The critical aspect of cycle´s definition in multiaxial fatigue conditions.

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ABSTRACT. *Multiaxial fatigue loading conditions are a key issue in several mechanical components. Studying the life of components subject to cyclic stresses is of utmost importance to avoid the unexpected failure of equipment, vehicles or structures. Among other parameters a correct definition of a fatigue cycle in multiaxial fatigue conditions appears crucial concerning the fatigue life issue. The objective of this work is an attempt to achieve a correct definition for a fatigue cycle in multiaxial loading conditions. A low alloy steel 42CrMo4 heat treated, quenched and tempered (500ºC) was used in this study. Based on achieved results a definition for a fatigue cycle in multiaxial fatigue loading conditions is proposed and some remarks are drawn.*

INTRODUCTION

In order to estimate materials fatigue strength the use of an equivalent stress is very appreciated due to the simplicity involved on the process, however this approach has some shortcomings related with loading blocks fatigue life estimations.
For multiaxial loading paths the equivalent stress approaches establishes a stress value representative of that loading path damage usually the maximum value [1]. This procedure leads to a “blind” approach which doesn’t consider what really happens along all loading period. For some simple cases those procedures work well, usually for reference ones, but for more complex loading histories the results are inconsistent.
Despite these results, loading blocks can be decomposed in simple loading cases where fatigue life estimation can be handled as a cumulative damage issue. This interpretation is not new, Miner´s rule on uniaxial cases establishes the material fatigue damage by identifying the different stress levels and associate them with the inherent number of cycles on the same loading block [2].
The critical aspect on loading block cumulative damage is centred on the equivalent stress or damage parameter used and on the cycle counting method which in turn is related with the cycle definition.
Along years cycle counting methods have not been a very studied issue, especially on the multiaxial fatigue, the most known methods have several years being the rainflow method the most used one, which has acceptable results on uniaxial loading cases, however on multiaxial regime new approaches on cycle counting are very few [3, 4].

Stress level increasing, in many cases, is experimentally related with a decrease on the amount of cycles at failure time, the cycle definition is the same but the damage inherent to each cycle is different, thus cycle damage does not only concerns with cycle definition but also on stress level. This leads to conclude of the importance on the equivalent stress definition to establish a representative damage cycle counting.
Usually on testing machinery the cycle counting is determined by block summation, the machine input establishes the loading trajectory which is repeated until the specimen test is totally separated. This cycle counting method considers the load block as one cycle but the cycle damage unit for the same stress level is different. Thus if the stress level is the same and the fatigue life is quite different then the cycle definition must be analysed.
Under this context, it is studied on this paper three different loading blocks using two equivalent stress approaches and the ASTM version of rainflow cycle counting method [5], in order to establish a relation between block damage and cycle definition.

MATERIAL AND METHODS

Material

The 42CrMo4 quenched and tempered high strength steel was the material used in this work. The specimens used on the tests series were machined from rods with 25 mm of diameter, its dimensions are shown in Figure 1.

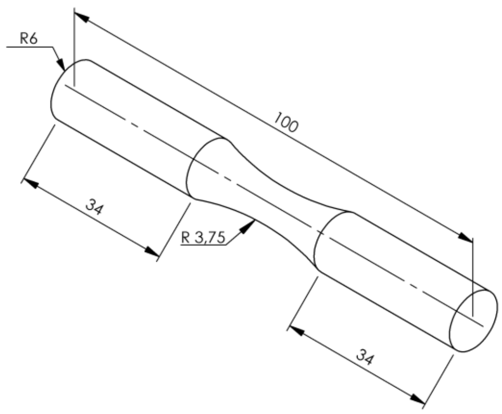

Figure 1. Specimen test geometry and dimensions for stress loading control.

Specimens were inspected and manually polished through sandpapers of decreasing grain size, from 200 to 1200. Fatigue experimental tests were carried out through a biaxial servo-hydraulic machine in order to study different sequential effects; the specimens were tested using three types of loading paths under axial and torsion combined loads, shown on Figure 2.

