Mechanosorptive behaviour of wood: a pseudo-elastic formulation

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ABSTRACT. *An analytical and numerical pseudo-elastic formulation for mechanosorptive behaviour of wood is investigated in this work in order to predict the crack appear in wood material. The relationship between the stress and the strain is introduced by the Hooke's law during the humidufication phases, and by the restricted Hooke's law during the drying process. In numerical examplaries, the sinusoidal stress and humidity loadings, the rigidity and the compliance analytic formulations are implemented in Matlab© software. Also, the mechanosorptive behaviour under a sinusoidal loading is proposed. The result shows the ability of the model to depict the mechanosorptive behaviour of wood material under variable loading and humidity. Simultaneously, a numerical pseudo elastic calculation is applied in order to modelled the crack initiation during natural drying process of a green wood slice.*

INTRODUCTION

Crack initiation and propagation in wood are strongly depending on mechanical effects due to variations of moisture content, ie. shrinkage and swelling [1, 2, 3, 4]. Indeed, the mechanosorptive behaviour, which is resulting from the interaction between the mechanical loading and the moisture content changes in wood, is the source of disorders in wood structures. Since the early 1960s, numbers of works were conducted to improve the knowledge and understanding of this phenomenon. A complication in mechanosorptive behaviour of wood is due to locking of strains during drying process under stress loading. This phenomenon is known as the hygro-lock effect [5].

Several models are based on the assumption of a partition between elastic strain and mechanosorptive strain in the course of time. In this paper, a dual approach of the pseudo-elastic mechanosorptive behaviour is developed, based on the assumption of strain partition (stiffness formulation) or stress partition (compliance formulation). A mechanosorptive stress and a mechanosorptive strain are introduced in order to account for the hygro-lock effect. Based on thermodynamics considerations, the analytical developments are conducted with the additional assumption of a linear relationship between these two variables.

The both approaches are generalized to any loading and any moisture content variations. The validation of the model is asserted by comparing numerical simulations worked out by means of the stiffness or the compliance approach. Exemplary stress-strain diagrams with variable moisture contents and stress loading functions as posted. Also, some examples of sinusoidal evolution of moisture content versus sinusoidal stress load with different phase shifts are discussed. Finally, the proposed model is used in order to predict crack initiation in green wood slice during a drying process.

ANALYTICAL FRAMWORKS

When wood is subjected to a mechanical load under constant or increasing moisture content, its behaviour depends on both the mechanical and the hydric solicitations. In this case, it follows the Hooke's law given by (1a)

$$a\text{: } \dot{\sigma} = E(w)\,\dot{\varepsilon} + \dot{E}(w)\,\varepsilon \text{ with } \dot{w} \geq 0\text{; and } b\text{: } \dot{\sigma} = E(w)\,\dot{\varepsilon} \text{ with } \dot{w} < 0 \qquad (1)$$

However, during a drying period, the behaviour law can be represented by the restricted Hooke's law (1b) [6]. In equation (1), $E(w)$ is the value of the elastic modulus corresponding to the actual moisture content. According to (1b), a stress lock occurs when a decreasing moisture content variation is imposed to the strained wood, figure 1.

