

# Influence of static loadings on load- and deformation-controlled torsional fatigue in steels

Koichiro Tomita<sup>1</sup> and Isao Ohkawa<sup>2</sup>

<sup>1</sup> Graduate Student, Division of Engineering, Hosei University Graduate School, 3-7-2 Kajino-cho, Koganei, Tokyo, 184-8584, JAPAN, Fax: 042-387-6121, e-mail: [koichiro.tomita.4v@stu.hosei.ac.jp](mailto:koichiro.tomita.4v@stu.hosei.ac.jp)

<sup>2</sup> Professor, Department of Mechanical Engineering, Faculty of Science and Engineering, Hosei University, 3-7-2 Kajino-cho, Koganei, Tokyo, 184-8584, JAPAN Fax: 042-387-6121, e-mail: [ohkawa@hosei.ac.jp](mailto:ohkawa@hosei.ac.jp)

**ABSTRACT.** *Fatigue tests were performed on smooth tubular specimens of steels with different deformation properties to investigate the effect of static loadings on torsional fatigue over the life from low to high cycle regions. For deformation-controlled test, the effect of static torque on the lifetime in low cycle regime was negligible due to mean stress relaxation after short number of cycles. For load-controlled test under fully-reversed torsion with static torque or tension in wide range of life, reduction of the fatigue life was revealed in both steels irrespective of the condition of static load. Variation of cyclic stress-strain response and accumulation of ratcheting strain were studied under various condition of static torque or tension. Based on the experimental results, a shear strain-based model including mean stress effect on critical plane was modified taking the influences of variation in cyclic strain and accumulation of ratcheting strain into consideration. The model can explain the influence of static load on torsional fatigue life of the present materials.*

## INTRODUCTION

Although the influence of static loadings on torsional fatigue strength has been investigated by many reserchers, the studies over a wide range of lifetime is rare. It is well known that initiation and growth of fatigue crack on critical planes are facilitated by superposition of static loadings. The addition of the static loadings may also affect cyclic stress-strain response and ratcheting deformation, especially in low cycle fatigue.

It is anticipated that the influences of the static loadings on the lifetime above mentioned can be different depending on deformation properties of the material and range of the fatigue life concerned besides the type of static load and control mode of the testing.

The objective of the present study is to investigate the influence of static loadings on torsional fatigue strength in the life region from low to high cycle fatigue. Faitigue tests were conducted on two kinds of steels with different deformation properties under load- and deformation-controlled fully-reversed torsion with static torque or tension.

Influence of the static loads on the fatigue life was estimated by a shear strain-based model modified incorporating the effects of cyclic and ratcheting strain on the lifetime.

## **MATERIALS AND TEST PROCEDURE**

The materials were structural medium carbon steel JIS S45C and austenitic stainless steel JIS SUS316L. Thin-walled tubular smooth specimens with outer diameter of 16 mm and inner diameter of 13.5mm over gage length of 15 mm were machined. After machining, S45C specimens were annealed at 850°C for 1 hour in vacuum. Mechanical properties of S45C are torsional yield stress  $\tau_y=214\text{MPa}$  and torsional strength  $\tau_B=442\text{MPa}$ . Those of SUS316L are 0.2 % proof stress  $\tau_{0.2}=281\text{MPa}$  and  $\tau_B=470\text{MPa}$ .

Deformation-controlled tests were conducted only in low cycle fatigue region. Static twist angle was superimposed on cyclic torsion keeping the ratio of minimum to maximum twist angles  $R_\theta=\theta_{\min}/\theta_{\max}$  at  $-0.5$  or  $0$ . On the other hand, load-controlled tests were performed under fully-reversed cyclic torsion with and without static torque in a wide life region from low to high cycle fatigue. In a high cycle fatigue region, the cyclic shear stress  $\tau_a$  was imposed holding the mean torsional stress  $\tau_m$  or mean tensile stress  $\sigma_m$  in constant values. The values of mean stress are  $\tau_m=55, 110\text{MPa}$  and  $\sigma_m=96, 176\text{MPa}$  for S45C and  $\tau_m=104\text{MPa}$  and  $\sigma_m=180\text{MPa}$  for SUS316L. In low cycle fatigue region of both materials, static torque or tensile load were applied keeping the ratio of minimum to maximum equivalent shear stress  $R_\tau=\tau_{\text{eqmin}}/\tau_{\text{eqmax}}$  at  $-0.9$  or  $-0.8$ .

