A multiaxial model for fatigue life estimation based on a combined deviatoric strain amplitude

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ABSTRACT. *A multiaxial model for fatigue life estimation based on a measure of combined deviatoric strain amplitudes, together with the amplitude and the mean value of the hydrostatic stress, is presented in this work. Assessment considered 166 proportional and non-proportional strain-controlled programs reported in the literature, including a number of cases with mean strains/stresses, for two' steels and two aluminum alloys. The resulting life estimates correlated well with the experiments, falling in most cases within a bandwith of factor two.*

INTRODUCTION

In the low cycle fatigue regime, many life estimates are based on critical plane approaches [1-3]. In these models, increase in normal stresses resulting from non-proportional hardening [4] has been considered to quantify the effect of non-proportional loading upon fatigue life. Other approaches to fatigue damage include the damage evolution law proposed by Jiang [5], the short crack model of Vormwald and co-workers [6], the modifed Coffin-Manson method by Susmel and co-workers [7] and the moment of inertia method by Meggiolaro and Castro [8] among others.

As an alternative to the currently available approaches, this paper proposes a model for fatigue life estimation, which presents, as its main feature, a measure of combined deviatoric strain amplitudes based on the concept of prismatic hull [9-11]. The influence of normal stresses on fatigue life is taken into account in terms of the amplitude and the mean value of the hydrostatic stress. Both strain and stress terms incorporate the effect of non-proportional loading upon the fatigue life.

FATIGUE MODEL

This section starts with the definition of a measure of deviatoric strain amplitude, assumed in this study as one of the driving forces responsible for crack nucleation in materials undergoing fatigue failure in shear mode.

For the sake of simplicity, the measure of strain amplitude is defined within the setting of plane stresses, where the matrix representation of the strain state is given by

$$\varepsilon = \begin{pmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & 0 \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}, \quad \text{where} \quad \varepsilon_z = \frac{1-2\nu}{E}(\sigma_x + \sigma_y) - (\varepsilon_x + \varepsilon_y). \tag{1}$$

The corresponding deviatoric strain tensor can be written in terms of an appropriate orthonormal basis, leading to its representation in vector form

$$e \ = \ (e_1 \quad e_2 \quad e_3) = \left(\frac{1}{\sqrt{6}}(2\varepsilon_x - \varepsilon_y - \varepsilon_z) \quad \frac{1}{\sqrt{2}}\,\gamma_{xy} \quad \frac{1}{\sqrt{2}}(\varepsilon_y - \varepsilon_z)\right). \tag{2}$$

From Eq. 2, strain histories in terms of its deviatoric components can be described as curves in R^3, as illustrated in Fig.1.a.