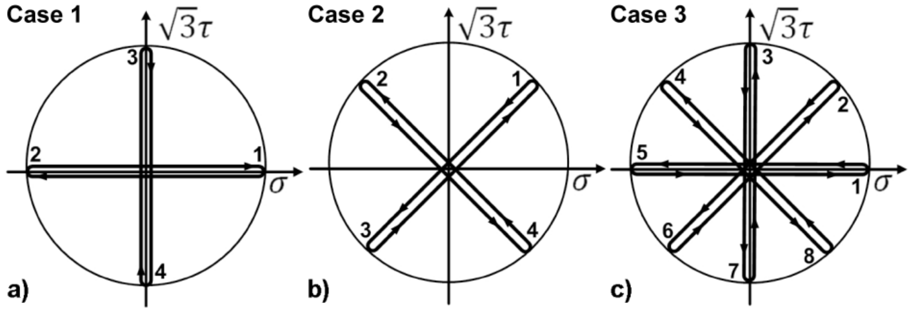

Figure 2. Selected multiaxial loading paths.

The loading sequence for each loading case is represented through the numbering sequence shown on the von Mises stress space, Fig. 2.

Theoretical Analysis

In order to study the fatigue damage inherent to each loading case it was selected two multiaxial fatigue approaches, i.e., the von Mises approach and the Stress Scale Factor (SSF) equivalent stress approach developed by the authors.

von Mises equivalent stress approach

The von Mises equivalent stress approach can be used with acceptable results on certain loadings paths, essentially proportional ones. During the block loading period this equivalent stress approach varies instantaneously being the damage parameter determined by considering the maximum value achieved on that period. Moreover this equivalent stress can be used to estimate fatigue life as shown on Equation 1.

$$\max_{block}\left(\sqrt{\sigma^2+3\tau^2}\right)=A\left(N_f\right)^b \quad (1)$$

where A and b are the power law regression variables obtained from uniaxial tension fatigue life results for the considered material.

SSF equivalent stress approach

The SSF equivalent stress approach was proposed by the authors, this approach considers that both the stress ratio amplitude and the stress loading level have a huge influence on the material fatigue strength response. These influences were accounted through the SSF parameter which transforms an axial damage into a shear one. With this equivalent stress approach it is also possible to estimate fatigue lives, see Equation 2.

$$\max_{block}\left(\tau+ssf\cdot\sigma\right)=B\left(N_f\right)^c \quad (2)$$

where B and c are the power law regression variables obtained from pure shear fatigue life results for the considered material. The ssf is calculated through Equation 3.

$$ssf\left(\sigma_a,\lambda\right)=a+b\cdot\sigma_a+c\cdot\sigma_a^{\ 2}+d\cdot\sigma_a^{\ 3}+f\cdot\lambda^2+g\cdot\lambda^3+h\cdot\lambda^4+i\cdot\lambda^5 \qquad (3)$$

Where σ_a is the axial component of the biaxial loading and λ is the stress amplitude ratio. The constants from "*a"* to "*i*" are determined through experimental tests.

Block Damage

To quantify the damage associated with each loading block, it was considered the experimental and estimated fatigue lives obtained from the equivalent stress approaches. As shown on previous subsections, the criterion to establish the damage parameter was the maximum value encountered along the loading block, however the approach does not consider how many times that maximum value occurs and does not have into account the loading path trajectory. The main idea around this investigation is to establish a *damage reference* associated with the maximum value achieved for the equivalent stress by determining the estimated fatigue life and then compare it with the experimental result as proposed on Equation 4.

$$\underset{Block}{Damage}=\frac{N_{f_{estimated}}}{N_{f_{experimental}}} \qquad (4)$$

For the selected loading blocks, which can be decomposed in other simplified loading paths, it is expected that the experimental fatigue life would be lesser than the estimated one; this difference can then be associated with the total block damage.
However remains to find out how many times lesser is the block fatigue life compared with the one estimated with the maximum equivalent stress verified on that same loading block. In order to estimate that difference without need to perform experimental tests the rain flow accuracy is investigated when applied to the selected equivalent stress approaches´ evolution along each loading block. With rainflow cycle counting method is calculated how many times the estimated fatigue life must be reduced due to block damage, see Equation 5; in this way both the loading history and the number of maximum equivalent stress occurrences during a loading block are taken into account on damage accounting. On Eq. 5, RF_{cycles} indicates the number of cycles encountered on the loading block with the rainflow method, $N_{f_{estimated}}$ indicates the number of cycles determined with block equivalent stress and $\underset{block}{N_f}$ and is the block estimated fatigue life.