The twist angle in gage length was estimated from measurement of the total twist angle between gripped ends using the relation between them examined in advance. The surface shear strain was calculated from the estimated twist angle in gage length.

Accumulation of ratcheting strain was observed through a measurement of relative displacement between micro-Vickers hardness impressions marked before the testing.

## **RESULTS AND DISCUSSION**

### ***Influence of static load on torsional fatigue life***

Fig.1 shows Coffin-Manson relation obtained from the deformation-controlled tests in low cycle fatigue region. Application of static twist angle showed no apparent influence on the torsional fatigue life in both materials. This is due to relaxation of the induced mean stress. Thus, the stress state rapidly approaches those in pure torsion before the cycle fraction  $N/N_f$  reaches about 0.1.

Fig.2 shows the results of load-controlled tests. The fatigue life was evaluated with plastic shear strain amplitude at half the life. The regression lines indicate the lifetime in pure torsion. Although the plots scattered due to dispersion of the measured strains, the fatigue life in both materials decreased with superimposed static loading regardless of the type of loading.

In the load-controlled test, the addition of static torsion or tension may exert various influences on torsional fatigue. From viewpoint of the deformation, the static load causes the variation of plastic shear strain and the reduction of ductility due to accumulation of ratcheting strain. With regard to the crack development, mean stress induced by application of static load promotes the crack growth on critical planes. In

particular, the influence on the cyclic creep deformation becomes prominent in low cycle fatigue regions [1]. In the present study, the effect of static load on plastic shear strain and ratcheting strain was examined on both materials.

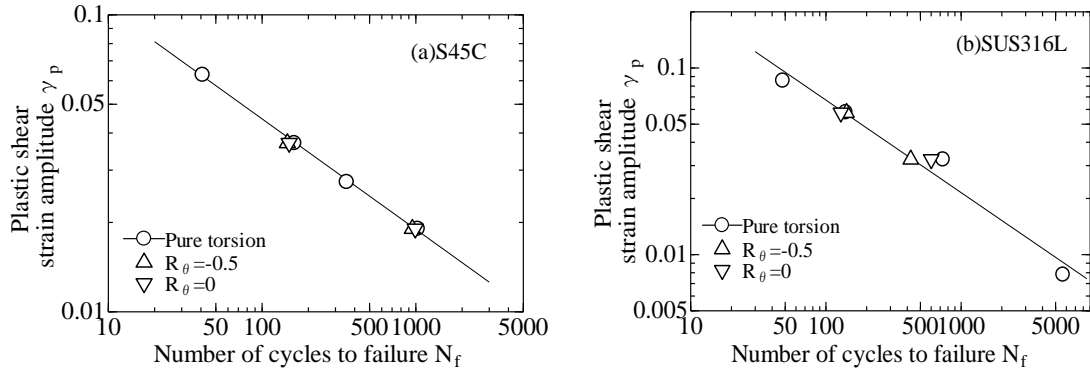


Figure1. Coffin-Manson plot for deformation controlled test: (a) S45C, (b) SUS316L.

