Incremental approach for the fatigue design under complex loading of metallic materials

D. Halm[1], Y. Nadot[1], Q. H. Vu[2]

[1] Institut PPRIME - CNRS UPR 3346 - University of Poitiers - ENSMA – Department Physics and Mechanics of Materials – Téléport 2 – 1 Avenue Clément Ader – BP 4019 – F 86961 FUTUROSCOPE CHASSENEUIL Cedex – France
damien.halm@ensma.fr
yves.nadot@ensma.fr
[2] Hanoi University of Science and Technology, 1 Dai Co Viet Road, HANOI, Vietnam
huyvq-ite@mail.hut.edu.vn

ABSTRACT. *This paper aims at presenting a design methodology for in-service complex variable fatigue loading on metallic engineering components. Whereas most of the approaches in the literature deal with variable loading by using counting techniques (for example, rainflow counting), the proposal at stake follows directly the load path over time history, and thus avoids any loading processing (i.e. transformation and loss of information). In order to overcome a purely phenomenological description, a two-scale damage model (macro – meso) integrates a multiaxial fatigue criterion and is formulated in the framework of thermodynamics of irreversible processes allowing to capture as closely as possible degradation mechanisms at mesoscopic scale as well as phase shift effect and non linear fatigue damage accumulation. The incremental formulation of the proposed model is an asset to deal with variable amplitude loadings in future works.*

INTRODUCTION

Fracture due to high cycle fatigue (HCF) is a frequently encountered issue in mechanical components. Numerous studies aim at providing reliable tools for computational mechanics of structures subjected to cyclic complex (multiaxial, non-proportional, variable amplitude) stress states. The complexity of the models results from the simultaneous influence of many factors, such as loading path, material microstructure, environment and surface effects. In HCF regime, the dominating role of plasticity and damage at the local scale (grain, slip bands) is widely admitted. This argument is the starting point for many multiscale models proposed in the literature. For example, Dang Van [1] introduces a microscopic approach using the elastic shakedown principle and the Schmid law to provide a non initiation criterion. Following this work, the models proposed by Papadopoulos [2] and Morel [3], [4], [5], allow determining fatigue lifetime for many loadings (even multiaxial with variable amplitude). However, these approaches do not explicitly refer to the damage phenomena at microscale and consider the cumulative plastic strain as the damage variable. In order to overcome this limit, two-scales (“meso / macro”) models can be found in the literature to predict both endurance limite and finite lifetime [6-9].

This study aims at providing a tool to predict lifetime for a metallic polycrystalline material subjected to high cycle complex multiaxial loadings. Two elements are associated:

(i) A simple multiaxial endurance criterion based on macroscopic stress invariants has been proposed by Vu et al. [10]. Besides the explicit influence of the hydrostatic stress, the presence of the mean value of the second invariant of the deviatoric stress allows capturing correctly the effects of the phase shift. The material parameters are identified from two fatigue limits (for example, fully reversed tension and torsion) and the ultimate stress. From the numerical point of view, the computational time appears negligible. This criterion is a pivotal ingredient to elaborate the plastic / damage model whose purpose is to predict finite fatigue lifetime.

(ii) A coupled plastic / damage incremental model at the mesoscale, initially proposed by Flacelière et al. [11] and impoved by Vu [12] is dedicated to HCF to predict both endurance limit and finite lifetime. The plastic strain and the hardening level at the grain scale are simulated by means of a plastic model incorporating the above mentioned fatigue criterion. The mechanisms responsible for fracture are coupled to plasticity via a damage model formulated in the framework of the irreversible processes. Progressive degradation at mesoscale is captured through a scalar damage variable which acts on the isotropic hardening. The effects of microstructure are also taken into account by splitting the damage evolution into two stages (initiation and propagation).The plasticity / damage model is validated by simulating complex loading fatigue tests (multiaxial, out of phase, block loading).

HYPOTHESES AND MODEL FORMULATION

In HCF, the main source of failure of metallic components is attributed to plasticity and damage at grain scale (micro/mesoscopic scale). However, loading on structure is applied at macroscopic scale, justifying the choice of a two-scale modelling framework. The crack initiation process is the result of a simple slip system within one or many grains. This assumption is consistent with observations carried out on different materials (Rasmussen and Pedersen [13]).

It is chosen to model a quite general plastic behaviour for polycrystalline metals, in particular for face-centered-cubic (fcc) metals. Damage mechanisms are related to the nucleation and growth of slip bands: micro-damage occurs from a pronounced localization level of plastic slip.

