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Threshold Conditions for Stage II Physically Short Cracks under Tension/Torsion Loading

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ABSTRACT: *Modified threshold conditions and equations for Stage II physically short crack growth are developed on the basis of Tresca and Rankine equivalent strain criteria and experimental push-pull data on short crack propagation in a medium carbon steel. Satisfactory correlation of Stage II calculated short crack behaviour and experimental data for torsion is ensured by fitting the formula for threshold condition to two limit cases, which are connected with the use of Rankine's equivalent strain criterion for fatigue limit strain range and Tresca's criterion for the high strain range regime. A modified threshold condition and an equation for Stage II short crack growth under tension/torsion loading have been derived from these equivalent strain criteria and two selected boundary threshold points, which are fitted to empirical torsion fatigue threshold results. The parameters and exponents of short crack growth equations are dependent on microstructure as well as on type of loading. The influence of biaxial strain ratio on non-propagating crack length is discussed, with regard to distance between major microstructurally barriers.*

Notation

a	crack length
a_f	non-propagating fatigue crack length
A, B, D_{th}	constants of crack growth equations
d	microstructural parameter
d_a	average ferrite grain size
da/dN	growth rates of short fatigue cracks
α, β	exponents of crack growth equations
$\Delta\gamma, \Delta\varepsilon$	shear and normal strain ranges
$\Delta\gamma_f, \Delta\varepsilon_f$	fatigue limit strain ranges
$\lambda = \Delta\gamma / \Delta\varepsilon$	strain ratio

Introduction

The consideration of two dominant phases of the fatigue failure process, Stage I and Stage II fatigue crack growth, proposed by Forsyth (1) plays a significant role in the understanding of damage accumulation. Certainly, an advance in up-date mechanics of fatigue crack propagation has been connected with an analytical description of microstructurally short and physically small cracks growth (2-4). Thus, damage accumulation has been interpreted as a failure process identified with the physical propagation of cracks. The general fatigue lifetime of a solid may be determined by integration of these crack propagation stages.

Equations which describe the propagation of microstructurally and physically short cracks (2,3) have been derived from experimental analysis of fatigue crack behaviour in medium carbon steel. Microstructurally short crack growth for Stage I can be presented by the function

$$\frac{da}{dN} = A\Delta\varepsilon^{\alpha}(d-a) \tag{1}$$

which will be equal to zero when a crack reaches a microstructural barrier. Here a is a crack length, A and α are material constants depending on the type of loading and $\Delta\varepsilon$ is the applied strain range. The microstructural parameter d refers to the possible distances between microstructural barriers.

Stage II short fatigue crack propagation follows and, being normal to the maximum principal strain, it exhibits mode I crack opening. A general equation for physically short crack growth may be written in the form

$$\frac{da}{dN} = B\Delta\varepsilon^{\beta}a - D_{th} \tag{2}$$

where B and β are material constants depending on the type of loading.

Parameter D_{th} represents the mechanical threshold for Stage II short cracks, providing a crack length a_{th} below which Stage II cracks cannot propagate. The microstructural barrier d represents a microstructural threshold, above which Stage I cracks cease growth.

The functions (1) and (2) for Stage I and Stage II short crack growth rates will be equal to zero at microstructural and mechanical threshold conditions respectively. There are two threshold conditions for a short cracks. The first threshold condition of Stage I is determined by microstructural barriers in the material. The second mechanical threshold for Stage II physically short crack growth is dependent on crack length as well as the applied strain range.

Apparently, the methodology of damage accumulation analysis based on crack propagation can be more complex for multiaxial modes of loading. Therefore, the issue addressed in this paper is to work out a two stage model of crack growth and a threshold criterion which will allow analytical equations for short crack propagation under torsion and combined tension/torsion fatigue loading to be obtained.

Modified equations for Stage I and Stage II short crack growth

Equations (1) and (2), which describe Stage I and Stage II short crack growth in material with an uniaxial stress state, for example, push-pull tests, can be transferred to the general case of multiaxial loading. The transformation is based on the reduction of a multiaxial cyclic strain state to an equivalent uniaxial strain range.

