Application of Stress-Based Multiaxial Fatigue Criteria for Proportional and Non-proportional Variable Amplitude Loadings for Laserbeam-Welded Thin Aluminium Joints

A. Bolchoun[1], J. Wiebesiek, H. Kaufmann, C. M. Sonsino

Fraunhofer Institute for Structural Durability and System Reliability LBF, Darmstadt, Germany

[1]**Contact person:** alexandre.bolchoun@lbf.fraunhofer.de

INTRODUCTION

The fatigue life assessment under multiaxial loadings is a rather complex task. There is a large number of criteria for such an assessment. Many of them are formulated for a constant amplitude loading. In the case of variable amplitudes an application of fatigue assessment criteria becomes more difficult. For many criteria there are no well-known detailed descriptions of such an application.

Therefore a simple algorithm which allows an application of critical plane and integral stress-based methods to variable proportional and non-proportional loadings is proposed. With this algorithm the well-known stress-based multiaxial fatigue hypotheses according to Findley [1, 2] and also the shear stress intensity hypothesis SIH [3, 4] (both originally proposed for estimation of the fatigue limit) are used for the evaluation of spectrum loadings. These hypotheses were applied to the finite fatigue strength assessment of thin-walled, overlapped laserbeam-welded aluminium joints made of the artificially hardened aluminium alloy AlSi1MgMn T6 (EN AW 6082 T6) and of the self-hardening alloy AlMg3.5Mn (EN AW 5042) [5]. Additionally, the fatigue strength evaluation of multiaxial spectrum loading was carried out by a modified Gough-Pollard algorithm as it is proposed in the IIW-recommendations for fatigue assessment of welds [6]. Since fatigue is a local process the used stress components are local stresses in the critical area at the weld root which are calculated by applying the notch stress concept with a reference radius of r_{ref} = 0.05 mm [7, 8, 9]. Fatigue life calculations using the criteria SSCH [10, 11] and EESH [12] were already performed in [5]. These two methods do not require computation of stress values in arbitrary cutting planes and hence the algorithm presented in this paper cannot be applied to them.

Further it is discussed if the hypotheses are suitable to describe the fatigue behaviour with sufficient precision and if some modifications are required.

SPECIMENS AND TESTING

In the investigation [5] fatigue tests with tube-tube laserbeam-welded joints were performed (Fig. 1). Two different materials were investigated: the self-hardening alloy AlMg3.5Mn (EN AW 5042) and the artificially hardened alloy AlSi1MgMn T6 (EN AW 6082 T6). Fatigue tests with constant as well as variable amplitudes were performed for uniaxial (pure axial loading, pure torsion) and for combined (axial+torsion) proportional and non-proportional loadings. For the tests with variable amplitudes a Gauß-distributed loading sequence with the load ratio $\overline{R} = -1$, length $L_S = 5 \cdot 10^4$ cycles and irregularity factor $I = 0.99$ was used. The experiments were carried out in a servo-hydraulic biaxial test rig at room temperature under load control with a testing frequency of 20 s^{-1}. Failure criterion was total rupture.

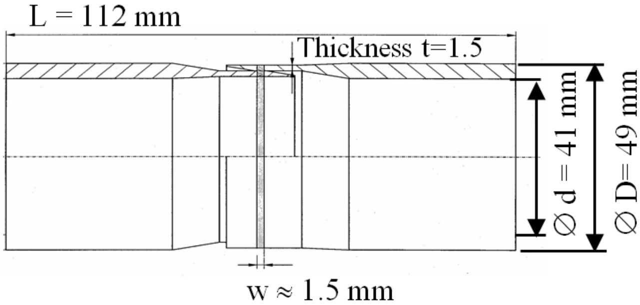

Figure 1. Specimen geometry

STRESS-BASED FATIGUE LIFE CRITERIA FOR MULTIAXIAL VARIABLE LOADS

A general plane stress state with the three load components $\sigma_x, \sigma_y, \tau_{xy}$, is considered, since crack initiation usually occurs on a free surface. The components are time dependent functions.