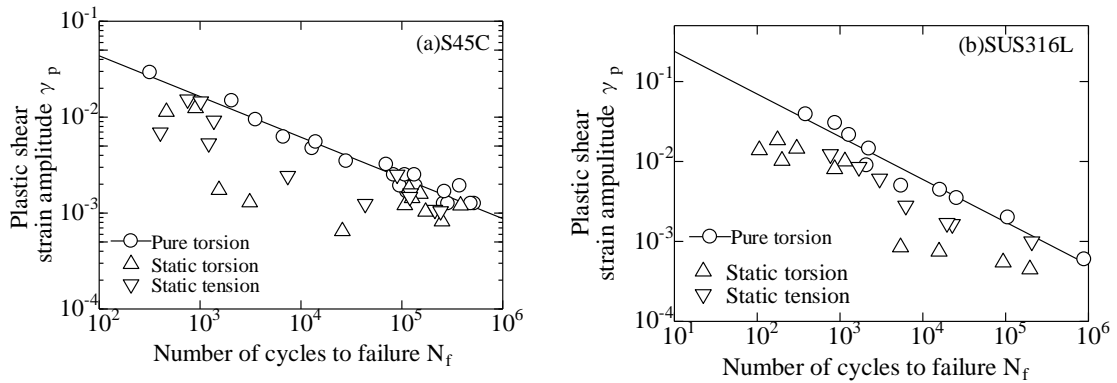


Figure2. Coffin-Manson plot for load controlled test: (a) S45C, (b) SUS316L.

### ***Influence of static load on cyclic stress-strain response***

Result of the strain measurement in pure torsion indicated that the cyclic hardening/softening properties are different between carbon steel S45C and austenitic stainless steel SUS316L [2]. Cyclic hardening was observed in S45C. On the other hand, SUS316L showed cyclic hardening in higher stress but cyclic softening in lower stress levels. For load-controlled tests in fully reversed torsion, stable cyclic shear stress-strain response can be expressed by the power-law relationship:

$$\tau_a = k_0 \gamma_{p0}^{n_0} \quad (1)$$

where  $\tau_a$  and  $\gamma_{p0}$  are cyclic shear stress amplitude and plastic shear strain amplitude at half the life. The values of cyclic strength coefficient  $k_0$  and cyclic strain hardening exponent  $n_0$  are 619.8MPa and 0.194 for S45C. These values for SUS316L are 511.8MPa and 0.132 respectively.

Several procedures have been proposed to evaluate the influence of mean tensile stress on cyclic stress-strain response under push-pull loading [3, 4]. In the present study, effect of static torsion  $\tau_m$  or tension  $\sigma_m$  on cyclic shear stress-strain response was estimated in the following manner. For constant values of the maximum equivalent shear stress  $\tau_{eqmax} = \tau_a + \tau_{eqm}$ , the relation between the logarithm of plastic shear strain amplitude  $\log \gamma_p$  and mean equivalent shear stress  $\tau_{eqm} = \sqrt{\tau_m^2 + \sigma_m^2} / \sqrt{3}$  assumes to be a family of straight lines having constant slope [4]. The logarithm of the intercept of these lines at zero mean stress  $\log \gamma_{p0}$  is obtained from the cyclic stress-strain relation in pure torsion expressed by Eq. (1):

$$\log \gamma_p = \log \gamma_{p0} + \alpha \tau_{eqm} \quad (2)$$

Here,  $\alpha$  is the slope of the straight lines. For constant values of  $\tau_{eqmax}$ , Eq. (2) can be expressed in the following form:

$$\log \left[ \frac{\gamma_p}{\left( \frac{\tau_{eqmax}}{k_0} \right)^{n_0}} \right] = \alpha \tau_{eqm} \quad (3)$$

Even if the actual strain measurement is not done under constant values of the maximum shear stress, the value of  $\alpha$  can be determined from slope of the regression line drawn to the plots of both sides of Eq. (3) for measured strains under arbitrary conditions of mean equivalent shear stress.

Fig.3 shows the results for addition of static torsion and tension all together. Although the plots scatter considerably, the value of  $\alpha$  obtained from the slope of the regression line was  $-0.01089$  for S45C and  $-0.01503$  for SUS316L.