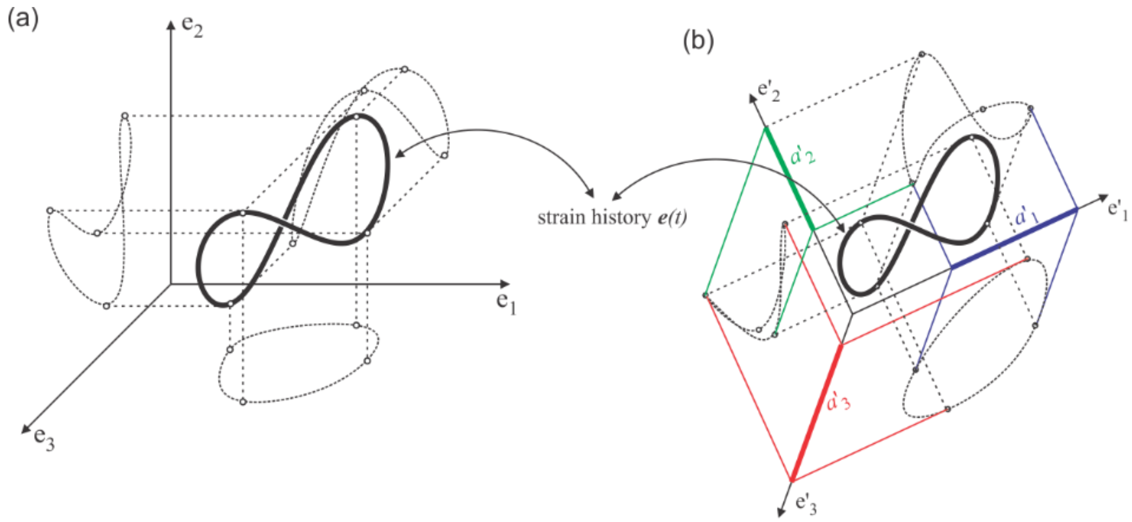

Figure 1: (a) Representation of history $e(t)$ of deviatoric strain in R^3; (b) Strain amplitudes a_1', a_2' and a_3' in a coordinate system with orientation $(\theta_1, \theta_2, \theta_3)$.

With respect to a coordinate system with orientation $(\theta_1, \theta_2, \theta_3)$, the strain components can be obtained from the standard transformation formula

$$\begin{pmatrix} e_1' \\ e_2' \\ e_3' \end{pmatrix} = \begin{pmatrix} c_2 c_3 & -s_1 s_2 c_3 + c_1 s_3 & c_1 s_2 c_3 + s_1 s_3 \\ -c_2 s_3 & s_1 s_2 s_3 + c_1 c_3 & -c_1 s_2 s_3 + s_1 c_3 \\ -s_2 & -s_1 c_2 & c_1 c_2 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}, \tag{3}$$

with $c_i = \cos\theta_i$, $s_i = \sin\theta_i$, $i = 1,2,3$. For each $(\theta_1, \theta_2, \theta_3)$-oriented frame, deviatoric strain amplitudes can be computed from

$$a_i' = \frac{1}{2}\left(\max_t e_i'(t) - \min_t e_i'(t)\right), \quad i = 1,2,3. \tag{4}$$

In this work, the *amplitude of the deviatoric strain history* is defined from the combination of deviatoric strain amplitudes in several directions as

$$\gamma_{dev} = \sqrt{2} \max_{(\theta_1,\theta_2,\theta_3)} \sqrt{a_1'^2 + a_2'^2 + a_3'^2}. \tag{5}$$

In addition to the amplitude of deviatoric strains defined in Eq. 5, the fatigue parameter proposed in the present study considers the effect of tractive normal stresses, which separate crack surfaces and reduce frictional forces, increasing stresses in crack tips and hence contributing to the reduction of fatigue life [4]. This mechanism has been accounted for by considering the maximum normal stress acting upon on a material plane [1-3] or the maximum hydrostatic stress [12-15] observed in a material point. The latter option is considered here, leading to the fatigue model expressed, for a given life N_f, as

$$\gamma_{dev} + \frac{\kappa}{G}(\sigma_{Ha} + \alpha\sigma_{Hm}) = f(N_f), \tag{6}$$

where σ_{Ha} is the amplitude of the hydrostatic stress, σ_{Hm} is its mean value, κ is the sensitivity of the material to hydrostatic stresses and G is the shear modulus. A value $\alpha = 3$ produced good life estimates for all the materials considered in the present study. Combination of Eq. 6 with torsional strain-life relation results in

$$\gamma_{dev} + \frac{\kappa}{G}(\sigma_{Ha} + \alpha\sigma_{Hm}) = \frac{\tau_f'}{G}(2N_f)^{b_0} + \gamma_f'(2N_f)^{c_0}, \tag{7}$$

where τ_f', b_0, γ_f' and c_0 are the cyclic properties of the material obtained from torsion tests. Fitting of the model to fully reversed axial test data leads to the following expression of κ as a function of life N_f,

$$\kappa = \mathrm{G}\frac{\left[\frac{\tau_f'}{G}(2N_f)^{b_0} + \gamma_f'(2N_f)^{c_0}\right] - \sqrt{3}\left[\frac{2}{3}(1+\nu)\frac{\sigma_f'}{E}(2N_f)^{b} + \varepsilon_f'(2N_f)^{c}\right]}{\frac{1}{3}\sigma_f'(2N_f)^{b}}, \tag{8}$$

where σ_f', b, ε_f' and c are the axial cyclic properties.