$$\underset{block}{N_f}=\frac{N_{f_{estimated}}}{RF_{cycles}} \qquad (5)$$

RESULTS AND DISCUSSION

Cycle count results

The rain flow method presented on the ASTM E-1049 [5] was selected to quantify the number of cycles associated with the loading block inherent to each loading case. Despite this method has been developed based on stress strain relation for uniaxial loading conditions, here it is tested considering the equivalent stress evolution along each loading block, i.e., the number of cycles is established by rainflow procedures applied to the equivalent stress evolution.

On Figs 3, 4 and 5 is shown the equivalent stress evolution along each loading case for the von Mises and SSF approaches.

The von Mises equivalent stress path is always positive, in contrast with the SSF approach which has positive and negative values. Due to this fact von Mises stress evolution indicates 1 cycle for each block branch, therefore to count the number of cycles associated with the loading block it is just needed to count the number of branches on the loading path trajectory; however in the case of ssf equivalent stress evolution the number of cycles to count is not so direct.

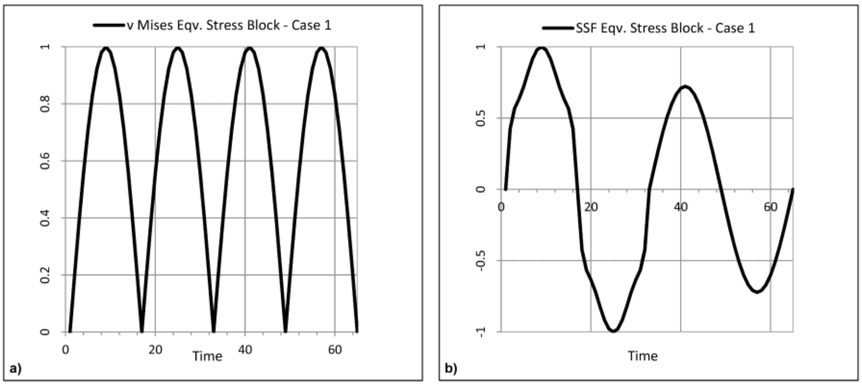

Figure 3. Equivalent stress evolution along a loading block for case 1: a) von Mises equivalent stress b) ssf equivalent stress.

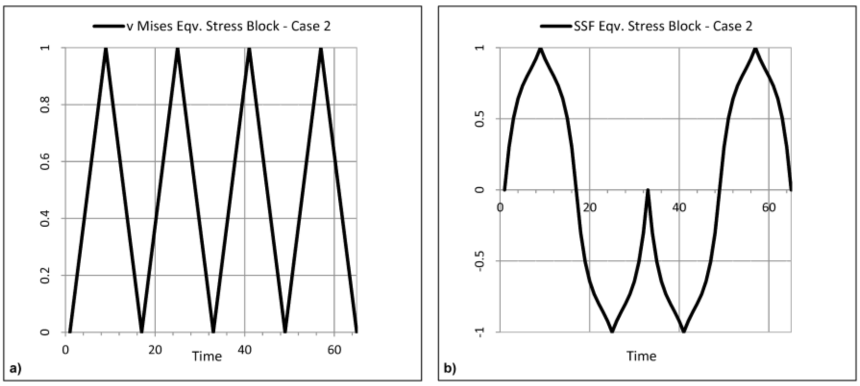

Figure 4. Equivalent stress evolution along a loading block for case 2, a) von Mises equivalent stress b) ssf equivalent stress.