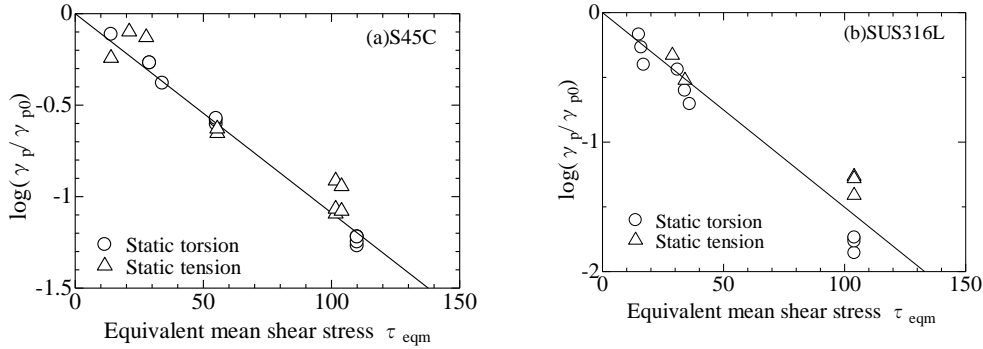


Figure3.  $\log(\gamma_p / \gamma_{p0})$  versus  $\tau_{eqm}$  for constant values of  $\tau_{eqmax}$ : (a) S45C, (b) SUS316L.

### ***Influence of static load on accumulation of ratcheting strain***

Fig.4 shows accumulation of the ratcheting strain in S45C with number of cycles. Addition of static torque and tensile load yields the shear and tensile ratcheting strains,  $\gamma_p$  and  $\epsilon_c$ , respectively. The accumulative process of the strain can be approximated by a straight line on double logarithmic diagram:

$$\gamma_c = \gamma_f \left( \frac{N}{N_f} \right)^{\beta_\tau} = A_\tau N^{\beta_\tau} \quad , \quad \varepsilon_c = \varepsilon_f \left( \frac{N}{N_f} \right)^{\beta_\sigma} = A_\sigma N^{\beta_\sigma} \quad (4)$$

In general, parameters  $A_\tau$ ,  $\beta_\tau$ ,  $A_\sigma$  and  $\beta_\sigma$  in the equations will vary depending on both cyclic shear stress amplitude  $\tau_a$  and mean equivalent shear stress  $\tau_{eqm}$  or the maximum equivalent shear stress  $\tau_{eqmax}$ . Here, noting that the plastic shear strain for addition of static load,  $\gamma_p$  also depends on the same stress variables (Eq. (3)), the parameters in Eq. (4) can be expressed in terms of the plastic shear strain  $\gamma_p$ . Thus, the relation between the parameter  $A_\tau$ ,  $A_\sigma$  and the logarithm of plastic shear strain amplitude  $\log \gamma_p$  for constant values of the maximum equivalent shear stress  $\tau_{eqmax}$  assumes to be expressed by a family of straight lines with constant slope. For zero mean equivalent stress  $\tau_{eqm}$ , namely  $\tau_{eqmax} = \tau_a$ , the values of  $A_\tau$  and  $A_\sigma$  suppose to be zero in correspondence with no ratcheting strain. Therefore, using Eq. (2), the following expressions are obtained for constant values of  $\tau_{eqmax}$ :

$$A_\tau = B_\tau \log \left( \frac{\gamma_p}{\gamma_{p0}} \right) = B_\tau \alpha \tau_{eqm} \quad , \quad A_\sigma = B_\sigma \log \left( \frac{\gamma_p}{\gamma_{p0}} \right) = B_\sigma \alpha \tau_{eqm} \quad (5)$$

Fig. 5 shows the relation between  $A_\tau$ ,  $A_\sigma$  and  $\log (\gamma_p/\gamma_{p0})$  in S45C obtained from the strain measurement. The material constants  $B_\tau$  and  $B_\sigma$  in Eq. (5) are given by the inclination of the regression line passing through the origin. The values of  $B_\tau$  and  $B_\sigma$  were  $-2.842 \times 10^{-2}$  and  $-1.147 \times 10^{-2}$  respectively.

Fig. 6 illustrates the correlation of the parameters  $\beta_\tau$  and  $\beta_\sigma$  with the plastic shear strain for addition of static load  $\gamma_p$ . The relation can be approximated by a straight line on double logarithmic diagram. Using Eq. (2),  $\beta_\tau$  and  $\beta_\sigma$  are thus given by the following equations:

$$\beta_\tau = C_\tau + D_\tau \log \gamma_p = C_\tau + D_\tau \alpha \tau_{eqm} + D_\tau \log \left( \frac{\tau_{eqmax}}{k_0} \right)^{\frac{1}{n_0}} \quad ,$$

$$\beta_\sigma = C_\sigma + D_\sigma \log \gamma_p = C_\sigma + D_\sigma \alpha \tau_{eqm} + D_\sigma \log \left( \frac{\tau_{eqmax}}{k_0} \right)^{\frac{1}{n_0}} \quad (6)$$

