Effects of Biaxial Mean Stress on Critical Plane Orientations under Biaxial Fatigue Loading Conditions

Baimao LEI[1, a], Van-Xuan TRAN[2, b], Saïd TAHERI[2], Li WAN[1] and Yu ZHOU[1]

[1] Institute of Nuclear and New Energy Technology, Tsinghua University
Nengke Building, Tsinghua University, Haidian District, Beijing 100084 CHINA
[2] Laboratoire Mécanique des Structures Industrielles Durables LaMSID, UMR CNRS-EDF 2832. Department of Analysis in Mechanics and Acoustics, EDF R&D
1 avenue du Général de Gaulle 92141 Clamart cedex, FRANCE
[a] E-mail: lbm09@mails.tsinghua.edu.cn and [b] E-mail: van-xuan.tran@edf.fr

ABSTRACT. *Different crack orientations are observed at different locations in the crack network discovered in the old Residual Heat Removal (RHR) system of nuclear power plants. The crack orientations are thought to be possibly related to the orientations of the critical plane on which the accumulated fatigue damage is maximum during the crack initiation period. In this paper, the effects of biaxial mean stress on critical plane orientations under biaxial fatigue loading conditions are investigated based on a combined analytical and computational approach. The analytical solutions of critical plane orientations using Matake's and Fatemi-Socie's crack initiation criteria are first derived and validated by the computational results obtained from a specialised C/C++ program and Code_Aster® software. The analytical solutions are then adopted to study the effects of biaxial mean stress on critical plane orientations. According to the analytical solutions, the critical plane orientations appear to strongly depend on the dominating stress under biaxial fatigue loading and tend to a constant orientation when the mean stress is sufficiently large. Finally, the analytical critical plane orientations are correlated with the observed crack directions in the RHR system.*

INTRODUCTION

Thermal fatigue crazing was observed in the mixing zone of the T-section of the pipes in the old RHR system subjected to thermal fluctuations caused by the turbulent mixture of hot and cold flows [1]. The thermal crazing is characterized by a network of edge cracks that are shallow, dense, uni- and multidirectional as shown in Fig. 1 [2]. Even though the design of RHR has been modified for the last decade, a better understanding of the thermal fatigue crazing will contribute to enhance the system integrity and durability.

The total fatigue life consists of crack initiation life and crack propagation life. For the estimation of the crack initiation life, a common approach is to compute the fatigue damage at all points of structures/components following all possible plane orientations (at each point) to find the maximum damage [3]. The obtained location and associated plane giving the maximum damage are defined as the critical location and the critical plane, respectively. It is noted that the cracks are likely to be initiated at the critical location with the cracking plane following the critical plane.

The main thermo-mechanical problem under consideration is a thin pipe subjected to high cycle thermal fatigue loading. The radial stress in a thin pipe is generally negligible as compared to axial and circumferential stresses [4]. In this case, the stress state in a pipe under thermal loading can be considered as biaxial. As shown in Fig. 1, different crack orientations are observed at different locations, especially at locations of tensile residual stresses. These different observed cracking directions are possibly due to different orientations of the critical planes during crack initiation period. These different critical plane orientations are in turn thought to be correlated with the different values of the residual stress in different locations and directions [1, 2].

Note that residual stress contributes significantly to the mean stress under fatigue loadings. Also, an exact determination of residual stresses for components in service is quite challenging. Therefore, the effects of biaxial mean stress on the critical plane orientations under biaxial fatigue loading conditions are investigated using a combined analytical and computational approach in this paper.

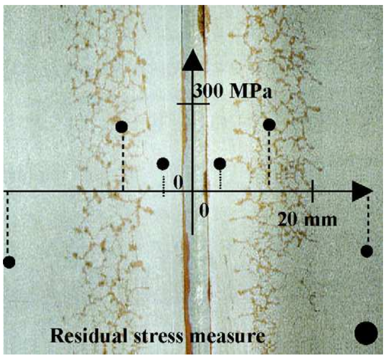

Figure 1. Thermal crazing in the old RHR system [2].

