# A Rapid Estimation Method of the Fatigue Cyclic Behavior in a Confined Plasticity zone : application to a double-notched specimen

**S. Plessis[1,2], C. Doudard[2], D. Thevenet[2], S. Calloch[2], M. Cazuguel[1]**

[1] DCNS–DNA\ING\DCSE\CSB, rue Choiseul 56311 Lorient Cedex
[2] LBMS Ensta-Bretagne, 2 rue François Verny 29200 Brest

sarah.plessis@ensta-bretagne.fr

***ABSTRACT.*** *Welded assemblies are commonly used in the shipbuilding industry. The welding process provides many possibilities and advantages for the design of complex structures from simple geometric elements. (e.g., plates, tubes…). They are usually key elements which contribute to the integrity of the structure and are subjected to complex and intense cyclic loading. Besides, welded assemblies contain geometrical discontinuities, which generate local stress and strain concentrations, and more particularly along the welding seam. Due to confined plasticity, nucleation of fatigue cracks can occur in these highly stressed regions and their growth could lead to fracture. Shipbuilding engineering and design department therefore need to have efficient methods and resources to predict the fatigue behavior of welded assemblies. Fatigue analysis requires calculation of elastic-plastic stresses and strains at the critical points of the structure (points of initiation for the fatigue crack). Elastic-plastic finite element analyses could be performed but since the computational expense is prohibitive, other quick estimation methods were developed. The local elastic-plastic behavior can be estimated using the purely elastic solution, (for instance Neuber's rule). However, the lack accuracy for multiaxial loadings in these methods led to a new approach based on homogenization models which is used in this study. The stress and strain histories thus determined will be compared to finite element computations and to experimental results on double notched specimen whose material is a shipbuilding steel. Its behavior law is known from previous testing.*

## INTRODUCTION

Welded assemblies are commonly used in shipbuilding industry. They are subject to confined plasticity and thus to fatigue cracks. Knowing stress and strain histories is necessary to determine this fatigue life, especially at the critical points of the structure. To avoid long non linear finite element computations, rapid estimation methods of the elasto-plastic strain and stress (such as Neuber) are used. Nevertheless, these methods lack accuracy. A new approach based on homogeneization techniques was developped by T.Herbland in 2009. In this work, the results predicted by this new approach will be compared to a complete finite element analysis. First, this simplified method will be introduced, then compared to others rapid estimation methods (Neuber, Molski-Glinka) and to finite element analysis on the example of a double-notched specimen (simple material behavior law). Finally, the simplified method will be tested on a double-notched specimen which behavior law corresponds to a shipbuilding steel.

## PRESENTATION OF THE METHOD ET COMPARISON WITH NEUBER'S AND MOLSKI-GLINKA'S METHODS

### *Presentation of the method*

Rapid estimation methods of the elastolastic stress tensor $\underline{\sigma}$ and strain tensor $\underline{\varepsilon}$ in a confined plasticity zone consist in determining these fields from elastic stress $\underline{\sigma}_M$ and elastic strain $\underline{\varepsilon}_M$. Herbland (2009) thought of assimilating this problem to an homogeneization problem (Kröner, 2005) : the inclusion becomes the confined plasticity zone and the infinite elastic matrix around this inclusion becomes the elastic zone around the confined plasticiy zone (Fig. 1 where $K_T$ is the stress concentration factor). The elastoplastic stress tensor $\underline{\sigma}$ (at the critical point of the structure) is given by Eq. 1

$$\underline{\sigma} = \underline{\sigma}_M - \underline{\underline{L}} : \underline{\varepsilon}^p \, . \tag{1}$$

For a one-dimension problem (for example a flat double-notched specimen) Eq.1 becomes Eq.2 where L' is a parameter that needs to be identified (every other coefficients of the $\underline{\underline{L}}$ tensor are equals to zero).

$$\sigma = \sigma_M - L' \varepsilon^p \, . \tag{2}$$

L' is identified on a FEM analysis but only on a monotonic loading. Many relationships have been tested int he literature to determine ($\underline{\sigma}$,$\underline{\varepsilon}$) from ($\underline{\sigma}_M$,$\underline{\varepsilon}_M$), for example Neuber's (1961) in Eq. 3 or Molski-Glinka's (1981) in Eq. 4.

$$\sigma_N \varepsilon_N = \sigma_M \varepsilon_M \, , \tag{3}$$

$$2 \int \sigma_{MG} \varepsilon_{MG} = \sigma_M \varepsilon_M \, . \tag{4}$$

These two methods don't require the identification of a parameter.