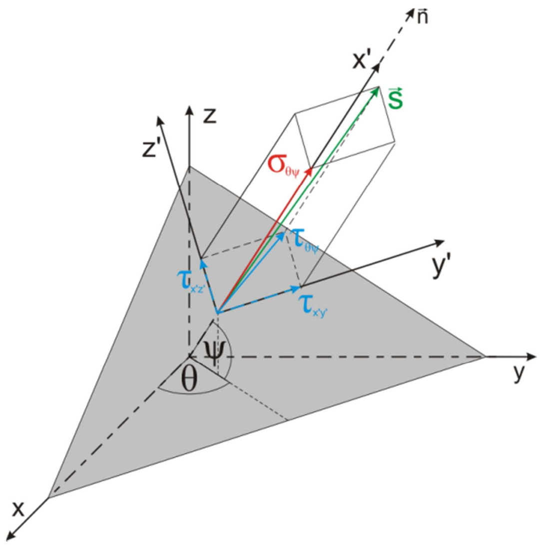

Figure 2. Position of the plane and it’s normal vector

Application of stress-based fatigue life criteria (integral as well as critical plane methods) often requires a notion of the normal stress amplitude and the shearing stress amplitude in any given plane. For a variable amplitude load such values can be easily computed if only planes orthogonal to the free surface are considered or for proportional loadings (that is the components σ_x, σ_y, τ_x are all in-phase oscillations). However the situation is much more difficult if arbitrary oriented planes are taken into account in the case of variable multiaxial non-proportional load (“arbitrary load”). A plane in space is defined by its normal direction, which is in turn defined by two angles θ, ψ (Fig. 2). In such a plane shear stress is a vector and there is no obvious definition of shearing stress amplitude $\tau_{\theta\psi,a}$. Hence there is no obvious way to apply many of the stress-based hypotheses. Application to the arbitrary loads is often discussed in the literature but no thorough description, which could be used to implement an algorithm, is known to the authors. Some hints can be found for instance in [13].

The normal stress amplitude $\sigma_{\theta\psi,a}$ in a plane given by the angles θ, ψ can be computed as the damage equivalent constant amplitude using rainflow counting and Palmgren-Miner-rule against the axial SN-curve (in this paper Palmgren-Miner elementary, i.e. no knee point of the SN-curve is assumed; the only exception is the computation according to the IIW design code). The main difficulty is to define a shear stress amplitude. For an arbitrary load the shear stress vector follows a curve and hence does not have a designated direction. In order to overcome this difficulty the following is proposed: an origin is fixed and the curve is projected onto each direction in the plane (Fig. 3). Such projection can be seen as a sequence of inversion points. Hence rainflow counting can be performed and the equivalent constant amplitude can be computed against the torsion SN-curve using Palmgren-Miner rule. Then the maximum of the equivalent amplitude over all directions is the shearing stress amplitude $\tau_{\theta\psi,a}$. Alternatively the shear stress amplitude in the plane can be obtained as an integral over all possible directions (cf. [4]).

