On the Use of the Modified Wöhler Curve Method to perform the Fatigue Assessment of Welded Joints subjected to Constant and Variable Amplitude Multiaxial Fatigue Loading

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ABSTRACT. *The present paper reports on the in-field procedure specifically devised to apply the Modified Wöhler Curve Method (MWCM) along with the Theory of Critical Distances (TCD) to estimate fatigue lifetime of steel and aluminium welded joints subjected to in-service constant (CA) and variable amplitude (VA) multiaxial fatigue loading. The accuracy and reliability of the MWCM was systematically checked through a large number of experimental results taken from the literature and generated by testing, under CA/VA biaxial nominal loading, welded samples having different geometry. Such a systematic validation exercise allowed us to prove that our method is successful in designing welded joints against CA/VA multiaxial fatigue. This result is very interesting because it suggests that our approach can be used in situations of practical interest by performing a fatigue assessment which fully complies with the recommendations of the available Standard Codes.*

INTRODUCTION

Other than those approaches suggested by the available Standard Codes and Recommendations as being adopted in situations of practical interest to design welded joints against fatigue, examination of the state of the art shows that, in recent years, several attempts have been made in order to devise alternative design techniques taking full advantage of local quantities. Amongst the different approaches which have been proposed so far, and somehow validated through appropriate experimental results, certainly the Reference Radius concept [1], the N-SIF approach [2], the Strain Energy Density parameter [3], and the TCD [4] deserve to be mentioned explicitly.

In this complex scenario, over the last decade we have made a systematic effort in order to formalise and validate a novel approach based on the combined use of the MWCM and the TCD to estimate finite lifetime of welded joints under CA multiaxial fatigue loading [5, 6]. In the present paper, such an approach is further generalised in order to make it suitable for performing the fatigue assessment of steel and aluminium welded joints subjected to in-service VA multiaxial fatigue loading.