The model is based on a multiscale approach (Dang Van [1]). The crystal hardening modelling takes into account three successive phases: hardening, saturation and softening (Papadopoulos [2]). The original idea consists of the competition, at the grain scale, between meso-plasticity and local damage during the sample lifetime.

The link between the stress at macroscale $\underline{\underline{\Sigma}}$ and at microscale $\underline{\underline{\sigma}}$ is given by the classical Lin-Taylor localization relationship $\underline{\underline{\sigma}} = \underline{\underline{\Sigma}} - 2\mu\underline{\underline{\varepsilon}}^p$ (where μ is the shear modulus and $\underline{\underline{\varepsilon}}^p$ the plastic strain at the grain scale). The model is formulated in the rigorous framework of irreversible thermodynamics with internal variables for isothermal, time-independent and small deformation transformations. The thermodynamic formulation assumes the existence of a state potential. The evolution of internal variables is governed by: (i) one (or many) threshold function(s) responding to the question "when do plasticity and damage evolve?" and (ii) one (or many) dissipation potential(s) indicating, by assumption of normality, how progress this mechanisms (Germain et al. [14]). Table 1 summarizes the expression of the different ingredients of the model, which are described below.

Together with the plastic strain $\underline{\underline{\varepsilon}}^p$, isotropic hardening (p) and kinematic hardening ($\underline{\underline{\alpha}}$) variables, two distinct scalar damage variables (d and β) are used to account for the evolution of cracking at the grain scale. The first variable (d) is "the damage effect variable" acting on the degradation of the mechanical strength of crystal that affects directly the isotropic hardening. The second variable (β), by analogy with the cumulated plastic strain (p), plays the role of cumulated damage, accounting for the storage of energy (friction effect on the crack faces). This variable is similar to that used by Murakami and Kamiya [15].

The specific free energy ω of a grain combines the influence of plasticity and damage. The state laws (namely the elastic stress $\underline{\underline{\sigma}}$, the backstress $\underline{\underline{x}}$, the isotropic hardening force r, the damage driving forces F^d and k) are obtained by differentiation of ω.

Regarding the plasticity evolution laws, the proposed plastic yield surface at grain scale is consistent with the work by Vu et al. [10]: beside the classical second invariant J_2, an extra term $J_{2,mean}$ captures the phase shift effects. It is defined as $J_{2,mean} = \frac{1}{T}\int_0^T \sqrt{\frac{1}{2}\underline{\underline{S}}^a(t):\underline{\underline{S}}^a(t)}\, dt$, *i.e.* the mean value of the second invariant of the amplitude $\underline{\underline{S}}^a$ of the deviatoric part of the applied stress $\underline{\underline{\Sigma}}$ over a period T. The influence of the hydrostatic stress is taken into account through a specific term, namely I_f, depending on the ultimate strength R_m and on the mean value $I_{1,m}$ and the amplitude $I_{1,a}$ of the first invariant of $\underline{\underline{\Sigma}}$ (cf. Vu et al. [10]). The evolution of the plastic internal variables $\underline{\underline{\varepsilon}}^p$, $\underline{\underline{\alpha}}$ and p have been found to follow the normality rule with respect to a dissipation potential F different from f (non associated plasticity).

As far as damage evolution is concerned, the model has to reflect the non-linearity of the damage evolution as well as the difference of damage behavior between tension and torsion loadings. A unique yield function h governs the damage threshold. The model considers the damage initiation and propagation phases and thus two dissipation potentials H_1 and H_2 allow distinguishing both phases. The condition of initiation /

propagation transition is based on the value of the variable *d* by using a material parameter d_p. A material parameter d_c is introduced to represent the critical value (at failure) of the damage effect variable.

Table 2 provides the meaning of each material parameter used in Table 1.

The terms $\dot{\lambda}^p$ and $\dot{\lambda}^d$ stand respectively for the plastic and damage multipliers. Note that, for the case of pure torsion loading, the damage rates in both phases (initiation / propagation) differ only in the coefficient *a*. For tension loading, the propagation phase rate is accelerated by the hydrostatic stress. An appropriate adjustment of the coefficients *a* and *b* allows taking into account the difference in damage kinetics in both phases.