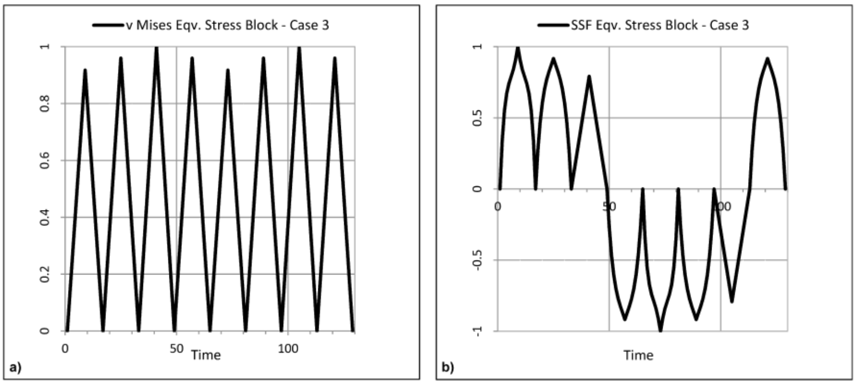

Figure 5. Equivalent stress evolution along a loading block for case 3, a) von Mises equivalent stress b) ssf equivalent stress.

On Table 1 is shown the rainflow results for the cycle counting inherent to each loading block and the loading block damage in average associated with each equivalent stress and experimental results.

Table 1. Cycle counting and block damage for each loading case.

	RainFlow ASTM		Damage Block	
	v Mises	SSF	v Mises	SSF
Case 1	4	2	1.2	1.9
Case 2	4	3	0.1	4.5
Case 3	8	7	1.9	9.1

The rainflow method applied to the von Mises approach leads to equivalent block cycle counting equal to the number of branches encountered on the loading path trajectory on the stress space, as shown in Figure 2. However for the ssf equivalent stress approach the rainflow procedure gives different cycle counting results, as can be seen on the second column of Table 1.

On third and fourth columns is shown the block damage in average for each loading case and equivalent stress approach, respectively. Regarding the von Mises results the block damage on loading cases 1 and 3 is greater than 1.0 which means that the damage caused by the loading block is greater than the one caused by the *reference damage* given by the equivalent stress applied to the uniaxial trend line.

However in case 2 the block damage is lower than 1.0 which is in contradition with the initial premise which establishes that the damage caused by the loading block is greater than the one caused by the maximum equivalent stress achieved on that loading path.

On the other hand for the SSF approach the block damage is greater than 1 for all considered cases being consistent with the premise adopted on this investigation. Concerning the SSF block damage results can be concluded that the loading blocks 1, 2 and 3 are 1.9, 4.5 and 9.1 times more damaging than the respectively *reference damage*; relating these data with the rainflow counting results on SSF equivalent stress, can be concluded that, for the SSF approach, the determined number of cycles is very alike with the block damage level, i.e., 2.5 cycles for 1.9 on loading case 1, 3 cycles for 4.5 on case 2 and 7 cycles for 9.1 on case 3. For the von Mises approach the number of cycles determined is very far from the inherent block damage level which leads to conclude that the von Mises approach associated with rainflow procedure is unsuitable to capture fatigue damage associated with complex loading paths.

Fatigue life correlation

On Figs. 6, 7 and 8 are shown the fatigue life correlation results for the selected loading cases. In each graph is shown with square symbol the fatigue life correlation without rainflow correction and with circle symbol the results with rainflow correction, which is the block fatigue life correlation. The loading block fatigue life correlation using the ssf approach and rainflow methodology gives satisfactory results with few results outside boundaries. However on von Mises approach in particular at loading case 2, the results fail completely, see Figure 7 a). Moreover, the von Mises results under uncorrected approach gave the same damage level for the case 1 and 3 loading blocks.