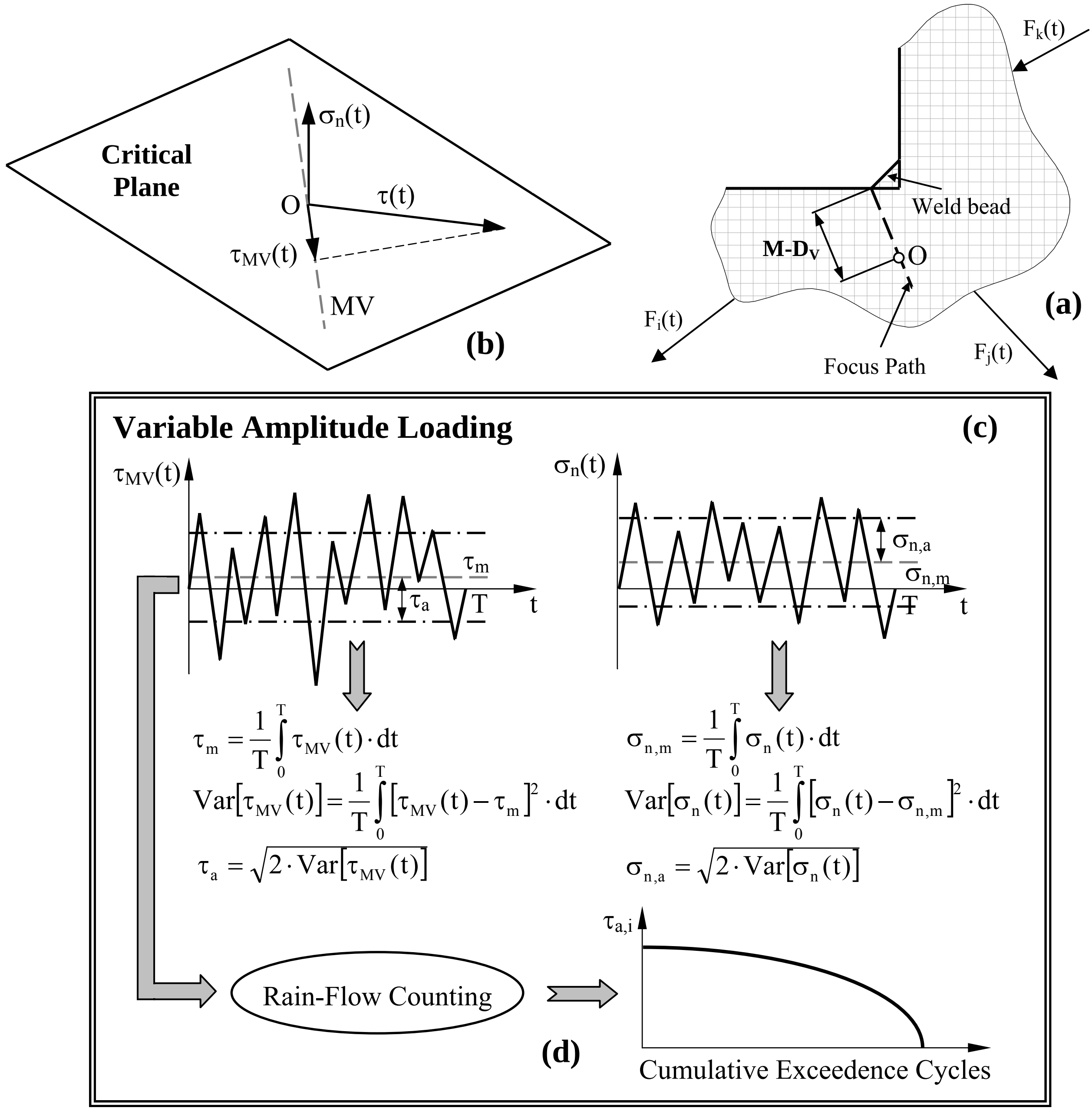

Figure 1. Adopted definitions to calculate the amplitude and the mean value of the stress components relative to the critical plane under VA loading.

CRITICAL PLANE STRESS COMPONENTS AND CYCLE COUNTING UNDER VARIABLE AMPLITUDE MULTIAXIAL FATIGUE LOADING

The MWCM is a bi-parametrical critical plane approach whose formalisation takes as a starting point the assumption that fatigue damage, under both VA and CA loading, reaches its maximum value on that plane (i.e., the so-called critical plane) experiencing the maximum shear stress amplitude.

By initially focussing attention solely on the CA problem, examination of the state of the art shows that different definitions [7] can successfully be adopted to calculate the maximum shear stress amplitude, τ_a. Even if such classical definitions are seen to be successful in calculating τ_a, their in-field usage becomes extremely time consuming when complex and long load histories are involved. To overcome the above problem, in recent years we have devised an alternative definition based on the maximum variance concept [8]. In more detail, the Maximum Variance Method (MVM) [7, 8] postulates that the critical plane can be defined as that plain containing the direction (passing through the assumed critical point) that experiences the maximum variance of the resolved shear stress, $\tau_{MV}(t)$ – see Figures 1a and 1b. From a practical point of view, the most remarkable peculiarity of the MVM is that, as soon as the variance and co-variance terms of the stress components at the critical location are known, the computational time required to determine the orientation of the critical plane does not depend on the length of the input load history being assessed [8]. Further, thanks to the specific features of the MVM, such a method can be used to determine the orientation of the critical plane not only under CA, but also under VA multiaxial fatigue loading [8]. Turning back to the CA problem, if the component sketched in Figure 1a is initially assumed to be subjected to a system of cyclic forces resulting in a CA stress state at critical point O, as soon as the orientation of the critical plane is known through the direction experiencing the maximum variance of the resolved shear stress (Fig. 1b), the amplitude, τ_a, and the mean value, τ_m, of the shear stress relative to the critical plane can directly be calculated as follows:

$$\tau_a = \frac{1}{2}\left(\tau_{MV,max} - \tau_{MV,min}\right);\ \tau_m = \frac{1}{2}\left(\tau_{MV,max} + \tau_{MV,min}\right) \qquad (1)$$

where $\tau_{MV,max}$ and $\tau_{MV,min}$ are the maximum and minimum value of $\tau_{MV}(t)$, respectively.

In a similar way, the amplitude, $\sigma_{n,a}$, and the mean value, $\sigma_{n,m}$, of the stress perpendicular to the critical plane, $\sigma_n(t)$, turn out to be:

$$\sigma_{n,a} = \frac{1}{2}\left(\sigma_{n,max} - \sigma_{n,min}\right); \sigma_{n,m} = \frac{1}{2}\left(\sigma_{n,max} + \sigma_{n,min}\right), \qquad (2)$$

$\sigma_{n,max}$ and $\sigma_{n,min}$ being the maximum and minimum value of $\sigma_n(t)$ during the loading cycle, respectively.