ASSESSMENT

Assessment of the proposed model was performed by producing life estimates for a total amount of 166 strain controlled axial-torsional loading programs applied on two steels –

S460N [16] and stainless Type 304 [17] | and two aluminum alloys – 7075-T651 [18] and 6061-T6 [17]. Table 1 lists the monotonic mechanical properties of these materials.

Table 1: Monotonic mechanical properties

Material	S460N	7075-T651	AISI 304	6061-T6
Ref.	[16]	[18]	[17]	[17]
E (GPa)	208.5	71.7	200.0	77.0
G (GPa)	80.2	27.5	82.0	28.9
ν	0.30	0.306	0.22	0.33
σ_Y (MPa)	500	501	200	253

Figure 2 illustrates the strain loading programs for S460N steel and 7075-T651 Al, while Fig. 3 sketches the ones for 6061-T6 Al and Type 304. Stress paths were approximated by considering their shapes to be the same as those from the strain paths, with stress amplitudes as reported in the references. For loading programs with mean strains, stress relaxation to zero was considered in the corresponding stress components. Table 2 lists the axial and torsional cyclic properties of the materials. Data for fully reversed torsion tests were not available in Reference [17] for neither 304 stainless steel nor 60601-T6 Al alloy and hence, in these cases, torsional cyclic properties were approximated by

$$\tau_f' = \frac{\sigma_f'}{\sqrt{3}}, \quad b_0 = b, \quad \gamma_f' = \sqrt{3}\,\varepsilon_f', \quad c_0 = c. \tag{9}$$

Table 2: Cyclic properties

Material	σ_f' (MPa)	b	ε_f'	c	τ_f' (MPa)	b_0	γ_f'	c_0
S460N	1005	-0.0975	0.142	-0.483	476	-0.0749	0.316	-0.469
7075-T651	1127	-0.121	0.984	-0.896	606	-0.0960	2.42	-1.03
AISI 304	800	-0.0960	0.0426	-0.307	462	-0.0960	0.0738	-0.307
6061-T6	565	-0.0969	0.376	-0.703	326	-0.0969	0.651	-0.703

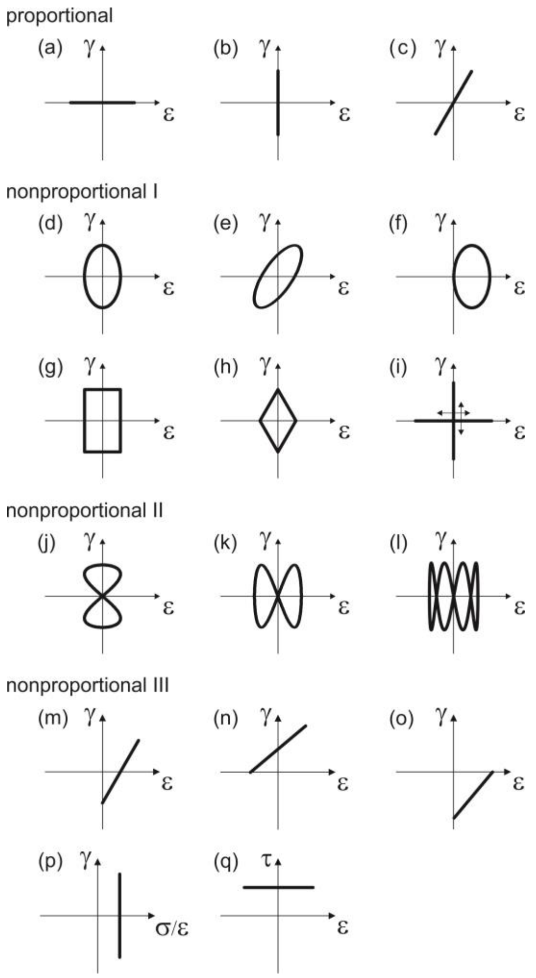

Figure 2: Strain paths for SAE 1045HR, S460N steels and 7075-T561 Al.