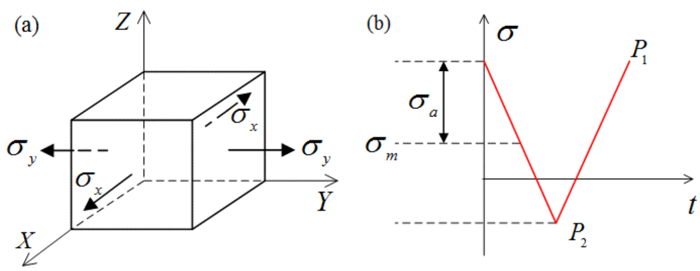

Figure 2. (a) Loading conditions on an elementary cube (b) Triangular loading histories.

ANALYTICAL AND COMPUTATIONAL MODELS

Loading conditions and materials

In this study, an elementary cube subjected to a biaxial proportional loading that represents applied thermal load at a particular material point (or at the critical location) in a tubular specimen is used in the computations as shown in Fig. 2(a). Three important parameters, the ratio of stress amplitudes, denoted as λ, and the ratios of mean stresses in the two directions, denoted as α_x and α_y respectively, are defined in Eq. 1. Here, $\sigma_{a,x}$, $\sigma_{m,x}$, $\sigma_{a,y}$ and $\sigma_{m,y}$ represent the amplitudes and mean values of stresses in x and y directions respectively. The loading histories are shown in Fig. 2(b).

$$\lambda = \frac{\sigma_{a,y}}{\sigma_{a,x}},\ \alpha_x = \frac{\sigma_{m,x}}{\sigma_{a,x}},\ \alpha_y = \frac{\sigma_{m,y}}{\sigma_{a,y}} \tag{1}$$

An isotropic elastic material with the Young's modulus $E = 193000\text{MPa}$ and Poisson's ratio $\nu = 0.3$ is used in this study.

Crack initiation criteria

Matake's criterion

An equivalent stress as defined in Eq. 2 is introduced to transform the original Matake's criterion into a damage accumulation model [6].

$$\sigma_{eq}\left(\vec{n}\right) = \frac{\Delta\tau\left(\vec{n}\right)}{2} + a\max\left\{0, N_{\max}\left(\vec{n}\right)\right\} \tag{2}$$

where a is a material constant. $\vec{n}$ represents the normal vector of the plane as shown in Fig. 3(a) and could be represented as $\vec{n} = \left(n_x, n_y, n_z\right) = \left(\cos\varphi_x, \cos\varphi_y, \cos\varphi_z\right)$, where φ_x, φ_y and φ_z represent angles between the normal vector and axes. Note that n_x, n_y and n_z represent the corresponding direction cosines. Also, it is noted that $n_x^2 + n_y^2 + n_z^2 = 1$.

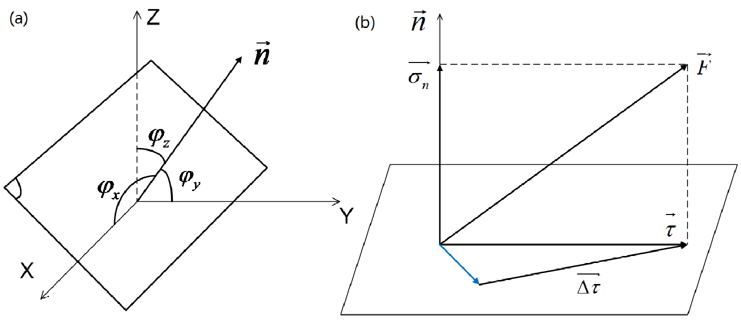

Figure 3. (a) A plane represented by $\vec{n}$
(b) Projection of stress on this plane

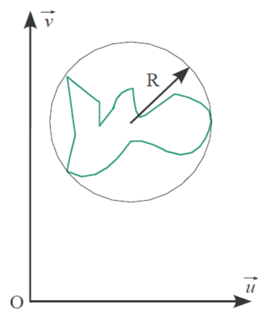

Figure 4. MCC Method [6]

In Eq. 2, $\Delta\tau$ represents the amplitude of shear stress and $N_{\max}$ represents the maximum normal stress on the plane represented by $\vec{n}$ (Fig. 3(b)) during the loading cycle. For periodic loads, the minimum circumscribed circle (MCC) method is used to compute $\Delta\tau/2$, which is computed as the radius of the circle R as shown in Fig. 4 [6]. This equivalent stress will then be used to estimate the number of cycles at rupture by using a corresponding fatigue life curve. Maximum damage is therefore reached on a plane of maximum σ_{eq}.