Table 1. Constitutive equations of the proposed model

Localization law : $\underline{\underline{\sigma}} = \underline{\underline{\Sigma}} - 2\mu\underline{\underline{\varepsilon}}^p$	
Free energy at the grain scale $\omega = \frac{1}{2}\left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p\right):\underline{\underline{\underline{C}}}:\left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p\right) + \frac{1}{2}c\underline{\underline{\alpha}}:\underline{\underline{\alpha}} + \tilde{r}_\infty p\exp(-sd) + \frac{\tilde{r}_\infty}{g}\exp(-gp)\exp(-sd) + \frac{1}{2}q\beta^2$	
Plasticity at the grain scale	Damage
$\underline{\underline{\sigma}} = \frac{\partial\omega}{\partial\underline{\underline{\varepsilon}}^e} = \underline{\underline{\underline{C}}}:\underline{\underline{\varepsilon}}^e$	$F_d = -\frac{\partial\omega}{\partial d} = \tilde{r}_\infty s\exp(-sd)\left(p + \frac{\exp(-gp)}{g}\right)$
$\underline{\underline{x}} = \frac{\partial\omega}{\partial\underline{\underline{\alpha}}} = c\underline{\underline{\alpha}} = c\underline{\underline{\varepsilon}}^p$	$k = \frac{\partial\omega}{\partial\beta} = q\beta$
$r = \frac{\partial\omega}{\partial p} = \tilde{r}_\infty\left(1 - \exp(-gp)\right)\exp(-sd)$	
Plasticity evolution laws (non - associated)	
$f\left(\underline{\underline{\sigma}},\underline{\underline{x}},r\right) = \sqrt{\gamma_1 J_2^2\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right) + \gamma_2 J_{2,mean}^2 + \gamma_3 I_f\left(I_{1,a}, I_{1,m}\right)} - (r + r_0) \le 0$	
$F\left(\underline{\underline{\sigma}},\underline{\underline{x}},r\right) = J_2\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right) - r$	
$\dot{\underline{\underline{\alpha}}} = \dot{\underline{\underline{\varepsilon}}}^p = -\dot{\lambda}^p\left(\frac{\partial F}{\partial\underline{\underline{x}}}\right) = \frac{1}{2}\dot{\lambda}^p\frac{\underline{\underline{s}} - \underline{\underline{x}}}{J_2\left(\underline{\underline{\sigma}} - \underline{\underline{x}}\right)}$	$\dot{p} = -\dot{\lambda}^p\left(\frac{\partial F}{\partial r}\right) = \dot{\lambda}^p$
Damage evolution laws (non - associated)	
$h(F_d, k) = F_d - (k + k_0) \le 0$	
Initiation phase	Propagation phase
$H_1(F_d, k) = aF_d - k \quad (0 < a \le 1)$	$H_2(F_d, k; \sigma_h) = F_d\left(1 + b\langle\sigma_h\rangle\right) - k \quad (b > 0)$
$\dot{d} = \dot{\lambda}^d\left(\frac{\partial H_1}{\partial F_d}\right) = a\dot{\lambda}^d$	$\dot{d} = \dot{\lambda}^d\left(\frac{\partial H_2}{\partial F_d}\right) = \dot{\lambda}^d\left(1 + b\langle\sigma_h\rangle\right)$
$\dot{\beta} = -\dot{\lambda}^d\left(\frac{\partial H_1}{\partial k}\right) = \dot{\lambda}^d$	$\dot{\beta} = -\dot{\lambda}^d\left(\frac{\partial H_2}{\partial k}\right) = \dot{\lambda}^d$

The proposed model requires the identification of thirteen parameters. The parameters involved in the plastic yield ($\gamma_1, \gamma_2, \gamma_3$, $r_0 + \tilde{r}_\infty$) are identified from two fatigue limits and from the ultimate strength R_m. The hardening moduli c and g are identified from cyclic hardening curves. The identification of the damage parameters are carried out using two S-N curves (under reversed torsion and reversed tension) and one curve giving the crack length evolution.

Table 2. Parameters used in the proposed model

Elastic parameter	
$\underset{\equiv}{C}$	Stiffness tensor
μ	Shear modulus
Plasticity parameters	
c	Kinematic hardening modulus
r_0	Initial yield stress
$r_0 + \tilde{r}_\infty$	Saturation yield stress of plastic flow
g	Hardening modulus
$\gamma_1, \gamma_2, \gamma_3$	Multiaxial fatigue criterion parameters
Damage parameters	
k_0	Initial damage threshold
s	Sensitivity of damage effect variable
q	Evolution modulus of the damage threshold
a	Damage rate coefficient in initiation phase
b	Sensitivity of damage rate wrt. hydrostatic stress in propagation phase
d_p	Damage threshold of propagation phase
d_c	Critical value of the damage effect variable

ASSESSMENT OF THE MODEL CAPABILITY

The set of material parameters has been identified for the 1045 steel (Vu [12]). Table 3 provides the value of the parameters. The model is tested by simulating S-N curves under out-of-phase tension / torsion lodings and block loadings. Figure 1 shows the comparison experiment / simulation corresponding to a 90° shift between the periodic tension and torsion loadings. The stress ratio between the torsion applied stress amplitude Σ_{xya} and the tension applied stress amplitude Σ_{xa} is *0.5*. For this case, the predictions of the proposed model are conservative and the prediction error in terms of applied stress is about 10%. It appears that the maximum prediction of the fatigue limit of the model plays a crucial role in the global quality of prediction. This highlights the interest of the use of the improved plastic yield surface (see Table 1).