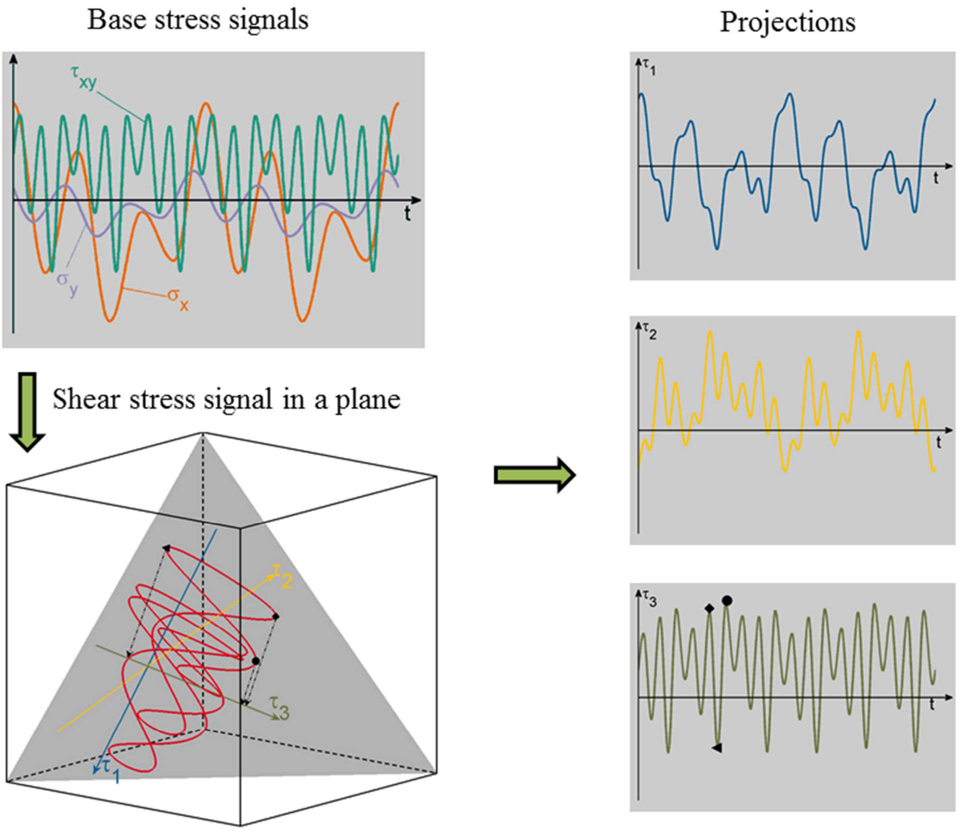

Figure 3. Shear stress curve in a plane and its projections on different directions

Findley-Criterion

Findley introduced his fatigue criterion [1] for a torsion cyclic loading combined with in-phase bending. It is a critical plane method. In each plane the sum of the shear stress amplitude and of the scaled maximum normal stress is computed. The maximum of the normal stress is taken over a whole cycle. Findley formulated his criterion as follows:

$$f = \max_{\theta,\psi}\left(\tau_{\theta\psi,a} + k\sigma_{\theta\psi,max}\right), \quad (1)$$

where $\tau_{\theta\psi,a}$ is the shear stress amplitude and $\sigma_{\theta\psi,max}$ is the maximum normal stress in a plane given by the angles θ and ψ, k is the Findley-parameter. The critical plane is the one, where the maximum of the expression $\tau_{\theta\psi,a} + k\sigma_{\theta\psi,max}$ is attained.

The parameter k is a measure for how sensitive is a certain material to the normal stresses. In [2] it is proposed to put $k \in [0.2, 0.3]$ for ductile materials. The value k can also be determined from the fatigue limit values at pure torsion and pure axial loading [2]. For this paper the Findley-parameter k is computed using uniaxial fatigue values at $\mathrm{N} = 10^7$, i.e.

under the knee point of both the axial and the torsion SN-curves (Figs. 5, 6). It can also be assumed that k depends on the number of cycles N, in this case k can be determined iteratively using a numerical root finding method such as regula falsi.
In the literature it is described how to apply Findley-criterion to combined loads with constant amplitudes in both proportional and non-proportional cases. An application to a variable loading sequence is presented in the current paper.
In order to apply the Findley-method to loads with constant as well as variable amplitudes, the following situation is considered: stress components are given by

$$\sigma_x = \sigma, \qquad \sigma_y = \nu\sigma, \qquad \tau_{xy} = \tau \tag{2}$$

corresponding to a plane stress state as it occurs in a notch under combined loading. The value ν is the Poisson's ratio. In the case of constant amplitudes the components are given by the cosinus oscillations:

$$\sigma = \sigma(t) = \sigma_m + \sigma_a \cos \omega t, \tag{3}$$

$$\tau = \tau(t) = \tau_m + \tau_a \cos(\omega t + \varphi_\tau). \tag{4}$$

By σ_m and τ_m mean stresses are denoted, σ_a and τ_a are amplitudes and φ_τ is the phase shift between the two loading signals σ and τ, in this paper $\varphi_\tau = 0°$ (proportional case) and $\varphi_\tau = 90°$ (non-proportional case) are considered.
The equivalent shear stress amplitude $\tau_{\theta\psi,a}$ in a plane for the Findley-criterion can be computed using the method presented in the previous section. A major difficulty is to define the respective maximum normal stress $\sigma_{\theta\psi,max}$. This can be done if it is taken into account that for the stress ratio $\overline{\mathrm{R}} = -1$ (i.e. $\sigma_m = \tau_m = 0$) the amplitude of the normal stress is equal to the maximum normal stress $\sigma_{\theta\psi,a} = \sigma_{\theta\psi,max}$. If for a certain rainflow cycle the stress ratio is not equal -1 a mean stress transformation can be performed.
For variable amplitudes the modification of the rainflow counting method according to [14] can also be used. This method assignes to each counted cycle of the shear stress signal in a plane the corresponding maximum normal stress value. The counting result consists of pairs of the form (shear stress cycle, maximum normal stress). Each pair is evaluated against the reference SN-curve using the Palmgren-Miner rule with $D_{PM} = 1.0$. The Findley-criterion evaluates proportional constant amplitude loads more damaging than the non-proportional ones. The counting method [14] can result in a reversal of this behaviour for variable loads.