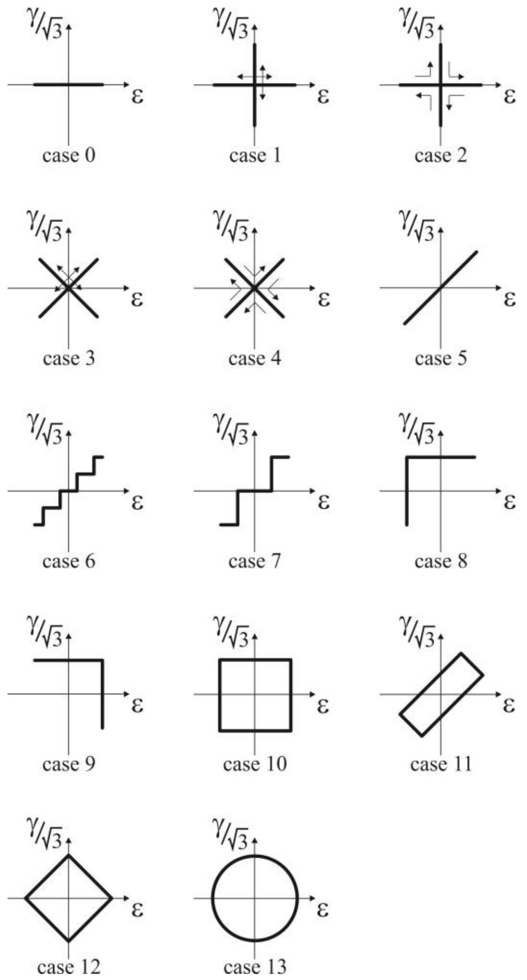

Figure 3: Strain paths for 6061-T6 Al and Type 304 stainless steel.

Figures 4-7 compare estimated lives with observed ones. Most results (84%) are within a factor two bandwidth (bounded by dotted lines) while 14% fall within a factor of three. A small number of results (only 2%) are within a factor of five bandwidth but in the conservative side.

DISCUSSION

The proposed model considers a combination of deviatoric strain amplitudes, rather than shear strain amplitudes defined on critical planes. Consequently, in the present study both strain and stress terms accounts for the effect of non-proportionality upon fatigue life. This can be illustrated by considering, for instance, the proportional and rectangular

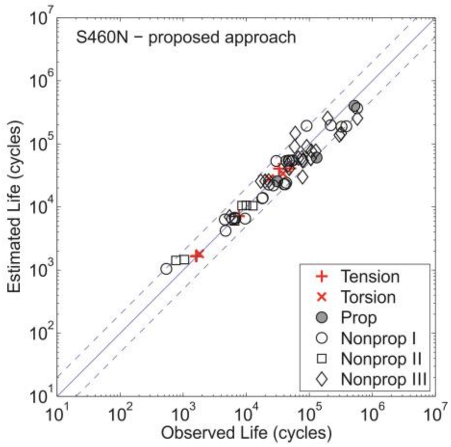

Figure 4: Estimated versus observed lives for S460N steel.

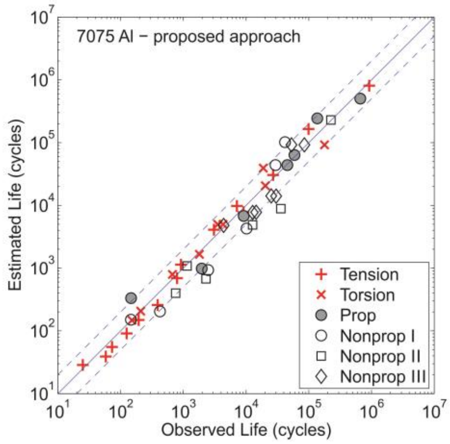

Figure 5: Estimated versus observed lives for 7075 Al alloy.