Assume now that the component of Figure 1a is subjected to a complex system of time-variable forces resulting in a stress state at point O whose components vary randomly in the time interval [0, T]. According to the MVM [8], the critical plane can be determined also in such circumstances by directly locating that plane containing the direction, MV, experiencing the maximum variance of resolved shear stress. As soon as the orientation of the critical plane is known, the mean value, $\tau_{n,m}$, and the amplitude, τ_a, of the shear stress relative to the critical plane take on the following values (Fig. 1c):

$$\tau_{n,m} = \frac{1}{T}\int_0^T \tau_{MV}(t)\cdot dt;\ \tau_a = \sqrt{2\cdot Var\left[\tau_{MV}(t)\right]} \Leftrightarrow Var\left[\tau_{MV}(t)\right] = \frac{1}{T}\int_0^T \left[\tau_{MV}(t) - \tau_m\right]^2 \cdot dt \qquad (3)$$

By following the same strategy as above, the equivalent amplitude and the mean value of normal stress $\sigma_n(t)$ take on the following values [8] (Fig. 1c):

$$\sigma_{n,m} = \frac{1}{T}\int_0^T \sigma_n(t)\cdot dt\,;\ \sigma_{n,a} = \sqrt{2\cdot \mathrm{Var}[\sigma_n(t)]} \Leftrightarrow \mathrm{Var}[\sigma_n(t)] = \frac{1}{T}\int_0^T [\sigma_n(t) - \sigma_{n,m}]^2 \cdot dt \quad (4)$$

Another problem which has to be addressed explicitly is the way of performing the cycle counting under VA uniaxial/multiaxial fatigue loading when the fatigue assessment is performed through the MWCM. In particular, since, as said above, under CA fatigue loading, the MWCM takes as its starting point the assumption that fatigue damage reaches its maximum value on the plane of maximum shear stress amplitude [7], it is logical to hypothesise that, under VA fatigue loading, resolved shear stress $\tau_{MV}(t)$ is the stress channel to be post-processed in order to efficiently count fatigue cycles. Owing to the fact that, by definition, $\tau_{MV}(t)$ is a monodimensional quantity, the cycle counting can then be performed according to the classical Three-Point Rain Flow Method: by so doing, from the counted shear stress cycles, the corresponding cumulative spectrum can directly be built and subsequently used to estimate the fatigue damage content associated with the assessed load history (Fig. 1d).

To conclude, it is worth observing that the available Standard Codes and Recommendations usually address the problem of designing weldments against fatigue in terms of ranges. Accordingly, the ranges of the stress quantities relative to the critical plane can be calculated as follows:

$$\Delta\tau = 2\cdot\tau_a\,;\ \Delta\sigma_n = 2\cdot\sigma_{n,a} \quad (5)$$

where the amplitudes of the two relevant stress components have to be calculated according to the definitions reviewed above, that is, by distinguishing between constant and variable amplitude situations.

THE MODIFIED WÖHLER CURVE METHOD TO DESIGN WELDED CONNECTIONS AGAINST VA MULTIAXIAL FATIGUE

In the present section the way of using the MWCM to perform the multiaxial fatigue assessment of welded joints is investigated by specifically considering VA multiaxial fatigue situations, the CA problem being a simpler sub-case.

Consider then a welded joint damaged by a complex system of cyclic forces: as schematically shown in Figure 1a, the stress state to be used to determine the necessary stress quantities relative the critical plane has to be determined, along the bisector, at distance from the weld toe apex (or the weld root apex) equal to M-D_V, such a critical distance being equal to 0.5 mm and to 0.075 mm for steel and aluminium welded joints, respectively [5, 6].

The MWCM estimates the fatigue damage extent associated with the assessed load history through the ranges of the stress components relative to the critical plane, the combined effect of the shear and normal stress being taken into account by means of the following stress ratio [5-7, 9]:

$$\rho_w = \frac{\Delta\sigma_n}{\Delta\tau} \quad (6)$$

The most relevant peculiarity of the above stress quantity is that, thanks to the way it is defined, ρ_w is seen to be sensitive to the degree of multiaxiality and non-proportionality of the stress state at the assessed critical point: for instance, ρ_w is equal to unity under uniaxial fatigue loading, whereas it is invariably equal to zero under torsion [7]. Intentionally, the critical plane stress ratio is instead insensitive to the presence of non-zero mean stresses: this suggests that ρ_w as defined above can be used solely to perform the fatigue assessment of weldments working in the as-welded condition. On the contrary, in stress relieved welded joints, the effect of non-zero mean stresses cannot be disregarded and the presence of superimposed static stresses is usually taken into account through appropriate enhancement factors [1], their in-field usage being explained below in great detail.