Shear Stress Intensity Hypothesis (SIH)
SIH as it is introduced by Zenner et al. [3, 4] is an integral hypothesis, which is capable to take mean stresses into account. Fatigue failure occurs if the following condition is satisfied:

$$a\tau_{va}^2 + b\sigma_{va}^2 + m\tau_{vm}^2 + n\sigma_{vm} = \sigma_W^2. \tag{5}$$

The left-hand side of the equation (5) can be interpreted as a square of an equivalent stress σ_v^2. Value σ_W is the fatigue limit (which in reality often does not exist [15]) under fully reversed axial loading. Values $m, n, \tau_{vm}^2, \sigma_{vm}$ are introduced in order to deal with mean stresses and are ignored in this paper, since almost every loading cycle has the stress ratio near $\overline{\mathrm{R}} = -1$. Parameters a, b are defined in the papers [3, 4] as follows:

$$a = \frac{1}{5}\left[3\left(\frac{\sigma_W}{\tau_W}\right)^2 - 4\right], \qquad b = \frac{1}{5}\left[6 - 2\left(\frac{\sigma_W}{\tau_W}\right)^2\right] \tag{6}$$

By τ_W the fatigue limit under fully reversed torsion loading is denoted. With the angles θ, ψ as shown in Fig. 2 the integral values τ_{va}^2, σ_{va}^2 are given by the expressions:

$$\tau_{va}^2 = \frac{15}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\pi} \tau_{\theta\psi,a}^2 \cos\psi \, d\theta d\psi, \tag{7}$$

$$\sigma_{va}^2 = \frac{15}{4\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\pi} \sigma_{\theta\psi,a}^2 \cos\psi \, d\theta d\psi, \tag{8}$$

The situation discussed in this paper is slightly different from the one in the papers [3, 4]. A notched specimen is considered, so the stress components are as defined by (2). In this setting parameters a and b have the following form:

$$a = \frac{1}{5(1+\nu)^2}\left[(3\nu^2 + 2\nu + 3)\left(\frac{\sigma_W}{\tau_W}\right)^2 - 4\right], \tag{9}$$

$$b = \frac{2}{5(1+\nu)^2}\left[3 - (\nu^2 - \nu + 1)\left(\frac{\sigma_W}{\tau_W}\right)^2\right]. \tag{10}$$

Values σ_W and τ_W refer to $N = 10^7$, that is at the same number of cycles as for the Findley-criterion. In order to obtain sensible values of the parameters a, b, the inequality

$$\sqrt{\frac{3}{\nu^2 - \nu + 1}} > \frac{\sigma_W}{\tau_W} > \frac{2}{\sqrt{3\nu^2 + 2\nu + 3}} \tag{11}$$

must hold (cf. [4]). Analogous to the Findley-criterion there is a possibility to make the parameters dependent on N and so to take different slopes of the axial and torsion SN-curves into account.

Application of SIH for constant amplitudes requires knowledge of the normal stress and torsion stress amplitudes $\sigma_{\theta\psi,a}, \tau_{\theta\psi,a}$ in a plane with a normal vector defined by the angles θ, ψ. Their computation is ommited here. Similar computations can be found e.g. in [13].

Then the values τ_{va}^2 and σ_{va}^2 can be obtained by numerical integration.

For variable amplitudes the method described above is used.

IIW-Recommendations

The Basis for the IIW design code [6] is the Gough-Pollard criterion [16], which corresponds to the von Mises hypothesis when the multiaxial damage parameter $D_{MA} = 1.0$ is used:

$$\left(\frac{\sigma_{a,comb}}{\sigma_{a,SN\ pure\ axial}(N)}\right)^2 + \left(\frac{\tau_{a,comb}}{\tau_{a,SN\ pure\ torsion}(N)}\right)^2 \leq D_{MA} \tag{12}$$

This criterion comprises a physically found consideration of shear and normal stress interaction. Calculating lifetimes with this method must be done numerically.

The modification of this method, used for the IIW design code, is given by the alteration of the right-hand side D_{MA}, dependent on the material and on whether the loading is proportional or not. If semi-ductile material states are to be assessed, which usually show no difference in fatigue life proportional and non-proportional loadings, $D_{MA} = 1.0$ can be applied for both cases. If assessment shall be done for ductile material states, which usually show a decrease of fatigue life under non-proportional loadings in comparison to proportional loadings, the right-hand side must be reduced. For such cases the IIW-recommendations based on the investigations [18] suggest to use a conservative damage parameter $D_{MA} = 0.5$. Thus, in this case the allowable loads are reduced by a factor of $\sqrt{2}$.