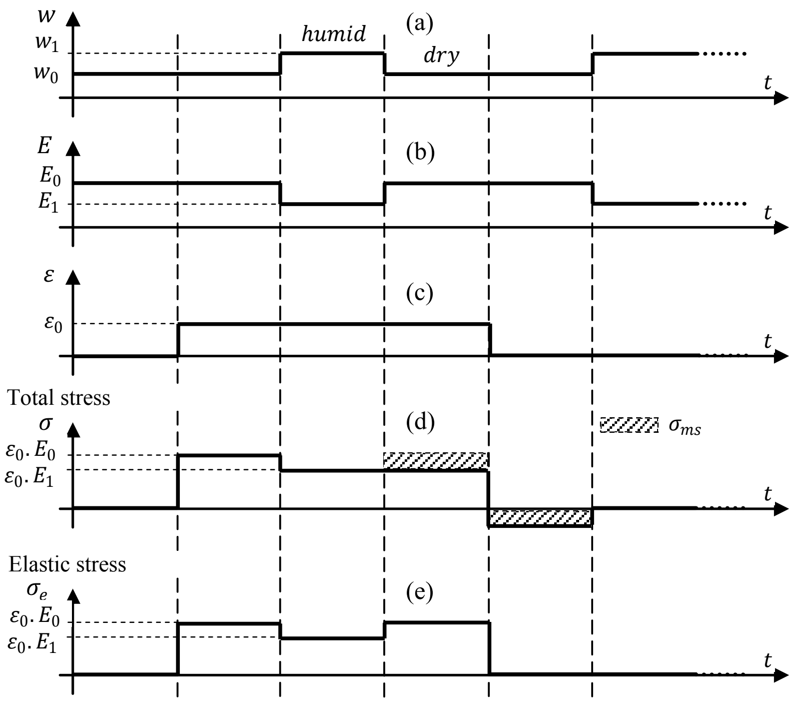

Figure 1. Mechanosorptive behaviour

Based on figure 1, we propose to partition the actual stress as the sum of an elastic stress σ_e and a mechanosorptive stress σ_{ms}, as follows

$$\sigma = \sigma_e + \sigma_{ms} \tag{2}$$

We consider an isotherm transformation (for the sake of clarity, the hydric strain is omitted in the following). Based on usual assumptions of thermodynamics and continuum mechanics, a relationship is proposed between the mechanosorptive stress σ_{ms} and the related internal variable ε_{ms}, as follows

$$\sigma_{ms} = E(w)\,\varepsilon_{ms} \tag{3}$$

The approach is generalised for any imposed strain variation, by subdividing the total strain in a sum of finite increments $\Delta\varepsilon_i$. After analytical developments, the mechanosorptive behaviour law obtained in stiffness form is given by

$$\sigma = \sum_{i=0}^{N} H(t-t_i).\overline{E}_i.\Delta\varepsilon_i; \qquad \varepsilon = \sum_{i=0}^{N} H(t-t_i).\frac{1}{\overline{E}_i}.\Delta\sigma_i \tag{4}$$

where $\overline{E}_i$ is the elastic modulus corresponding to the moisture content $\overline{w}_i$ at the beginning of the drying phase for the strain increment $\Delta\varepsilon_i$, figure 2. Similarly, the dual complaisance approach, based on the strain partition, is given by equation (4) [7].

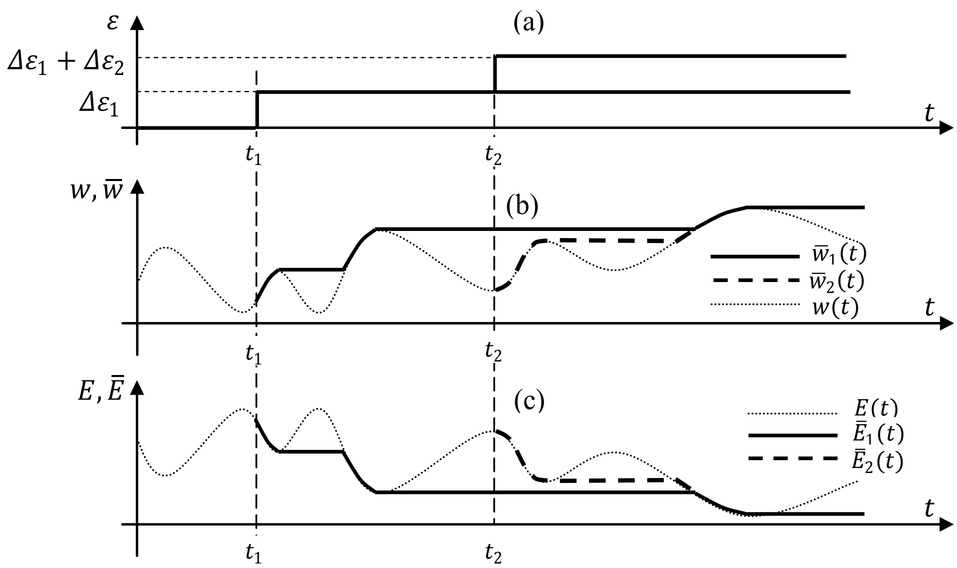

Figure 2. Variation of $\overline{E}_i$ and $\overline{w}_i$.

NUMERICAL RESULTS

In this section, two numericals examplaries is proposed. In this case, the sinusoidal stress and humidity loadings, the rigidity and the compliance analytic formulations are implemented in Matlab© software.

Compliance and rigidity approaches

Firstly, 3 imposed levels strain applied to wood material subjected to moisture content variation have considered. We suppose a linear relationship between moisture content w, figure 3 (b), and elastic modulus E, figure 3 (c).