As soon as ρ_w is known, the position of the pertinent modified Wöhler curve has to be determined (Fig. 2d), where the negative inverse slope, $k_\tau(\rho)$, and the reference shear stress range, $\Delta\tau_{Ref}(\rho_w)$, at $N_A=5\cdot10^6$ cycles to failure are suggested as being estimated as follows:

Steel Welded Joints P_S=97.7%

$$\begin{aligned} &k_\tau(\rho_w) = -2\cdot\rho_w + 5 \text{ for } \rho_w \le 1 \\ &k_\tau(\rho_w) = 3 \text{ for } \rho_w > 1 \\ &\Delta\tau_{A,Ref}(\rho_w) = -24\cdot\rho_w + 67 \text{ [MPa] for } \rho_w \le 2 \\ &\Delta\tau_{A,Ref}(\rho_w) = 19 \text{ [MPa] for } \rho_w > 2 \end{aligned} \quad (7)$$

Aluminium Welded Joints P_S=97.7%

$$\begin{aligned} &k_\tau(\rho_w) = -0.5\cdot\rho_w + 5 \text{ for } \rho_w \le 4 \\ &k_\tau(\rho_w) = 3 \text{ for } \rho_w > 4 \\ &\Delta\tau_{Ref}(\rho_w) = -5\cdot\rho_w + 28 \text{ [MPa] for } \rho_w \le 4 \\ &\Delta\tau_{Ref}(\rho_w) = 8 \text{ [MPa] for } \rho_w > 4 \end{aligned} \quad (8)$$

The $\Delta\tau_{Ref}$ vs. ρ_w relationships reported above are strictly valid solely to assess welded joints working in the as-welded condition. On the contrary, if the welded joint being designed is stress relieved, then a procedure similar to the one recommended by Eurocode 3 is proposed to be used [5]. In particular, an effective shear stress range is determined by adding the tensile part to 60% of the compressive portion of the shear stress range. Accordingly, by adopting a strategy similar to the one suggested by the IIW [10], a suitable shear stress enhancement factor, $f(\tau)$, can directly be calculated as follows:

$$f(\tau) = 1 \text{ for } (\tau_m - \tau_a) \ge 0\text{ ; } f(\tau) = \frac{2\tau_a}{|\tau_m + \tau_a| + 0.6|\tau_m - \tau_a|} \text{ for } (\tau_m - \tau_a) < 0 \quad (9)$$

Further, in order to properly take into account the damaging effect of those stress cycles of low stress amplitude (Fig. 2d), according to Haibach [11], the negative inverse slope has to be corrected in the long-life regime as follows:

$$m_\tau(\rho_w) = 2\cdot k_\tau(\rho_w) - 1 \quad (10)$$

where the knee point is recommended to be always taken at $N_{kp}=10^8$ cycles to failure.

By taking full advantage of the classical Rain-Flow method, the resolved shear stress cycles can now be counted (Fig. 2e) to build the corresponding load spectrum (Fig. 2f).