One difficulty of using different D_{MA} values for proportional- and non-proportional loading is that the uniaxial edge cases (e.g. pure axial loading) do not match, but this aspect has no relevance for practical applications.
Application of this simple algorithm to multiaxial spectrum loadings demands to first determine scalar values for each loading component that represent the whole spectrum. This is done by using the condition that the scalar value has to represent constant amplitude loading that causes the same damage as the given spectrum. By using this equation, normal and shear stress spectra are reduced to damage equivalent values:

$$X_{a,eq} = \sqrt[k_1]{\frac{1}{D_{PM}} \frac{\sum_{i=1}^{j} n_i \, X_{a,i}^{k_1} + X_{k,a}^{k_1-k_2} \sum_{i=j+1}^{m} n_i \, X_{a,i}^{k_2}}{\sum_{i=1}^{m} n_i}}. \quad (13)$$

In the latter expression D_{PM} is the Palmgren-Miner damage sum, i the index of the load level, k_1 the slope of the SN-curve above the knee point, k_2 the slope of the SN-curve under the knee point, n_i the number of load cycles with the i-th load level, m the total number of different load levels, j the index of the lowest load level above the knee point.
Modification of the calculated damage can be done by introducing an allowable Palmgren-Miner sum $D_{PM} < 1.0$ into the equation. IIW recommends to set $D_{PM} = 0.5$ for welded joints and the slope of the SN-curve below the knee point $k_2 = 2\,k_1 - 2$ is dependent on the slope above the knee point (Palmgren-Miner modified). The computations in this paper are performed in accordance with this recommendations. The computation was performed against the experimental SN-curves for 50% probability of survival (i.e. the same basis curves as for the other two methods), not against ones provided in the IIW design code. With the SN-curves from the IIW design code more conservative results would have been obtained.

COMPARISON AND CONCLUSIONS

For both alloys and the two combined loads (proportional, non-proportional) the computational Gaßner-curves are shown (Figs. 5, 6). In Figs. 7, 8 the fatigue life evaluations are presented for all tests (constant as well as variable amplitude loadings) carried out in [5]. The criteria of Findley and SIH both yield good results for the proportional case and strongly overestimate fatigue life under non-proportional loadings. The IIW-recommendations lead to a conservative assessment for both alloys, however it is required to adapt the multiaxial damage parameter D_{MA} for a non-proportional load.

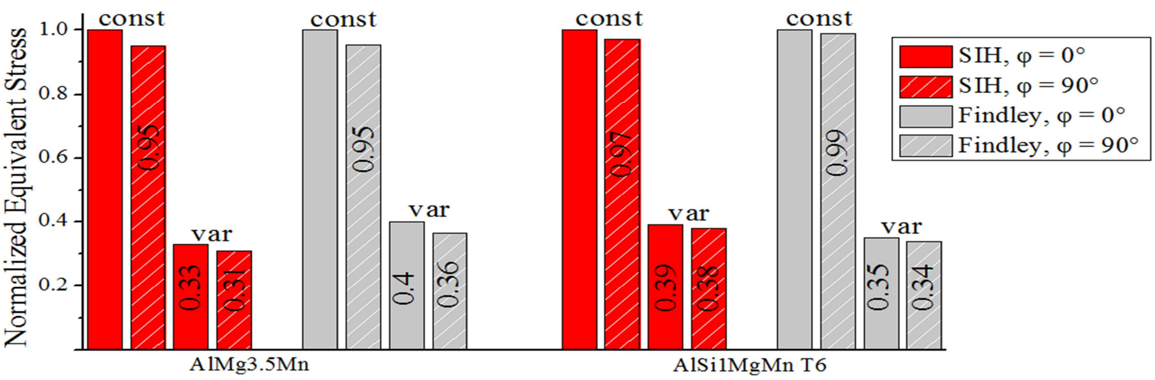

Figure 4. Ratios prop./nprop. for Findley-criterion and SIH. Values normalized with respect to the equivalent stress for proportional constant amplitude loading. Values for variable amplitudes apply to the loading sequence used in investigation [5]