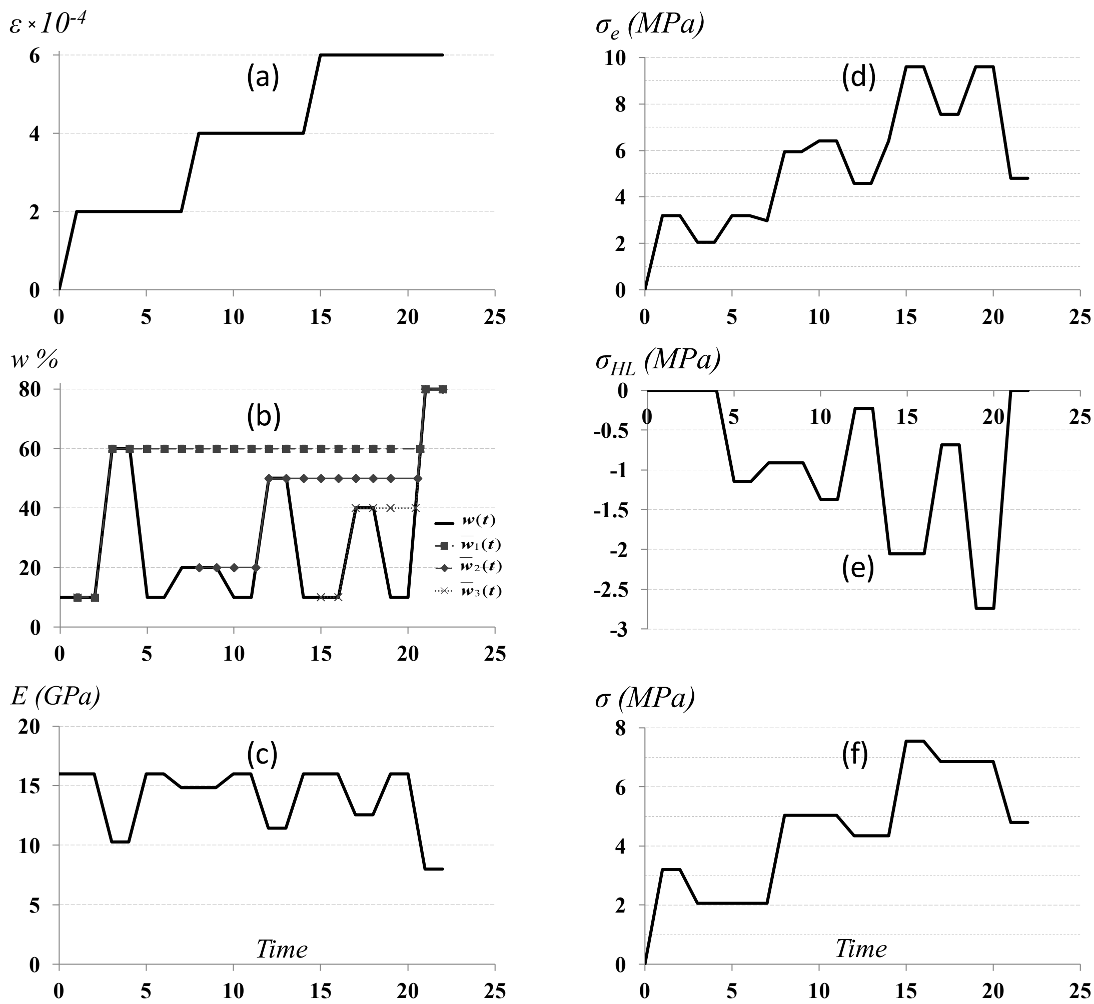

Figure 3. Variation versus time of : (a) strain loading ; (b) moisture contain ; (c) elasticity modulus ; (d) elastic stress ; (e) mechanosorptive stress ; (f) total stress.

Moisture variation is chosen such that $\bar{w}$ (or $\bar{E}$) is different for each loading level when it is calculated separately.

σ_e is calculated according to Hooke's low, figure 3. (d); σ_{ms} is given by (1), figure 3 (e), and σ is deduced by the sum of σ_e and σ_{ms} given by (3) or directly by (4), figure 3 (e). The sum of the stress responses σ_i corresponding to each strain loading level ε_i, is the same as that calculated by the direct formulation (4). This result proves that the superposition principle is correctly used in the numerical model.