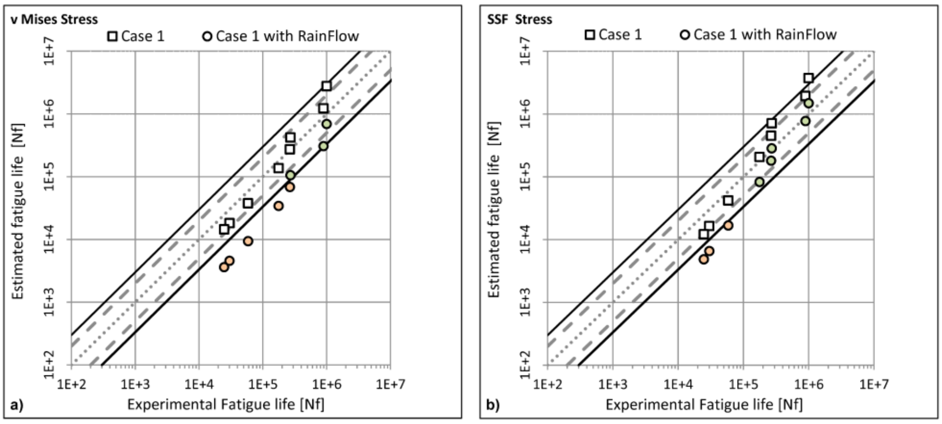

Figure 6. Fatigue life correlation for loading case 1: a) von Mises approach, b) SSF equivalent stress approach.

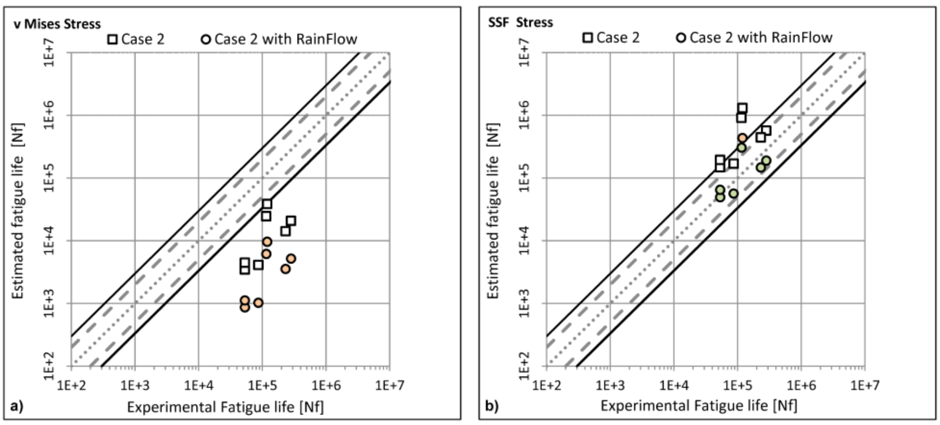

Figure 7. Fatigue life correlations for loading case 2: a) von Mises approach, b) SSF equivalent stress approach.

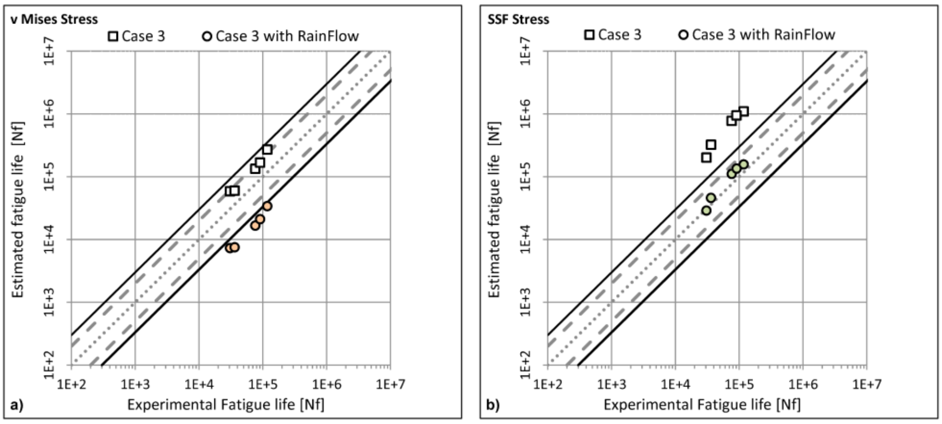

Figure 8. Fatigue life correlations for loading case 3: a) von Mises approach, b) SSF equivalent stress approach.

CONCLUSIONS

In this paper was implemented a new approach on loading block damage interpretation on a high strength steel, 42CrMo4. Two equivalent stress approaches, i.e., the von Mises and SSF equivalent stress were used in association with the rainflow ASTM E-1049 cycle counting method. Unlike the von Mises approach the SSF equivalent stress methodology gave consistent results on block damage explanation with good fatigue life correlations.

ACKNOWLEDGEMENTS

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