Fig. 4 shows, that both Findley and SIH evaluate the non-proportional case as slightly less damaging than the proportional one. The relation of the equivalent stress in proportional case

for the non-proportional loads for variable amplitudes is approximately the same as for the constant ones (cf. Fig. 4).
In order to assess fatigue life using Findley and SIH correctly for materials which live less under non-proportional loadings some sort of "non-proportionality" factor is required. This factor will evaluate to what extent a certain load is non-proportional and then used to weight the equivalent stresses. For introduction of such a factor the ratio of the equivalent stresses in the proportional and the non-proportional case computed for the constant amplitudes must be approximately the same as for variable amplitudes. This requirement is satisfied as can be seen in Fig. 4. Also ductility of the material must be taken into account [19], since for brittle materials a non-proportional loading is usually less damaging.
As soon as the non-proportionality factor is introduced both hypotheses (SIH, Findley) can be applied to arbitrary loads in a sensible way.

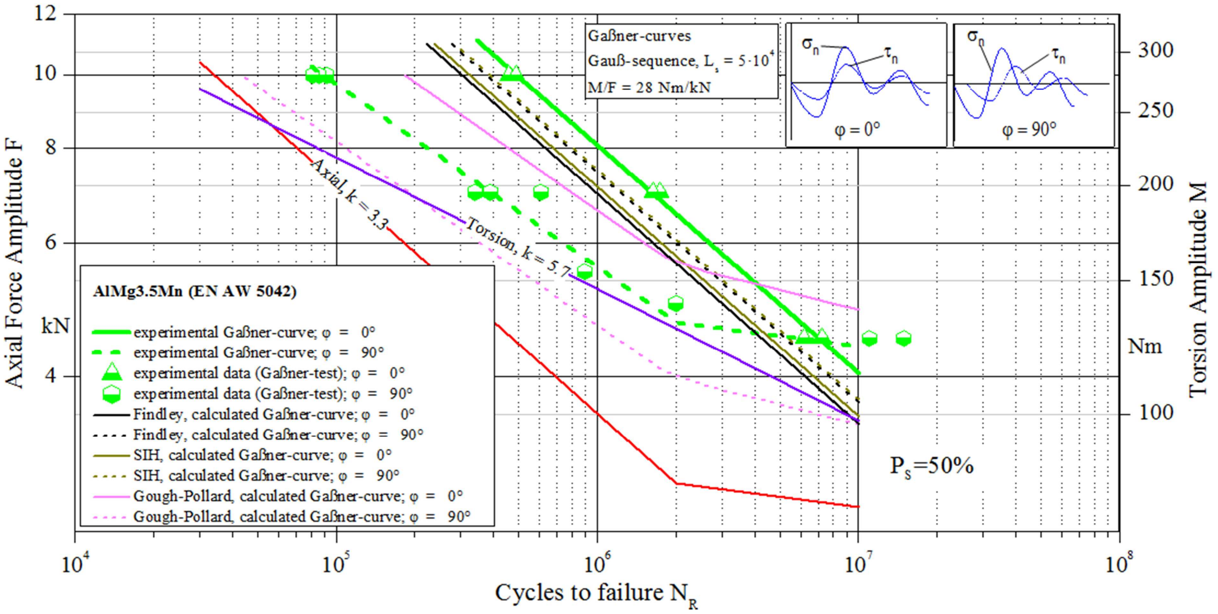

Figure 5. Gaßner-curves for the AlMg3.5Mn alloy calculated using different hypotheses, axial and torsion SN-curves are also shown. Ratio $\frac{M}{F} = 28$ for combined loadings