For S45C shown in Fig. 6, the material constants in the equations were  $C_\tau = 1.804$ ,  $D_\tau = 0.5433$ ,  $C_\sigma = 1.059$  and  $D_\sigma = 0.3116$ . The material constants in Eqs. (5) and (6) were determined for SUS316L in the same manner. These values were  $B_\tau = -5.287 \times 10^{-3}$ ,  $B_\sigma = -5.740 \times 10^{-7}$ ,  $C_\tau = 1.612$ ,  $D_\tau = 0.3794$ ,  $C_\sigma = 3.294$  and  $D_\sigma = 0.7248$ .

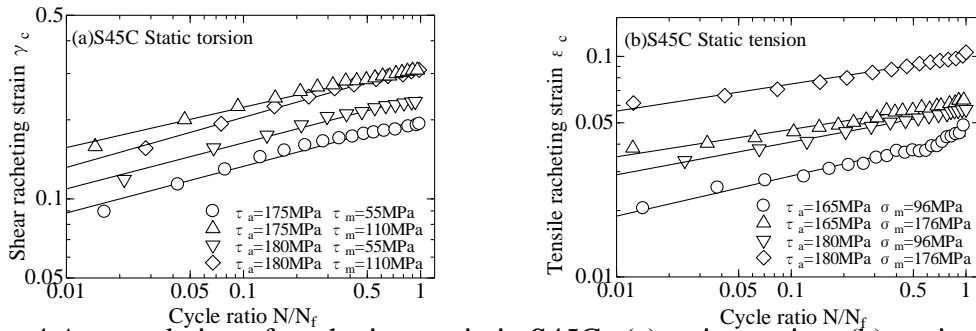


Figure.4 Accumulation of ratcheting strain in S45C: (a) static torsion, (b) static tension.

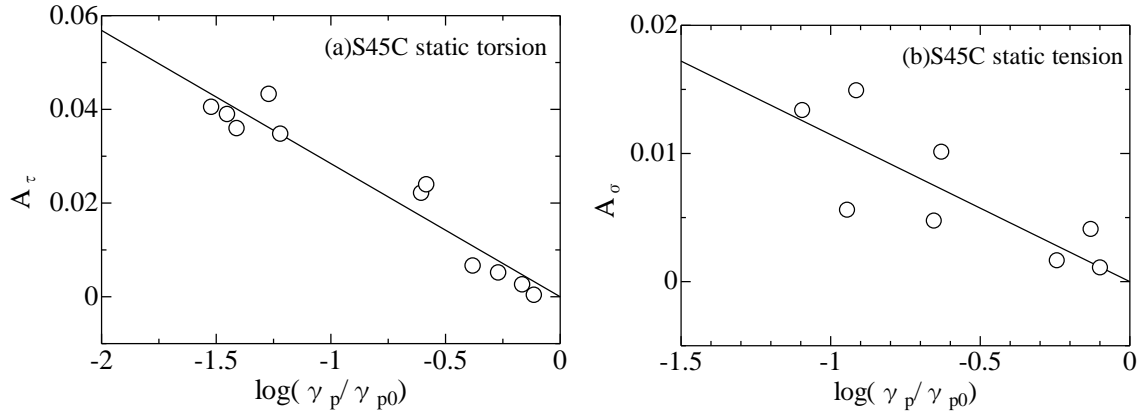


Figure5.  $A_\tau$  or  $A_\sigma$  versus  $\log(\gamma_p/\gamma_{p0})$  for constant values of  $\tau_{eqmax}$  (S45C).