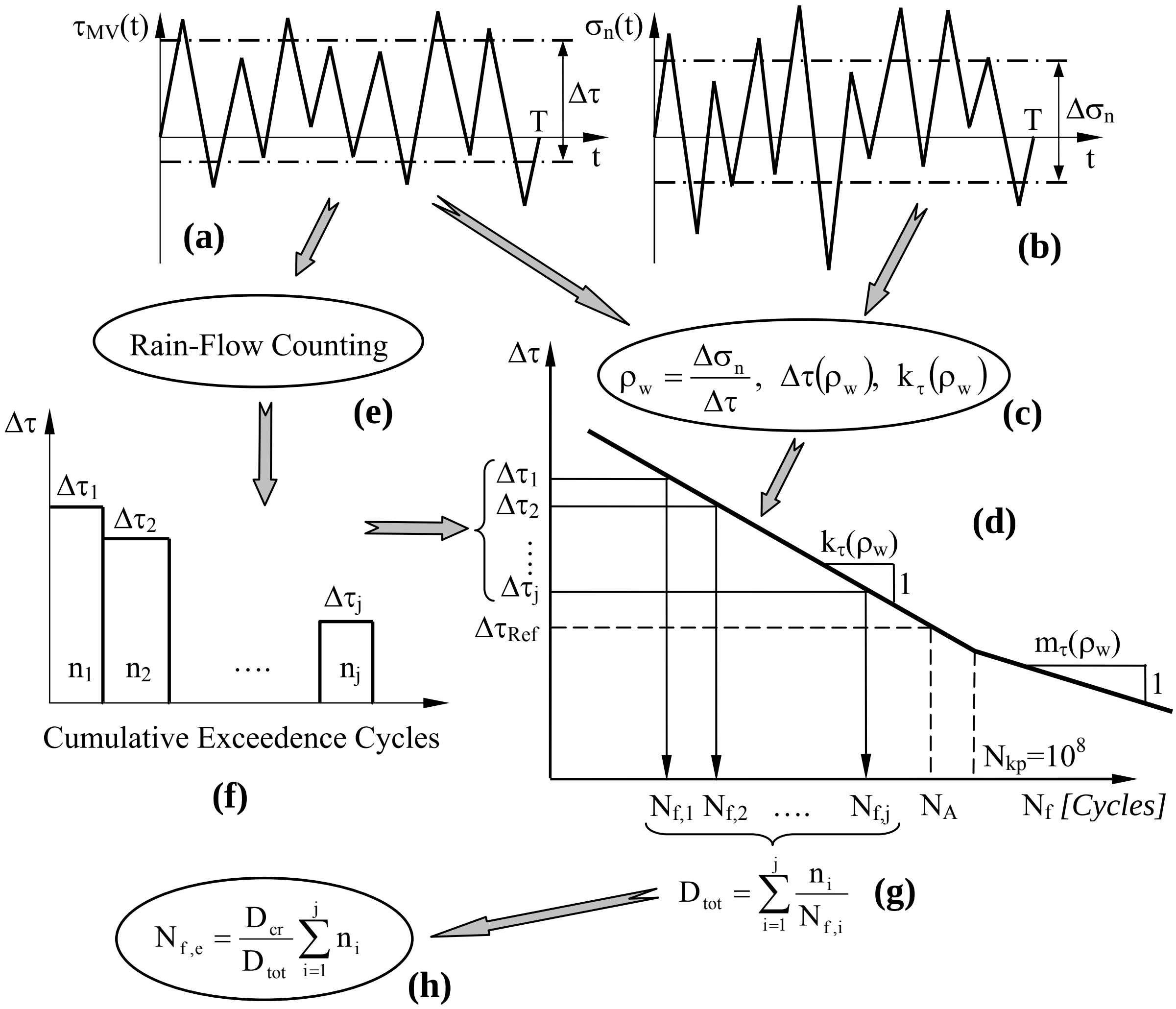

Figure 2. Design against VA multiaxial fatigue loading according to the MWCM.

Subsequently, the calculated load spectrum can directly be used, along with the adopted modified Wöhler curve, to evaluate the damage content associated with any counted shear stress cycles (Figs 2f and 2d), the estimated number of cycles to failure being equal to (Figs 2g and 2h):

$$D_{tot} = \sum_{i=1}^{j} \frac{n_i}{N_{f,i}} \Rightarrow N_{f,e} = \frac{D_{cr}}{D_{tot}} \sum_{i=1}^{j} n_i \ . \qquad (11)$$

Finally, it is worth observing that, as recommended by the IIW [10], the critical value of the damage sum, D_{cr}, is suggested as being taken invariably equal to 0.5.