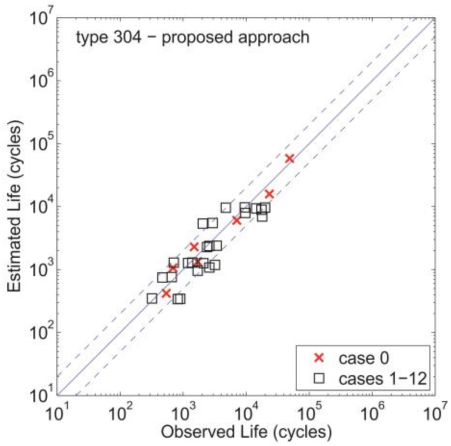

Figure 6: Estimated versus observed lives for type 304 stainless steel

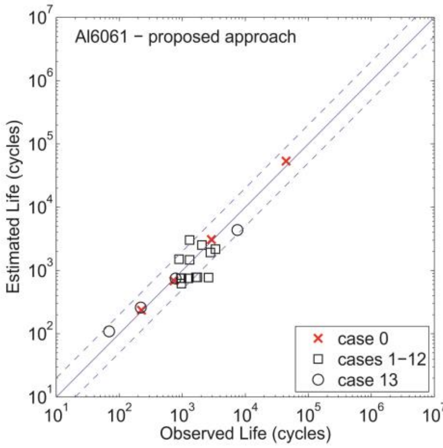

Figure 7: Estimated versus observed lives for 6061 Al alloy

strain paths represented by cases 5 and 10 from tests performed by Itoh on Type 304 stainless steel. The same strain ranges $\Delta\varepsilon = 0.8\%$ and $\Delta\gamma = 1.392\%$ were applied to both proportional and rectangular paths but the resulting fatigue lives were 3200 cycles for the former case and 320 cycles for the non-proportional one. In the deviatoric strain approach, the non-proportional nature of the rectangular loading path is characterized by an increment of 37% in the measure of strain amplitude (from 0.948% to 1.302%). Since $\kappa = 0$ due to the approximation (9), the increase in the measure of combined deviatoric strain amplitude was the only responsible for the change in life estimate from 1186 to 339 cycles. In other cases where the sensitivity parameter is not equal to zero, the hydrostatic stress term also plays a role in accounting for the effect of non-proportionality upon fatigue life.

Even though assessment relied on an approximate description of stress paths, the resulting life estimates correlated well with experimental observations. In cases where stress amplitudes are not available, an elasto-plastic model capable to accurately describe non-proportional cyclic hardening - as for instance the ones from References [5,6,19] should be employed.

The additive interaction of the combined deviatoric amplitude γ_{dev} and the hydrostatic stress terms provided satisfactory life estimates for the materials and loading conditions addressed in this work, but other interactions between these parameters could be necessary for other materials.

CONCLUSIONS

This paper introduced a multiaxial model for fatigue life estimation based on a combination of strain amplitudes in mutually orthogonal directions in the deviatoric space, together with hydrostatic stresses observed along a loading history. Both strain and stress terms account for the effect of non-proportional loading upon the fatigue life. The proposed model represents an alternative to the critical plane approach, with distinct measures of strains and stresses leading to equally satisfactory life estimates, in most cases within a factor of two bandwidth for the four materials (two steels and two aluminum alloys) and 166 loading (proportional and non-proportional) histories considered in this work. Assessment considering loading histories in the presence of mean stresses also correlated well with the observed lives. The deviatoric strain based approach is a suitable alternative to life estimation in the context of constant amplitude multiaxial loading. Situations involving variable amplitude loading as well as notches will be addressed in future works.

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