Fatemi-Socie's criterion

An equivalent strain as defined in Eq. 3 is introduced to transform the original Fatemi-Socie's criterion into a damage accumulation model [3].

$$\varepsilon_{eq}\left(\vec{n}\right)=\frac{\Delta\gamma\left(\vec{n}\right)}{2}\left(1+k\frac{\max\left\{0,N_{\max}\left(\vec{n}\right)\right\}}{S_y}\right) \tag{3}$$

where k is a material constant and S_y represents the yield stress. In this study, $S_y = 208\text{MPa}$ is adopted. $\Delta\gamma$ represents the amplitude of shear strain during the loading history. The minimum circumscribed circle method for strain histories is also used to compute $\Delta\gamma/2$ for periododic loads [6].

ANALYTICAL SOLUTIONS OF CRITICAL PLANE ORIENTATIONS

In Fig. 3(b), $\vec{F}$ represents the stress vector applied on an arbitrary plane of normal vector $\vec{n}$. Note that $\vec{F}=\boldsymbol{\sigma}\cdot\vec{n}$ and the normal stress $\overrightarrow{\sigma_n}=\left(\vec{F}\cdot\vec{n}\right)\vec{n}$ ($\boldsymbol{\sigma}$ represents the stress tensor). The shear stress is projected on this plane as $\vec{\tau}=\vec{F}-\overrightarrow{\sigma_n}$. From Eqs. 1, 2 and $n_x^2+n_y^2+n_z^2=1$, the equivalent stress for Matake's criterion could be written as

$$\sigma_{eq}(n_x,n_y)=\sigma_{a,x}\left\{\sqrt{n_x^2+\lambda^2 n_y^2-\left(n_x^2+\lambda n_y^2\right)^2}+a\max\left\{0,\left[\left(1+\alpha_x\right)n_x^2+\left(1+\alpha_y\right)\lambda n_y^2\right]\right\}\right\} \tag{4}$$

The critical plane orientation is obtained by finding the maximum value of the two-variable function $\sigma_{eq}(n_x,n_y)$. This leads to the solving of the following equation

$$\frac{\partial\sigma_{eq}}{\partial n_x}=\frac{\partial\sigma_{eq}}{\partial n_y}=0,\left(\frac{\partial^2\sigma_{eq}}{\partial n_x\partial n_y}\right)^2-\frac{\partial^2\sigma_{eq}}{\partial n_x^2}\cdot\frac{\partial^2\sigma_{eq}}{\partial n_y^2}<0,\frac{\partial^2\sigma_{eq}}{\partial n_x^2}<0 \tag{5}$$

Similar derivation process for obtaining the analytical solution using Fatemi-Socie's criterion is not repeated here. The analytical solutions of φ_x, φ_y and φ_z for the critical plane orientations are summarized in Table 1 as functions of $C_j\left(\alpha_i\right)$ with $j=M,F$ associated with Matake's and Fatemi-Socie's criteria, respectively. The expressions of $C_M\left(\alpha_i\right)$ and $C_F\left(\alpha_i\right)$ are defined in Eqs. 6 and 7, respectively. In these equations, $i=x,y$ represents the directions.

$$C_M\left(\alpha_i\right)=\sqrt{\frac{1+\sqrt{1-\frac{1}{a^2(1+\alpha_i)^2+1}}}{2}} \tag{6}$$

$$C_F(\alpha_i)=\sqrt{\frac{3-\frac{2S_y}{k\sigma_{a,i}\left(1+\alpha_i\right)}+\sqrt{\left(1+\frac{2S_y}{k\sigma_{a,i}\left(1+\alpha_i\right)}\right)^2+8}}{8}} \tag{7}$$

A representative value of $\sigma_{a,x}=200\text{MPa}$, which represents the magnitude of stress variation computed for the RHR system [5], is fixed in this study. A complete parametric study requires to vary the three parameters λ, α_x and α_y. For the convenience of discussion, only special cases of ($\alpha_x=\alpha_y$, λ varies) and ($\lambda=1$, α_x and α_y vary) are presented in this paper.