Table 3. Identified parameter for the 1045 steel

μ (MPa)	γ_1	γ_2	γ_3 (MPa)	r_0 (MPa)	$\tilde{r}_\infty$ (MPa)	c (MPa)	g
70000	0.65	0.8636	39	160	9	2000	0.1

k_0 (MPa)	s	q (MPa)	a	b	d_p	d_c
40	0.1	300	0.4	0.09	0.1	1

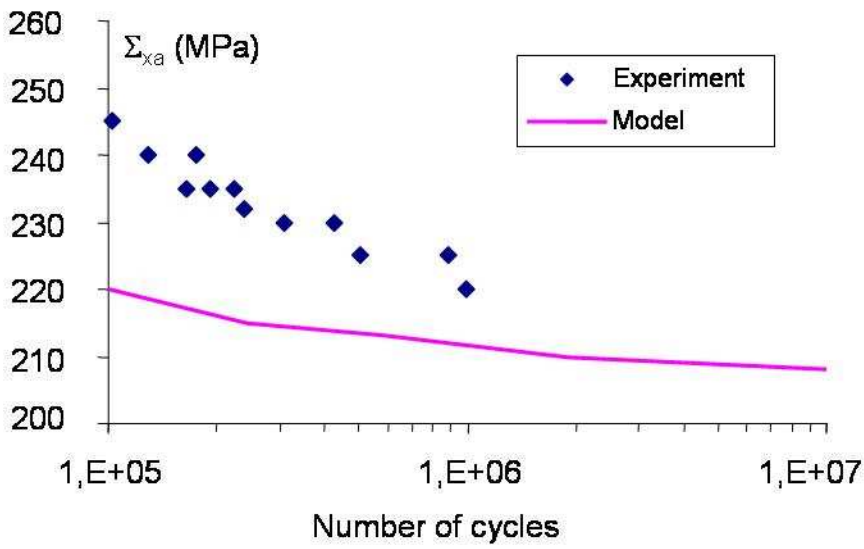

Figure 1. S - N predicted curve and experimental points for 90° out-of-phase reversed tension - torsion

The model is also tested by simulating blocks of different nature: tension followed by torsion (tension/torsion) and torsion followed by tension (torsion/tension). The applied stress amplitudes are of 250 MPa in tension and of 185 MPa in torsion. The corresponding experiments at those amplitude levels under reversed tension and reversed torsion would give the same fatigue lifetime of 3.2 10^5 cycles. The lifetimes predicted by the model are also in the order of 3.2 10^5 cycles ($N_{f,Ten}$ = 3.27 10^5 cycles for tension, $N_{f,Tor}$ = 3.24 10^5 cycles for torsion).

The predictions of the model and the experimental points for the block loadings tension - torsion and torsion - tension are shown in Figure 2. It appears that the model predicts an important nonlinear accumulation for the block loading of different nature. The estimated lifetime of the model is higher than the lifetime obtained from the linear accumulation of Palmgren - Miner in the case of tension - torsion ($\sum n/N_f > 1$) and lower in the case of torsion - tension ($\sum n/N_f < 1$). For the case of 1045 steel in this study, the model reproduces correctly the nonlinear accumulation associated with the loading tension - torsion, the predictions for the case of torsion - tension are less accurate but remain conservative.