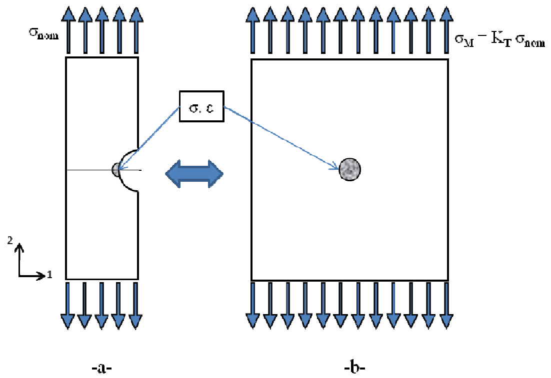


Figure 1. Assimilation between confined plasticity and Homogeneization

### *Application and comparison with other rapid estimation methods and FEM*

A monotonic loading on a flat double notched specimen is used to illustrate the simplified method and compare its results with Neuber's and Molski-Glinka's methods. The specimen's geometry and mesh are represented on Fig. 2. : 6115 nodes for 5944 quadrangle with a plane stress hypothesis.

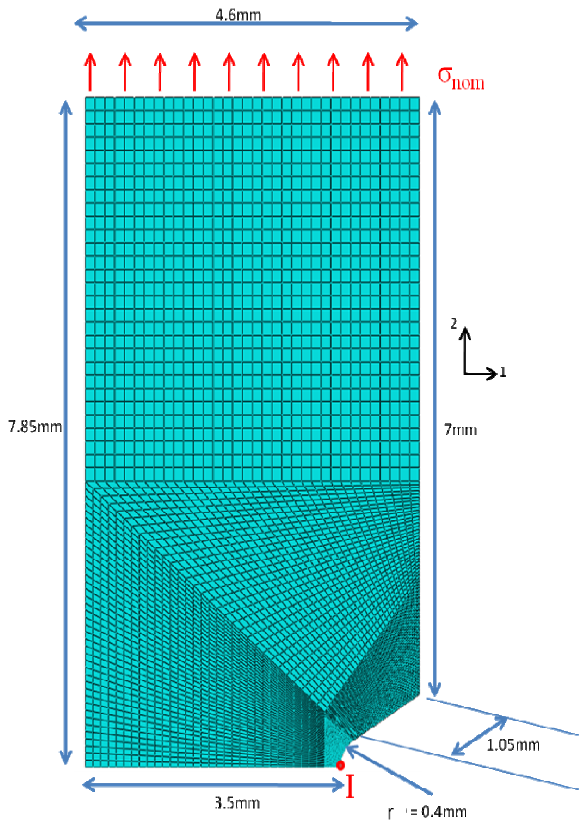


Figure 2. Geometry and Mesh of the specimen

The behavior law parameters correspond to a non linear kinematic hardening (Tab. 1). The equations used in the model are reminded in Tab.2.

Table 1. Behavior law parameters

| E | ν | $R_e$ | C | γ |
|---|---|---|---|---|
| 200 GPa | 0,3 | 250 MPa | 312 500 MPa | 1250 |

Table 2. Equations of the model

| | |
|---|---|
| Elasticity area | $f = J_2(\underline{\sigma} - \underline{X}_T) - R_e - R \le 0$ |
| Evolution law of R (isotropic hardening) | $\dot{R} = b*(Q-R)\,\dot{p}$ |
| Evolution law of $\underline{X}$ (kinematic hardening) | $\dot{\underline{X}}_i = \frac{2}{3} C_i \dot{\underline{\varepsilon}}^p - \gamma_i \underline{X}_i \dot{p} \quad \underline{X} = \sum_i \underline{X}_i$ |

A monotonic loading of 180 MPa is applied to the specimen. To compare the errors made by the classical simplified methods (Neuber's and M-G's), their responses are shown on Fig. 3 in the ($\varepsilon_p$, $\sigma_M$-σ) plan. The FEM response is also represented. A significant error can be seen : Neuber's method overstimates the stress response whereas M-G's method underestimates it. The Herbland's simplified method which requires the identification of L' gives a satisfactory result.