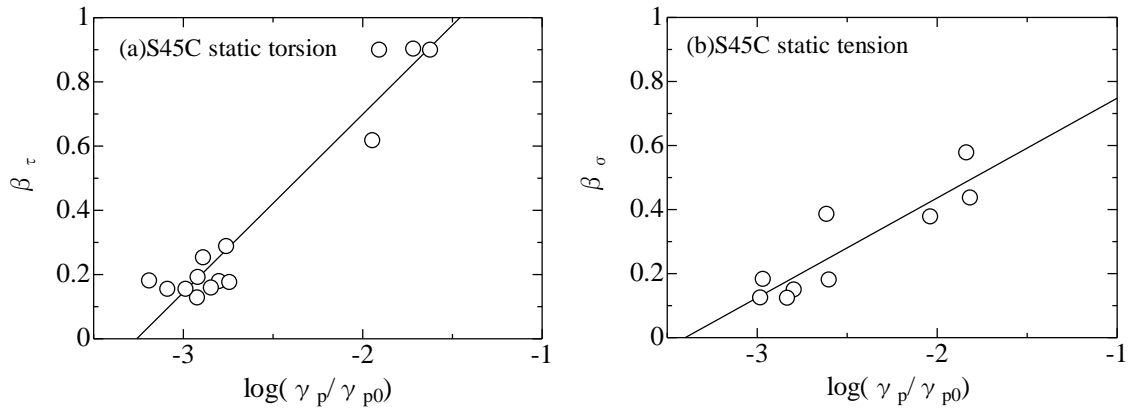


Figure6.  $\beta_\tau$  or  $\beta_\sigma$  versus  $\log(\gamma_p/\gamma_{p0})$  for constant values of  $\tau_{eqmax}$  (S45C).

### ***Influence of static load on torsional fatigue life***

Various criteria have been proposed to estimate the influence of static torsion or tension on the fatigue strength under cyclic torsion [5-7]. Here, a shear strain-based model including mean stress effect on the crack growth planes was employed as the framework for life estimation. Then, the influences of variation in cyclic strain and accumulation of ratcheting strain were incorporated into the model.

The relation between plastic shear strain amplitude and fatigue life in pure torsion is given by:

$$\gamma_{p0} N_f^a = H \quad (7)$$

where  $a$  and  $H$  are material constants. The values of  $a$  and  $H$  are  $-0.4541$ ,  $0.4021$  for S45C and  $-0.5701$ ,  $1.138$  for SUS316L respectively.

For application of static loading, Eq. (7) can be modified as follows:

$$\left(\gamma_p + k \frac{\sigma_m}{E}\right) N_f^a = H - f N_f^b \quad (8)$$

Here, effect of mean shear stress on the critical plane was neglected. E is Young's modulus. Constant k denotes effect of mean tensile stress acting on the crack growth plane and can be determined by fitting the pure torsion data to fully-reversed axial loading data. f and b are the parameters concerning influence of ratcheting deformation on the fatigue life. They relate to the maximum and mean equivalent shear stresses  $\tau_{eqmax}$  and  $\tau_{eqm}$  and are also regarded as the function of plastic shear strain  $\gamma_p$  according to Eqs. (5) and (6).

Although considerable amount of ratcheting strain accumulates at failure even in lower stress levels due to large number of stress cycles, influence of the ratcheting deformation seems to be more prominent in higher stresses and shorter lifetime region.

Thus the reduction of the fatigue strength assumes to depend on an accumulation rate rather than amount of the ratcheting strain. The accumulation rate of the ratcheting strain varies with number of cycles but quantities  $\gamma_f/N_f$  and  $\varepsilon_f/N_f$  were conveniently employed as the representative of accumulation rate. If the exponent b is equated with a in Eq. (8), the relation between f and these quantities can be then approximated by straight lines in on double logarithmic diagram. Therefore, the function f has the forms

$$f = J_\tau \left(\frac{\gamma_f}{N_f}\right)^{m_\tau} \text{ for static torsion and } f = J_\sigma \left(\frac{\varepsilon_f}{N_f}\right)^{m_\sigma} \text{ for static tension} \quad (9)$$

The values of the constants are  $J_\tau = 0.63$ ,  $m_\tau = 1.94$ ,  $J_\sigma = 1.83$  and  $m_\sigma = 0.57$  for S45C and  $J_\tau = 0.993$ ,  $m_\tau = 0.54$ ,  $J_\sigma = 18.92$  and  $m_\sigma = 0.757$  for SUS316L respectively. Furthermore, the following relations for  $\gamma_f/N_f$  and  $\varepsilon_f/N_f$  were found from regression analysis including manipulation of Eq. (5):

$$\frac{\gamma_f}{N_f} = Q_\tau B_\tau \alpha \tau_{eqm} \gamma_p^{r_\tau} \text{ and } \frac{\varepsilon_f}{N_f} = Q_\sigma B_\sigma \alpha \tau_{eqm} \gamma_p^{r_\sigma} \quad (10)$$