Table 1. Analytical solutions of critical plane orientations

Case	Loading Condition		φ_x	φ_y	φ_z
(a)	$\lambda=0$		$\arccos C_j(\alpha_x)$	$\cos^2\varphi_y+\cos^2\varphi_z=1-C_j^2(\alpha_x)$	
(b)	$0<\lambda<1$	$\alpha_x=\alpha_y$	$\arccos C_j(\alpha_x)$	$90°$	$\arccos\sqrt{1-C_j^2(\alpha_x)}$
(c)	$\lambda>1$	$\alpha_x=\alpha_y$	$90°$	$\arccos C_j(\alpha_y)$	$\arccos\sqrt{1-C_j^2(\alpha_y)}$
(d)	$\lambda=1$	$\alpha_x>\alpha_y$	$\arccos C_j(\alpha_x)$	$90°$	$\arccos\sqrt{1-C_j^2(\alpha_x)}$
(e)		$\alpha_x=\alpha_y$	$\cos^2\varphi_x+\cos^2\varphi_y=C_j^2(\alpha_x)$		$\arccos\sqrt{1-C_j^2(\alpha_x)}$
(f)		$\alpha_x<\alpha_y$	$90°$	$\arccos C_j(\alpha_y)$	$\arccos\sqrt{1-C_j^2(\alpha_y)}$

As listed in Table 1, critical plane orientations are determined by the dominating stress with either the larger mean stress (same amplitude) or larger amplitude (same mean stress ratio). When stresses in the two directions are identical (case (e) in Table 1), many possible critical planes exist.

The analytical critical plane orientations are sketched in red as shown in Fig. 5. It is noted that for $\lambda=0$ (uniaxial loading), φ_x becomes 45° when $N_{\max}\leq 0$. In this case, the plane of maximum damage is also the plane of maximum shear stress. In more general cases, φ_x is different from 45° due to the influence of the mean stress. For cases (b) and (d), the critical plane is parallel to the Y axis. For cases (c) and (f), the critical plane is parallel to the X axis. For the special case (e) when λ=1 and $\alpha_x = \alpha_y$, any combination of angles satisfying the condition $\cos^2\varphi_x + \cos^2\varphi_y = C_j^2(\alpha_x)$ is a possible sollution.

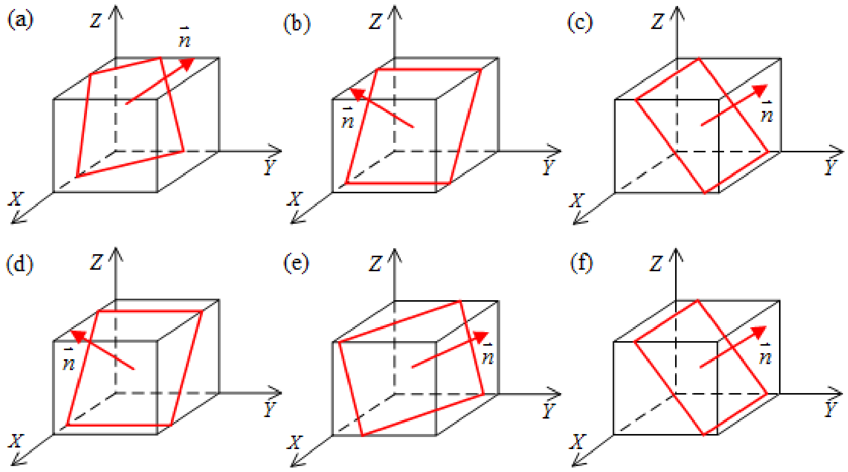

Figure 5. Visualizations of analytical solutions listed in Table 1

EFFECTS OF MEAN STRESS ON CRTICAL PLANE ORIENTATIONS

Computational results using a specialized C/C++ program and Code_Aster® software are first used to validate the analytical solutions as shown in Fig. 6 for a representative case (b) using Matake's criterion. Similar figure for Fatemi-Socie's criterion will be reported elsewhere. A good agreement between analytical and computational results confirms the analytical solutions and the robustness of Code_Aster® development.