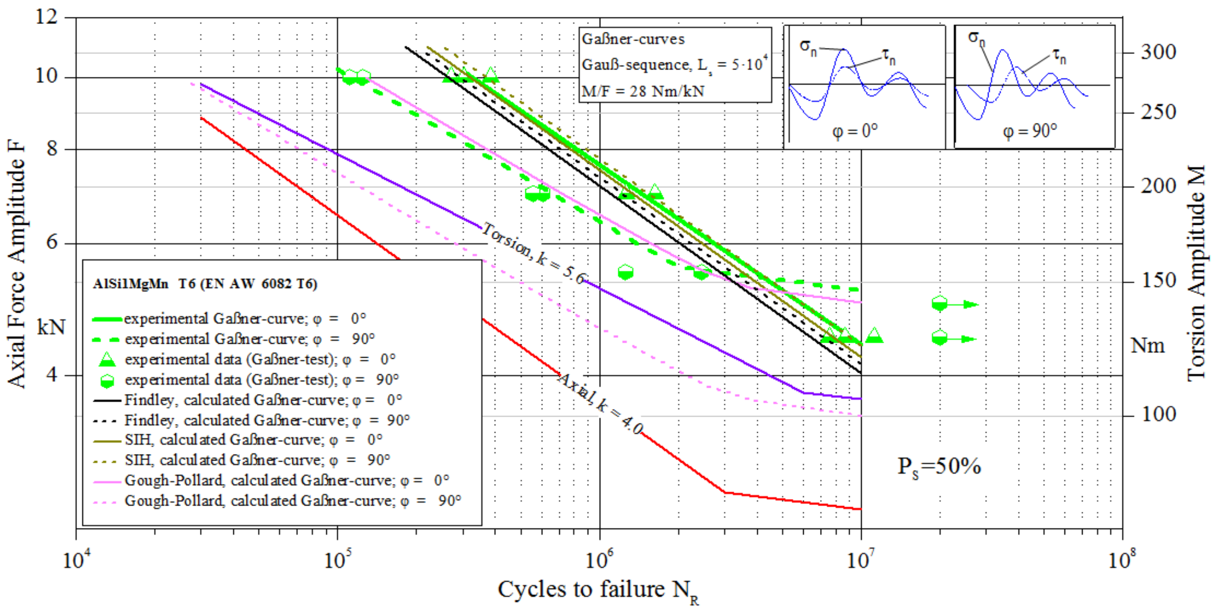

Figure 6. Gaßner-curves for the AlSi1MgMn alloy calculated using different hypotheses, axial and torsion SN-curves are also shown. Ratio $\frac{M}{F} = 28$ for combined loadings