From Eq. (3), the plastic shear strain  $\gamma_p$  in this equation can be expressed with the stresses  $\tau_{eqmax}$  and  $\tau_{eqm}$ . It is obvious that the average accumulation rate of ratcheting strain  $\gamma_f/N_f$  and  $\varepsilon_f/N_f$  become zero for zero mean equivalent shear stresses. The values of the constants are  $Q_\tau = 1.20 \times 10^6$ ,  $r_\tau = 3.59$ ,  $Q_\sigma = 4.33 \times 10^3$  and  $r_\sigma = 3.00$  for S45C and  $Q_\tau = 2.95 \times 10^3$ ,  $r_\tau = 1.95$ ,  $Q_\sigma = 4.42 \times 10^5$  and  $r_\sigma = 1.64$  for SUS316L respectively.

The fatigue life for addition of static torsion and tension is evaluated using Eqs. (8), (9) and (10). The results are shown in Fig. 7. The plots for various static loading conditions are in good agreement with the regression line for pure torsion over the whole range of lifetime concerned. Therefore, it was shown that the modified shear strain-based model, Eq. (8) could explain adequately the influence of static load on torsional fatigue life of the present materials.

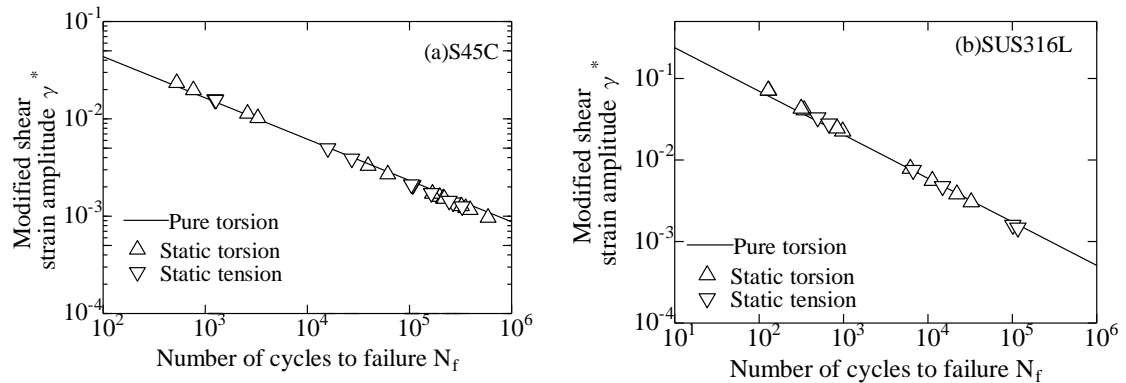


Figure 7. Modified Coffin-Manson plot for load controlled test: (a) S45C, (b) SUS316L.

## CONCLUSION

Influence of static loads on torsional fatigue was investigated in a wide range of lifetime. Load- and deformation-controlled fatigue tests were conducted on two kind of steel tubular specimens under fully-reversed torsion with superimposed static torque or tension. For deformation-controlled test with static twist angle, the fatigue life of the materials in low cycle region was almost unchanged due to relaxation of induced mean stress in early stage of the life. For load controlled tests of the materials, reduction of the fatigue life resulted from addition of static torque or tension in the life ranging from low to high cycle fatigue.

Shear strain-based model including mean stress effect on the critical plane was employed as a framework for estimation of the life in load-controlled tests. Cyclic and ratcheting strains under application of static load were estimated on the basis of measurement of the strains in various conditions of static torque or tension. Then, the model was modified incorporating the effects of variation in cyclic strain and accumulation of ratcheting strain on the fatigue life. It was shown that the modified model gave an explanation of the influence of static load on torsional fatigue life of the present materials.

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