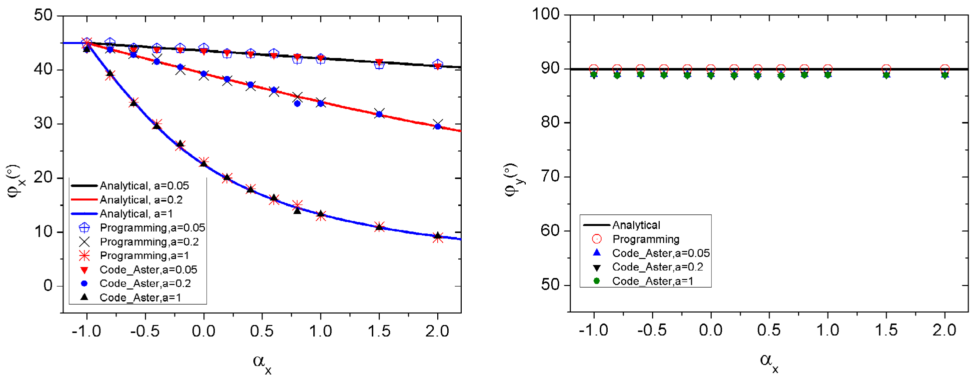

Figure 6. Critical plane orientation using Matake's criterion for case (b).

When the mean stress ratio $\alpha_x \le -1$, the maximum normal stress $N_{\max} \le 0$. In this case, the plane of maximum damage is also the plane of maximum shear stress ($\varphi_x = 45°$) based on Eqs. 2 and 3. When the mean stress ratio α_x increases and tends to sufficiently large, it could be induced from Eq. 6 that φ_x decreases and tends to 0 for Matake's criterion. Similarly, φ_x decreases and tends to 30° for Fatemi-Socie's criterion from Eq. 7. The ranges of φ_x for both criteria are shown in Fig. 7.

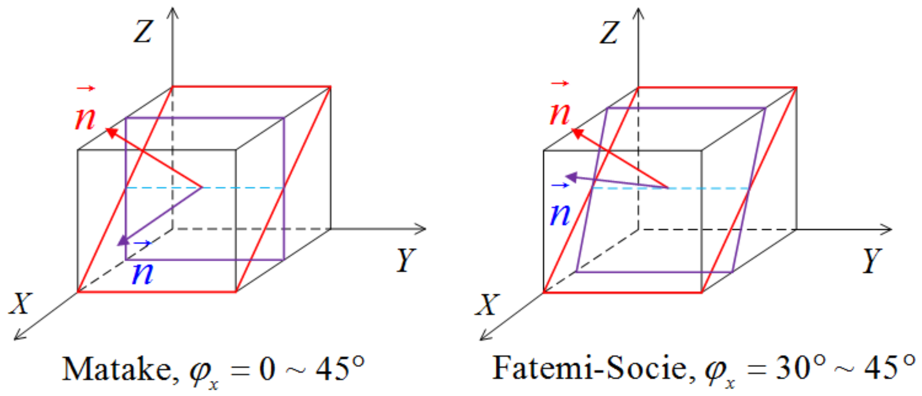

Figure 7. Ranges of the critical angle φ_x for case (b).

CORRELATION WITH OBSERVED CRACK DIRECTIONS AND DISCUSSIONS

The crack network in the old RHR system and the simulated residual stresses are shown in Fig. 8 [1]. Note that for a free-end thin pipe, the values of axial and circumferential stresses due to thermal expansion are nearly equal [4]. So $\lambda = 1$ is taken for the stresses induced by thermal loadings in the pipes of the old RHR system.