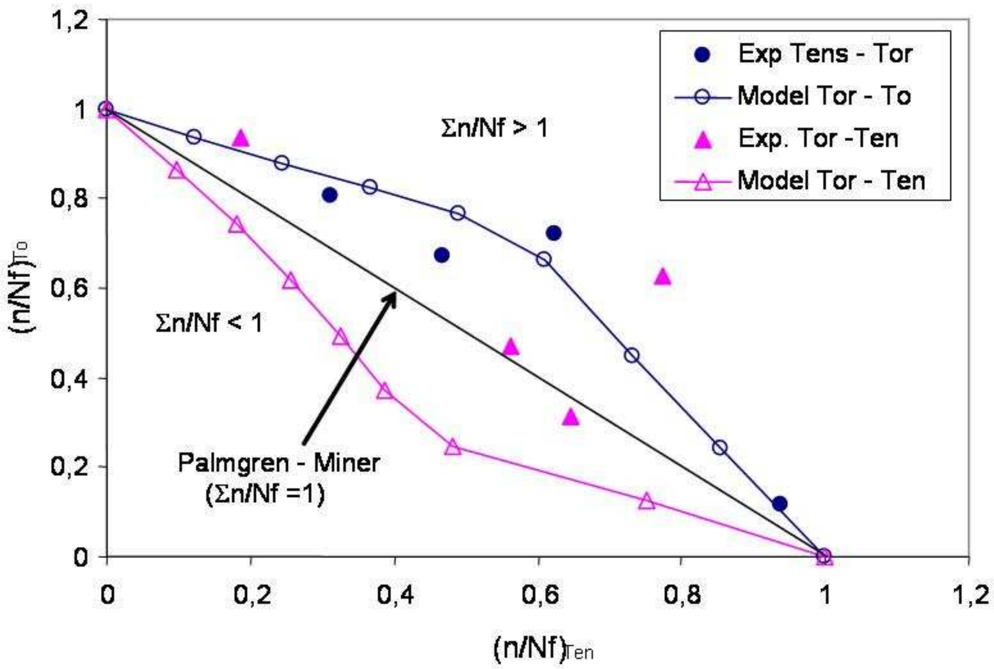

Figure 2. Block loadings Tension - Torsion and Torsion - Tension for 1045 steel – comparison experiment / simulation

CONCLUSION

The model proposed here aims at capturing the effect of out-of-phase loading and the effect of loading sequence (block loading). Concerning the out-of-phase effect, the main feature consists in the introduction of a complex plastic yield surface that derives from a multiaxial fatigue criterion, whose terms ($J_{2,mean}$, $I_{1,a}$, $I_{1,m}$) are easily calculated from a loading cycle.

Experimentally, the block loading leads to a nonlinear accumulation in terms of specimen lifetime. Since the specimen lifetime is estimated from the evolution of the variable of damage effect (*d*), the model captures the nonlinearity via the variation of the damage kinetics during each block. In the structure of the model, two sources of nonlinearity are identified. The first source is related to the difference of the damage evolution between tension mode loading and torsion mode loading. The second source comes from the introduction of two phases of damage propagation.

The incremental nature of the proposed model (damage evolujtion is calculated at each loading increment) is an asset to deal with more complex (f. ex. variable) loadings without any counting method. It could be coupled with cycle jump methods (Cojocaru and Karlsson [16]) in order to avoid prohibitive computer run time.

REFERENCES

1. Dang Van, K. (1973). *Sciences et Technique de l'Armement* **47**, 647-722
2. Papadopoulos, I. V. (1987). PhD thesis, Ecole Nationale des Ponts et Chaussées, Paris, France.
3. Morel, F. (1998). *Fatigue and Fracture of Engineering Materials and Structures* **21**,241-256.
4. Morel, F. (2000). *Int. J. Fatigue* **22**,101-119.
5. Morel, F. (2001). *Fatigue and Fracture of Engineering Materials and Structures*, **24**,153-164.
6. Lemaitre, J., Sermage, J. P., Desmorat, R. (1999). *Int. J. Fracture* **97**,67-81.
7. Doudard, C., Hild, F., Calloch, S. (2007). *Fatigue & Fracture of Engineering Materials & Structures* **30**,107-114.
8. Monchiet, V., Charkaluk, E., Kondo, D. (2008). *Mech. Res. Comm.* **35**,383-391.
9. McDowell, D.L., Dunne, F.P.E. (2010). *Int. J. Fatigue* **32**,1521-1542.
10. Vu, Q.H., Halm, D., Nadot, Y. (2010). *Int. J. Fatigue* **32**,1004-1014.
11. Flacelière, L., Morel, F., Dragon, A. (2007). Int. J. Damage Mech. **16**,473-509.
12. Vu, Q.H. (2009). PhD thesis, ENSMA, Poitiers, France.
13. Rasmussen, K., Pedersen, O. (1980). *Acta Metallurgica* **28**,1467-1478.
14. Germain, P., Nguyen, Q. S., Suquet, P. (1983). *ASME J. Applied Mech*. **50**,1010-1020.
15. Murakami, S., Kamiya, K. (1997). *Int. J. Mech. Sci*. **39**,473-486.
16. Cojocaru, D., Karlsson, A. (2006). *Int. J. Fatigue* **28**,1677-1689.