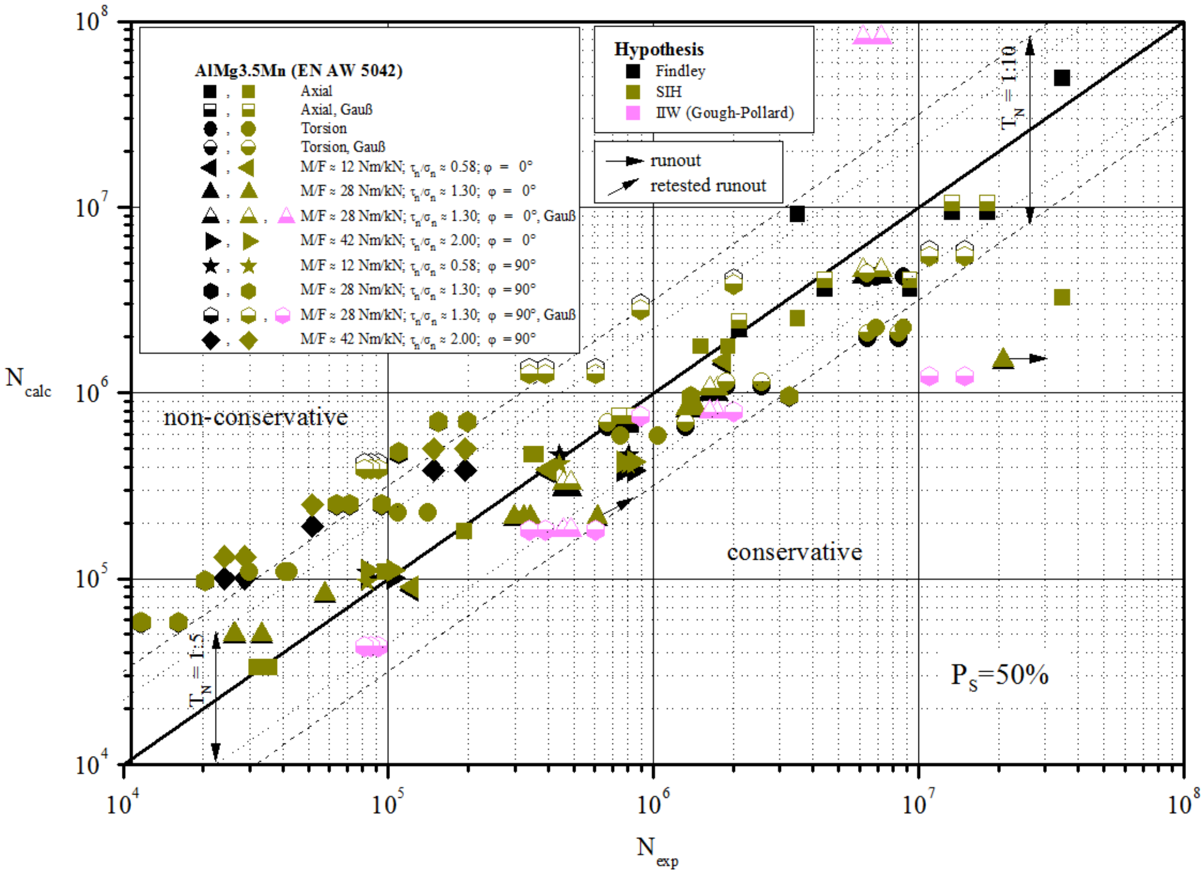

Figure 7. Experimental and calculated fatigue life for the AlMg3.5Mn alloy

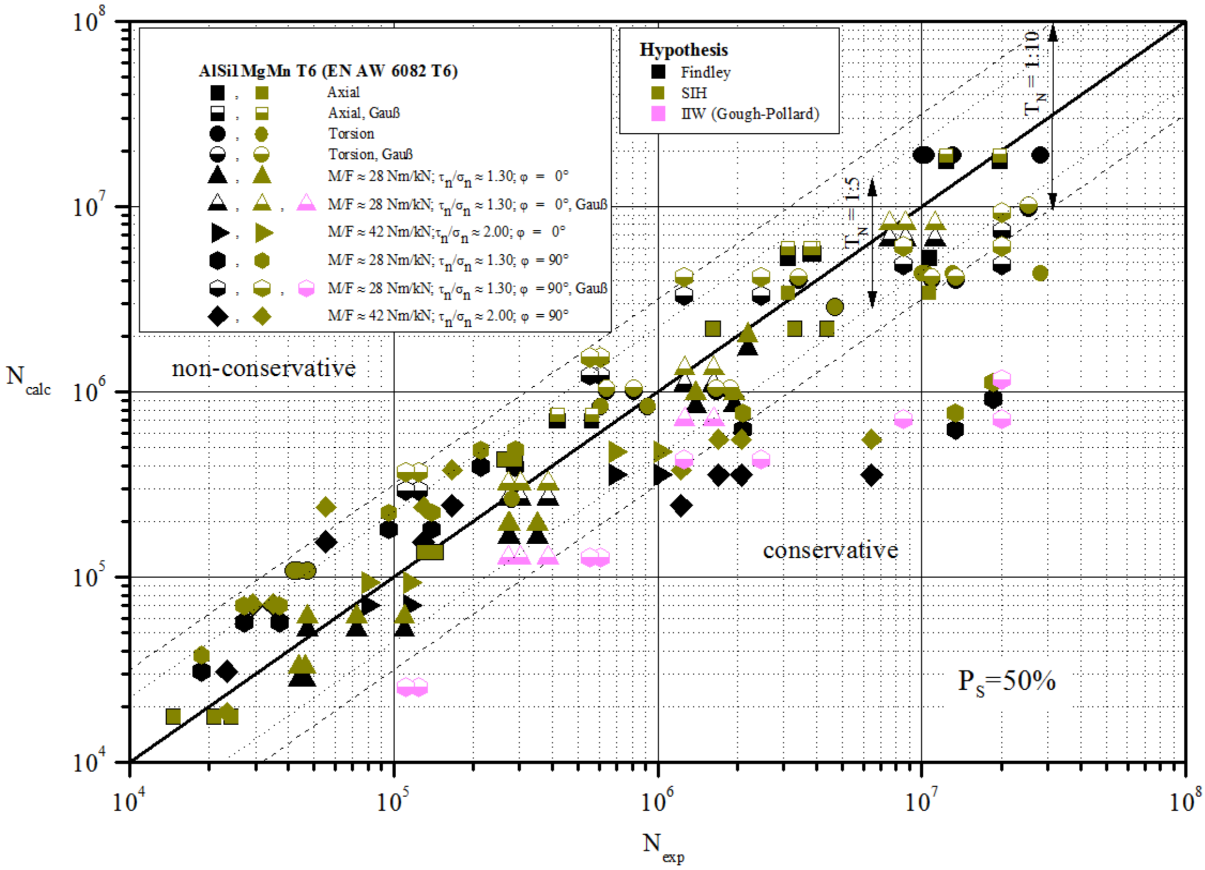

Figure 8. Experimental and calculated fatigue life for the AlSi1MgMn alloy

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