For the area near the weld tips, the cracks are mainly in the axial direction as shown in Fig. 8(a). It is observed from Fig. 8(b) that the axial residual stress σ_{res}^{axi} is generally smaller than the circumferential residual stress σ_{res}^{cir} near the weld. That leads to $\alpha_x < \alpha_y$. The analytical solution listed in Table 1 for case (f) predicts that the critical plane is parallel to the axial direction ($\varphi_x = 90°$). Far away from the weld, a multidirectional crack network is observed. As shown in Fig. 8(b), σ_{res}^{axi} could be larger, equal or smaller than σ_{res}^{cir} for the areas far away from the weld. The analytical solutions listed in Table 1 for cases (d), (e) and (f) predict that the critical plane could orient in different directions as functions of residual stresses. The prediction of critical plane orientation is therefore qualitatively consistent with the observed directions of edge cracks. The critical plane orientations may be therefore used to partially explain the cracking directions of thermal fatigue crazing.

Note that the crack propagation life depends strongly on the initial crack location and plane. As discussed earlier, it is likely that the crack is initiated at the critical location where the damage reaches maximum and the cracking plane could follow the critical plane orientation associated with this critical location as shown in Fig. 9. The analytical solutions of the critical plane orientations reported in this paper can be also used as a reference to justify the choice of the initial crack plane in the crack propagation analysis with the presence of residual stresses.

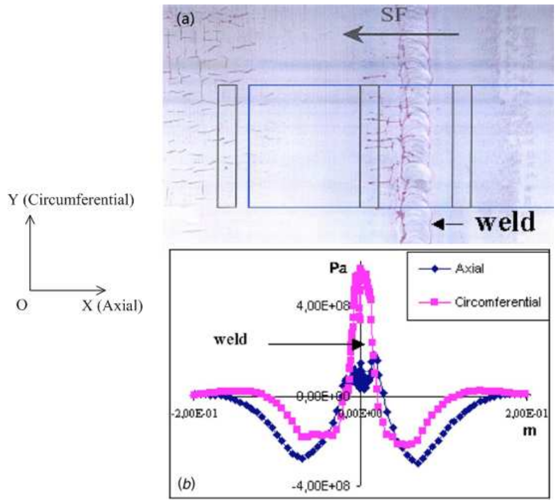

Figure 8. (a) Crack network and (b) simulated residual stresses in a RHR[1].

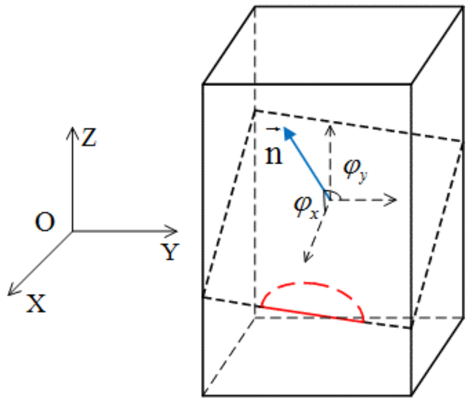

Figure 9. Initial crack plane for crack propagation models.

CONCLUSIONS

In this paper, the effects of biaxial mean stress on critical plane orientations under biaxial fatigue loading conditions are investigated based on a combined analytical and computational approach. According to the analytical and computational solutions, the critical plane orientations appear to strongly depend on the dominating stress under biaxial fatigue loading, and tend to a constant orientation when the mean stress is sufficiently large. Finally, the analytical critical plane orientations are correlated with the observed crack directions in the old RHR system.

REFERENCES

1. Taheri, S. (2007) *J. Press. Vess.-T. ASME* **129**, 400-410.
2. Sbitti, A., and Taheri, S. (2010) *Nucl. Eng. Des.* **240**, 30-38.
3. Tran, V. X., (2013) Code_Aster Reference Documentation, R7.04.04
4. Musi, S., and Beaud, F., (2003) *ASME PVP2003* -1997
5. Chapuliot, S., Gourdin, etc., (2005) *Nucl. Eng. Des.* **235**, 575-596.
6. Angles, J., Taheri, S., and Papaconstantinou, T., (2005) *SMiRT 18* W101-4