Cyclic loading

In this section, the mechanosorptive behaviour under a sinusoidal loading is investigated. The stiffness and the compliance analytic formulations are implemented in Matlab© software. Figure 3 shows the evolution of the total strain and his partitions (elastic and mechanosorptive) given by the model for a loading/humidity cycle

$$a\text{: } \sigma = \sigma_m + \sigma_a.\cos(\omega t + \phi) \text{ with } \sigma_m = \sigma_a = 3MPa\text{; and } b: w\% = 50 + 40.\cos\omega t \quad (5)$$

In equation (5a), σ_m is the amplitude, ω is the pulsation, and φ is the phase angle). Also, the cyclic humidity loading is given by equation (5b).

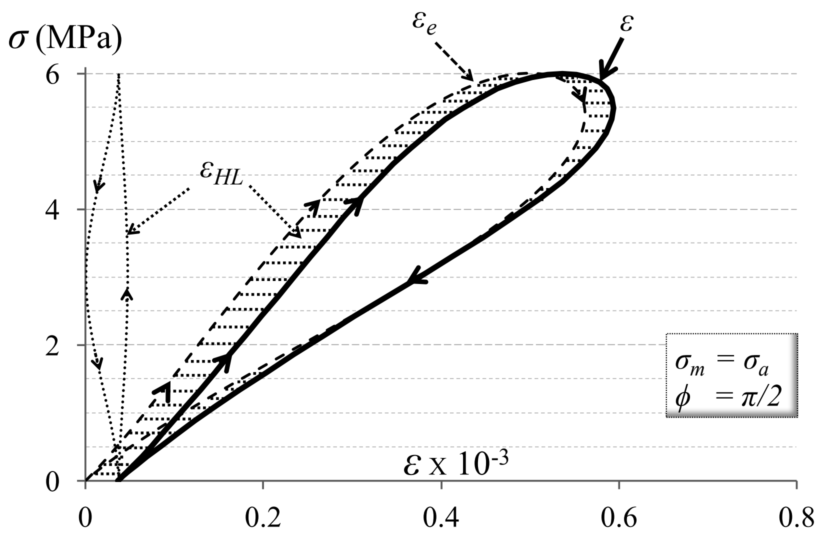

Figure 4. Stress-strain curves under sinusoidal loading/humidity cycles.

The stress-strain diagrams are shaped as stable hysteresis loops. ε_m appears at the beginning of the cycle (drying under stress); then it is released during the humidification phase. This result shows the ability of the model to depict the mechanosorptive behaviour of wood material under variable loading and humidity. Further work in progress is devoted to the expansion of the approach to the three-dimensional orthotropic case, and to the coupling with a viscoelastic model.

FRACTURE APPROACH APPLIED TO GREEN WOOD SLICE

Simultaneously, a numerical pseudo elastic calculation is applied in order to modelled the crack initiation during natural drying process. In figure 4, we observe that the crack appear is developed at the botton of the green slice. The experimental protocol and the environtmental conditions have describet by Moutou et al [8]. The mechanical behaviour is written in terms of strain rate $\dot{\underline{\underline{\varepsilon}}}(t)$ as follow:

$$\dot{\underline{\underline{\varepsilon}}}(t) = \mathbf{C}(t) \cdot \dot{\underline{\underline{\sigma}}}(t) + \underline{\underline{\alpha}} \cdot \dot{w}(t) \tag{6}$$

The total stress rate tensor is called $\dot{\underline{\underline{\sigma}}}$. $\mathbf{C}(t)$ is the four-order compliance tensor adapted for cylindrical orthotropic properties. $\underline{\underline{\alpha}}$ is the shrinkage-swelling expansion tensor. In the orthotropic reference frame, this tensor is diagonal. In this study, we assume a constant tensor in the hygroscopic domain. $\dot{w}(t)$ is the moisture content rate.

$$\mathbf{C}(w) = \begin{bmatrix} \dfrac{1}{E_R(w)} & -\dfrac{\nu_{RT}}{E_R(w)} & 0 \\ -\dfrac{\nu_{RT}}{E_R(w)} & \dfrac{1}{E_T(w)} & 0 \\ 0 & 0 & \dfrac{1}{G_{RT}(w)} \end{bmatrix} \quad \text{and } \underline{\underline{\alpha}} = \begin{bmatrix} \alpha_R & 0 & 0 \\ 0 & \alpha_T & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{7}$$