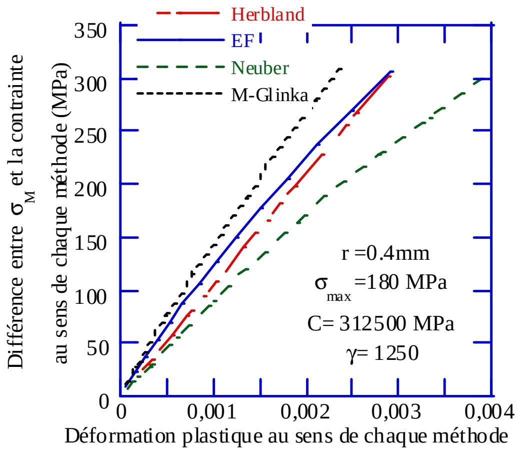


Figure 3. Comparison of the simplified methods on a monotonic loading

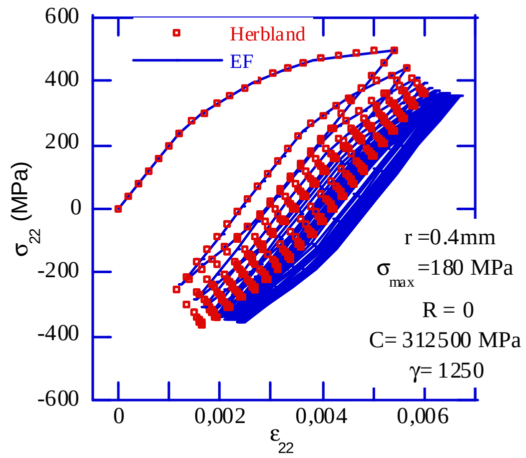


Figure 4. Comparison FEM/ Herbland's simplified method

The simplified method can be validated by comparing the predictions for a cyclic loading (R = 0) with the FEM results. On 30 cycles, the stress-strain response at the critical point I predicted by the Herbland's simplified method is close to the FEM response (Fig. 4). The mean stress relaxation is well described. Besides, it is essential to mention that the predictions of the simplified method are obtained in a few seconds whereas the non linear FEM analysis lasts fifteen minutes.

## APPLICATION AUX EPROUVETTES ENTAILLEES

### *Case study*

The simplified method was then applied to a shipbuilding steel double-notched specimen. The material parameters used in the FEM model have been identified on several cyclic testings on axisymmetric test-pieces (Fig. 5 and 6). They are described in Tab. 3.The geometry and the mesh of the specimen is represented on Fig.7.

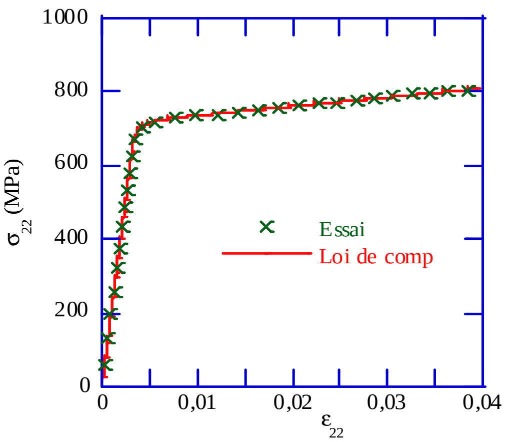


Figure 5. Comparison Testing/Identified Law (monotonic loading)

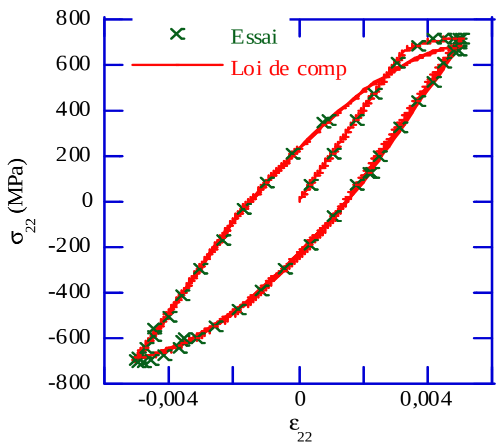


Figure 6. Comparison Testing/Identified Law (cyclic loading)