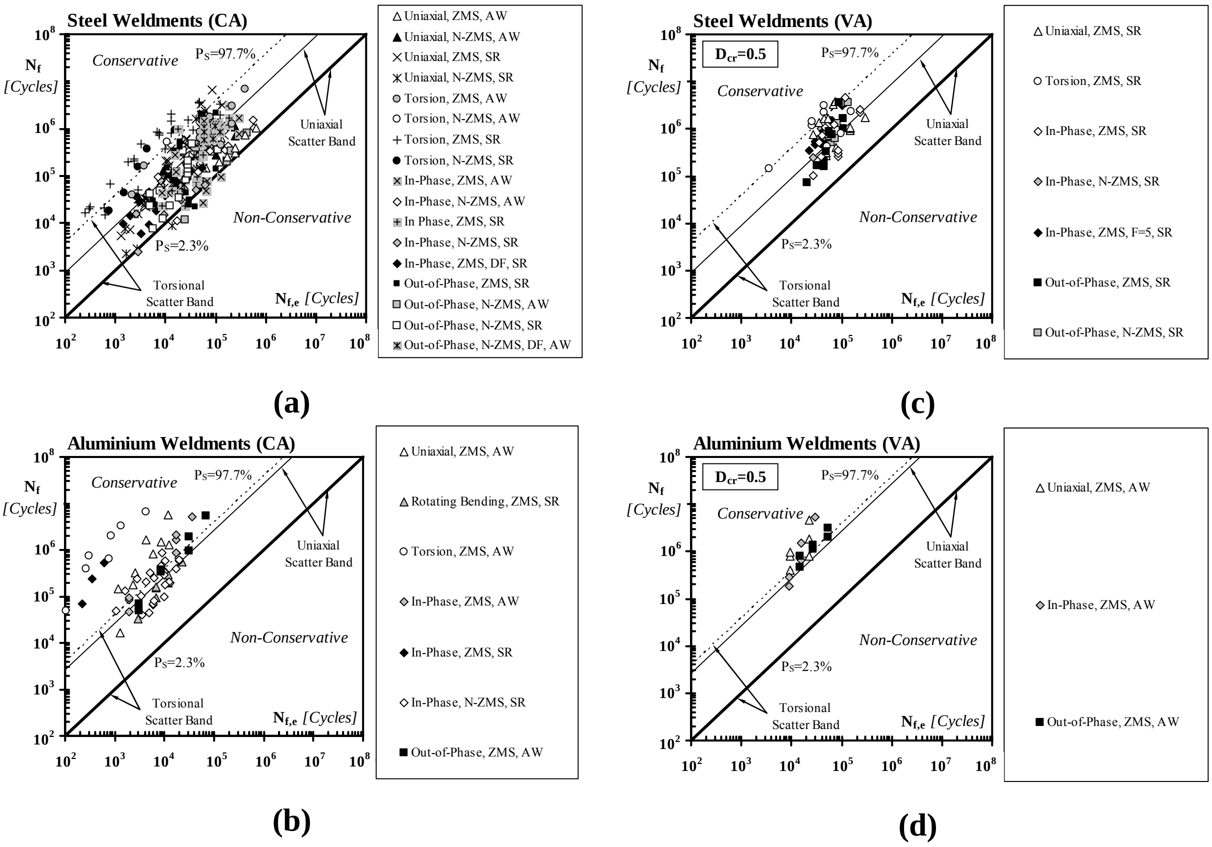

Figure 3. Accuracy of the MWCM applied along with the TCD in estimating fatigue lifetime of welded joints (ZMS = zero mean stress; N-ZMS = non-zero mean stress; AW = as-welded; SR = stress relieved; F= frequency ratio).

VALIDATION BY EXPERIMENTAL RESULTS

In order to check the accuracy of the MWCM applied along with the TCD in estimating fatigue lifetime under both CA and VA multiaxial fatigue loading a number of data sets were selected from the technical literature. In more detail, our approach was initially employed to estimate fatigue results generated, under CA multiaxial fatigue loading, by testing steel and aluminium welded samples having different geometry [12-22]. The considered VA results [12, 13, 23] were generated by adopting the classical LBF Gaussian spectrum having sequence length equal to $5 \cdot 10^4$ cycles [12].

In order to show the accuracy of our local approach, initially the error diagrams of Figures 3a and 3b report the predictions made (for a probability of survival, P_S, equal to 97.7%) by considering the results generated under CA multiaxial fatigue loading. The above charts should make it evident that our local method is capable of correctly evaluating the degree of multiaxiality and non-proportionality of the local stress field when they depend not only on the geometrical features of the assessed welded connection, but also on the specific characteristics of the investigated CA loading path. Finally, the charts of Figures 3c and 3d show the accuracy and reliability of our local method in estimating fatigue lifetime of welded joints subjected to VA multiaxial fatigue loading: this result suggests that our approach can safely be used to perform the fatigue assessment of welded joints subjected to in-service VA multiaxial fatigue loading by fully complying with the recommendations of the pertinent Standard Codes and Recommendations.

CONCLUSIONS

The MWCM is seen to be successful in estimating fatigue lifetime of both steel and aluminium weldments subjected to CA as well as to VA multiaxial fatigue loading: accordingly, it is a powerful candidate to be considered for being included amongst those method recommended by the pertinent standard codes.

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