The crack growth initiation phase can be stued using a criterion of elasticity limit like Tsai or Hill based on the combination of stress tensor components [9]. In a plane stress configuration, the criterion can take the form of the function f, equation (8a)

$$a: f = \frac{<\sigma_{RR}>^{+}}{\sigma_{RR}^{c}} + \frac{<\sigma_{TT}>^{+}}{\sigma_{TT}^{c}} + \frac{|\sigma_{RT}|}{\sigma_{RT}^{c}}, \text{with } b\text{: } f < 1: no \text{ crack}, \; f = 1\text{: crak initiation} \tag{8}$$

σ_{RR}^{c}, σ_{TT}^{c} and σ_{RT}^{c} are critical values for radial, tangential and shear stresses, respectively, corresponding to uniaxial elastic limit and fracture initiation assuming a brittle fracture process. The notation $<>^{+}$ designates the positive part of fields corresponding to tension parts. The criterion inducing the crack appearing can be written as equation (8b).

At the moisture content of 8%, figure 5 shows the mapping of the function f by noting a concentration in the crack growth zone vocinity. The radial compression state, shown in figure 5, can explain a local mechanical containment that hinders the process zone development front the crack tip. This fact can explain the crack tip arrest at the limit between the tension and compression zones. However, the proposed model doesn't take into account the mécanosorptif behaviour and complex loadings.

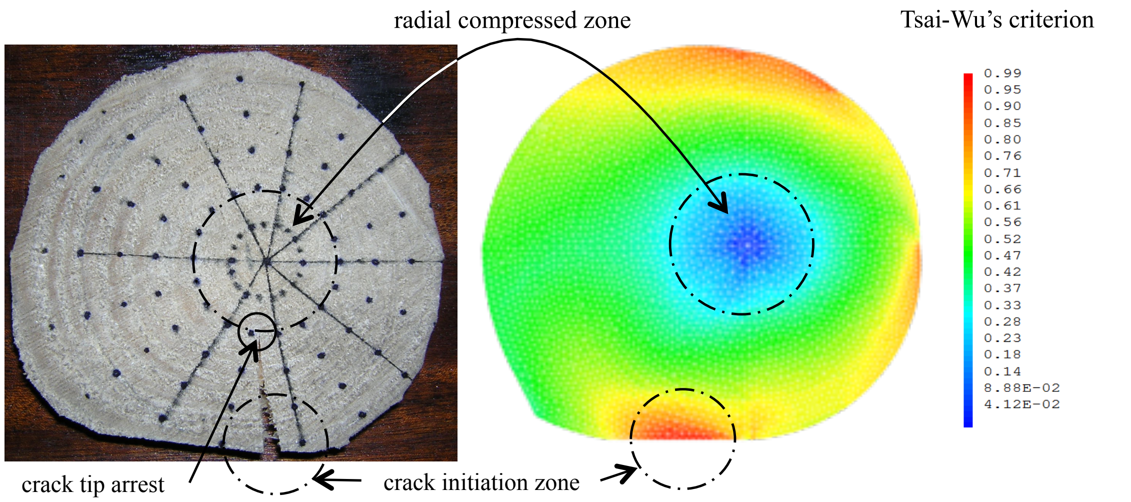

Figure 5. Detection of the crack growth initiation zone

CONCLUSIONS AND PERSPECTIVES

A new analytical stiffness approach of elastic mechanosorptive behaviour during the cycles of drying and humidification has been proposed in this paper. This approach is based on the stress partition combining an elastic and mechanosorptive stress. The exact development of this approach takes into account the effect of moisture content and the loading hereditary variations. Using an incremental formulation, some applications are modelled by Matlab software. The both approaches have been applied and discussed with variation of loading and moisture contain.

The results indicate that the sum of the stress responses corresponding to each strain loading level is the same as that given by the analytical formulation providing, in the same case, the use of superposition principle in the numerical model. Also, it is shown that the result of strain response in the compliance approach is the same of strain loading applied in the rigidity approach providing the efficiency of both methods. In the case of mechanosorptive behaviour under a sinusoidal loading the efficiency of the numerical approach to simulate drying process under complex loadings is justified.

Also, the strain and fracture processes due to the natural drying of a green wood slice are studied. The experimental protocol was completed with a numerical model, integrating elastic compliance changes versus moisture content. By developing a strength criterion based on Tsai-Wu or Hill approaches, the numerical model enabled us to focus on the localization of crack growth initiation. In the coming work, the model proposed in the firt and second sections of this work will be integrated in the crack approach in order to unterdand the collapse of wood structures under mechanosptive and complexe loadings.

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