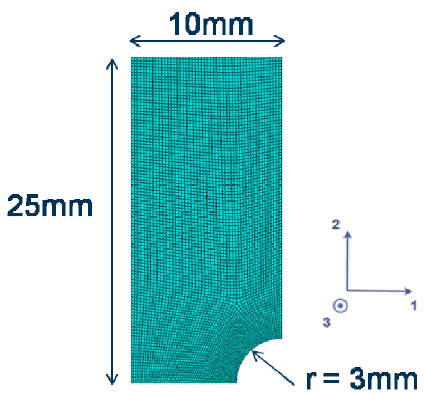


Figure 7. Geometry and mesh of the specimen

Table 3. Behavior law parameters

| E | ν | $R_e$ | $C_1$ | $\gamma_1$ | $C_2$ | $\gamma_2$ | $C_3$ | $\gamma_3$ | b | Q |
|---|---|---|---|---|---|---|---|---|---|---|
| 202 GPa | 0,3 | 540 MPa | 525 000 MPa | 3000 | 2600 MPa | 0 | 250 000 MPa | 1000 | 1000 | -250 |

***Comparison of the simplified method's and FEM cyclic behavior***

A comparison between testing and FEM has been made on a monotonic loading (Fig.8). The FEM monotonic behavior is very close to the experimental behavior. The FEM model described earlier allowed to identify the localization operator of this double-notched specimen (the monotonic behavior predicted by the simplified method is identical to the FEM response)

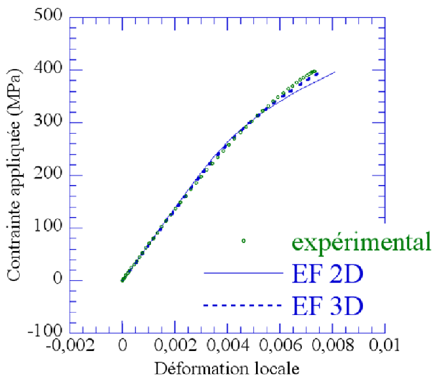


Figure 8. Comparison Experimental/FEM (monotonic loading)

Cyclic tensile tests have been realized ($R_\sigma$ = 0). A comparison between the cyclic behaviors at the notch root obtained by each method (FEM, simplified method, experimental) was made. The experimental strain was obtained thanks to a gauge at the notch root. The average strain amplitude (on the length of the gauge = 5 FEM elements) is described on Fig. 9 depending on the number of cycles : experimental and FEM. The strain amplitude at the critical point of the structure is represented on Fig. 10 : FEM (1 element) and simplified method. The different methods (FEM, simplified method, experimental) give similar results for the strain amplitude vs number of cycles. This point is essential for the fatigue life estimation in the next paragraph.

***Comparison between numerical fatigue life and experimental fatigue life***

Numerical analyses (FEM/simplified method) enable to access the stress and strain histories at the notch root and thus to the stabilized cycle. This is enough to know the numerical life fatigue of the specimen through two fatigue criteria : Morrow's criterion (Eq. 5) and Smith-Watson-Topper's (Eq. 6), where $\Delta\varepsilon_t/2$ is the total strain amplitude of the stabilized cycle, E the material Young's modulus, $N_r$ the number of cycles to failure. $\sigma'_f$, b, $\varepsilon'_f$, c are the coefficients of Manson-Coffin-Basquin (determined from the fatigue curve, $R_\varepsilon$ = -1), $\sigma_m$ is the mean stress and $\sigma_{max}$ the maximal stress of the stabilized cycle.

