_{\max 2}|e_{a2}|, & S_{\max 3}|e_{a3}| \\ or & & \\ S_{\min 1}|e_{a1}|, & S_{\min 2}|e_{a2}|, & S_{\min 3}|e_{a3}| \end{bmatrix} = f(2N_f) \tag{17}$$

For positive mean stresses and moderate negative mean stresses the original SWT parameter, Eq. (5), and the proposed deviatoric SWT_D parameter, Eq. (17), yield similar results. A significant difference between these two parameters exists for relatively large compressive stresses where the corresponding product of $S_{\max}|e_a| < S_{\min}|e_a|$.

COMPARISON OF THE DEVIATORIC SWT_D PARAMETER WITH EXPERIMENTAL DATA

For the purpose of assessing the proposed deviatoric SWT_D parameter, two sets of experimental data one for aluminum and one for steel have been chosen. These data consist various positive and negative mean stresses. Figure 5 shows the correlation between experimental data and the proposed deviatoric SWT_D parameter for (a) 7075-T651 Al [11] and (b) ASTM A723 steel [12].

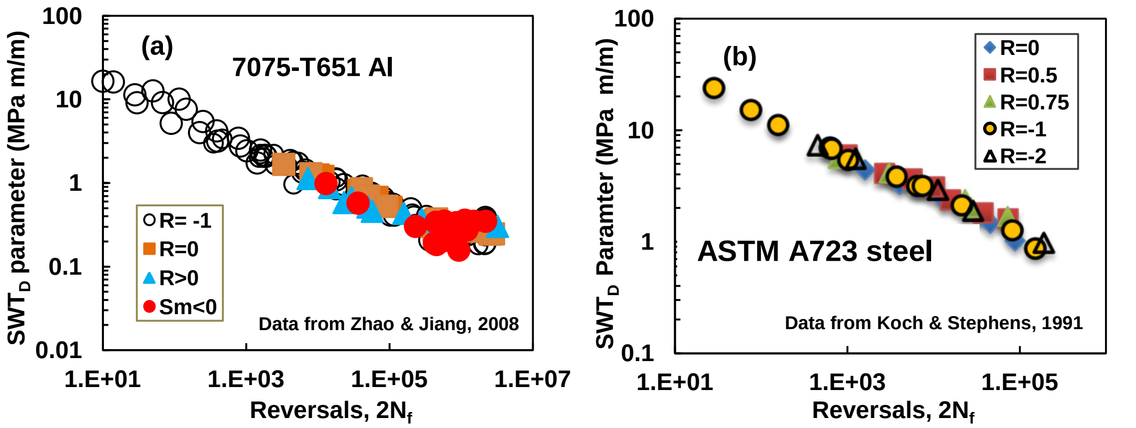


Figure 5 Correlation of experimental data with the deviatoric SWT_D parameter.

Figure 5 indicates a fairly good correlation of the SWT_D parameter with experimental data. Comparison of Fig. 2b with Fig.5a demonstrates that the SWT_D parameter is able to correlate the data, in particular for the high negative mean stresses.

CONCLUSIONS

A new energy based interpretation of the SWT parameter is presented in terms of the amplitude of the strain energy and the complementary strain energy densities supplemented by the strain energy density associated with the mean stress in the cycle. Then, a deviatoric formulation of the SWT parameter, called SWT_D is proposed to account for the mean stress effect on fatigue life, in particular for a high compressive mean stress. At LCF where plastic strain dominates the proposed SWT_D parameter is equivalent to Lorenzo and Laird parameter.

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