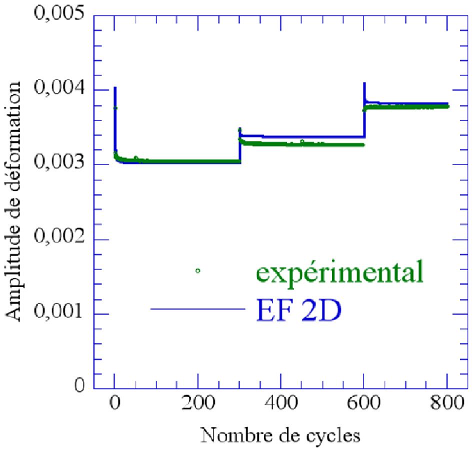


Figure 9. Comparison FEM/experimental (cyclic behavior)

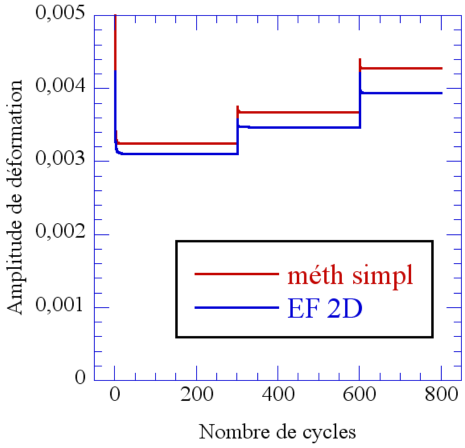


Figure 10. Comparison FEM/simplified method (cyclic behavior)

$$\frac{\Delta\varepsilon_t}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_r)^b + \varepsilon'_f (2N_r)^c \quad (5)$$

$$\frac{\Delta\varepsilon_t}{2}\sigma_{max} = \frac{\sigma'^{\,2}_f}{E}(2N_r)^{2b} + \sigma'_f \,\varepsilon'_f\,(2N_r)^{b+c}. \quad (6)$$

Fatigue tests with ACPD crack initiation detection have been realized on shipbuilding steel double-notched specimens (Fig. 11). There is a crack initiation when the increase of the tension is up to 0,03 Volts (Fig. 12). This criteria corresponds to a fissure length less than a millimeter. The experimental fatigue lives were compared to the numerical fatigue lives calculated earlier thanks to the two criteria (Fig. 13). The FEM model (and thus the simplified method) gives some satisfactory results close to experimental fatigue lives but non conservatives for the last three points.

## CONCLUSION

The simplified method (Herbland, 2009) needs only one FEM analysis to be applied : one elastoplastic monotonic analysis in order to determine the localizaion operator. Even if the simplified method needs this FEM analysis, it is less long than a complete non linear FEM analysis on dozens of cycles.
For the example of a shipbuilding steel double notched specimen, the FEM results obtained are satifactory compared to testings. The simplified method reproduces well the FEM behavior with a considerable gain of time négligeable (100 FEM cycles last 1h09min whereas the simplified method needs less than a minute).

Figure 11. ACPD Detection on a double notched specimen

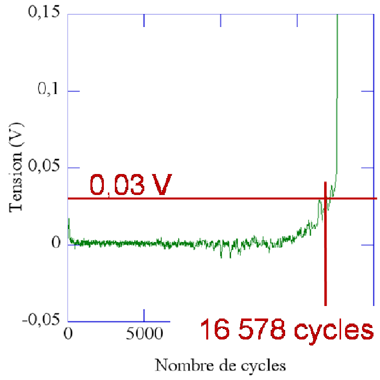


Figure 12. Tension during the fatigue test

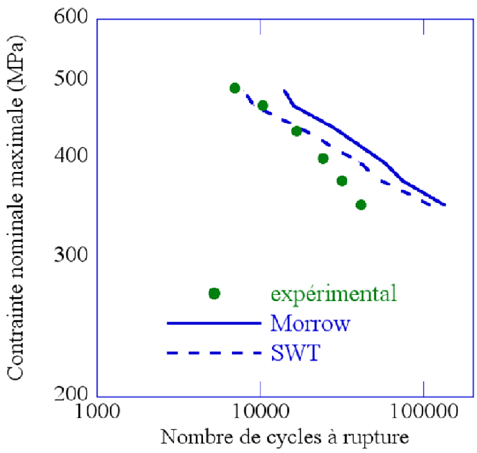


Figure 13. Comparison of fatigue lives Experimental/FEM

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