A widely used equivalent strain formula employed in plasticity theory is based on the Tresca or maximum shear criterion. In the low stress regime $(\sigma_{max} << \sigma_y)$ the Rankine failure criterion of maximum principal stress has been confirmed by biaxial fatigue studies. For proportional loading conditions this criterion may be extended to elastic-plastic conditions (5).

For combined tension-torsion loading the Tresca equivalent strain range formula can be written as

$$\Delta\varepsilon_{eqT} = \sqrt{\left[\Delta\varepsilon^2 + \Delta\gamma^2 / (1+\nu)^2\right]} \quad (3)$$

and Rankine's equivalent strain range formula becomes

$$\Delta\varepsilon_{eqR} = \left(\Delta\varepsilon_{eqT} + \Delta\varepsilon\right)/2 \quad (4)$$

where $\Delta\varepsilon$ and $\Delta\gamma$ are the axial and torsional component strain ranges, and ν is the elasto-plastic value of Poisson's ratio. Thus, general equations (1) and (2) of short crack propagation (Stage I and Stage II) may be re-written for combined loading in terms of torsional strain range components taking into account equations (3) and (4) and employing the strain ratio λ

$$\frac{da}{dN} = A_{eq}\Delta\gamma^{\alpha}(d-a) \text{ where } A_{eq} = A\left(\frac{1}{\lambda^2} + \frac{1}{(1+\nu)^2}\right)^{\alpha/2} \tag{5}$$

$$\frac{da}{dN} = B_{eq}\Delta\gamma^{\beta}a - D_{th} \tag{6}$$

where

$$B_{eq} = B\left(\frac{1}{\lambda^2} + \frac{1}{(1+\nu)^2}\right)^{\beta/2} \qquad \text{for Tresca}$$

$$B_{eq} = B\left[\frac{1}{2}\left(\frac{1}{\lambda^2} + \frac{1}{(1+\nu)^2}\right)^{1/2} + \frac{1}{2\lambda}\right]^{\beta} \qquad \text{for Rankine}$$

Here the parameter D_{th} is assumed to be a material constant. New constants A_{eq} and B_{eq} are determined by the parameters and exponents of equations (1) and (2), using the equivalent strain formulae.

A model of short microstructurally crack growth

The equation (5) and model of microstructurally short crack propagation was confirmed for a medium carbon steel under torsion and combined tension/torsion loading (6). Since Stage I fatigue crack propagation is a shear mode, the employment of Tresca's equivalent strain criterion and the equation (1) for microstructurally short crack growth under push-pull loading allows description of the observed behaviour of microstructurally short cracks under torsion and combined loading. The existence of differences in sizes of weak microstructural barriers and the mechanical threshold is the cause of the crack propagation by steps, connected with overcoming weak barriers (boundaries of ferrite grains), in accordance with the equation for Stage I crack growth. The average ferrite grain size in a medium carbon steel in the transverse direction is $d_a = 37\mu m$, with a standard deviation of 18 μm. The transcryctalline crack which becomes the short fatigue failure crack is that located in the largest ferrite grains, because it grows fastest. It is clear that the microstructural parameter d will change for each crack step. Parameter d represents an above average ferrite grain size $d_1 = 55\mu m$ for the first grain which is likely to initiate a Stage I crack. For the subsequent m crack steps of Stage I the microstructural parameter d will be of the form

$$\left.\begin{aligned} d_m &= d_1 + 2(m-1)d_a, \quad d_m < D \\ d_m &= D \end{aligned}\right\} \tag{7}$$

where D is the spacing for strong barriers, between pearlite grains. Thus D, the strong barrier, gives an absolute upper bound to weak barrier values d_m. For a crack in grain m, d is assumed to change from d_m to d_{m+1} when the crack length achieves a critical value of $0.95\,d_m$ (7). The transition of short crack propagation from Stage I to Stage II is determined by an equality of crack rates from equations (5) and (6), giving the transitional crack length a_{tr}.

The number of microstructurally short crack steps depends on a material's microstructure, and also on applied strain range because Stage I is limited by the transition of crack propagation to Stage II after the mechanical threshold has been exceeded.

Threshold criterion for Stage II short crack

Stage II short crack growth in a medium carbon steel subjected to torsional cyclic loading has been analysed, using the results of Zhang (8). The experimental results lied between two curves described by the Tresca and Rankine criteria (6). The Rankine and Tresca curves were derived from Zhang's push-pull crack growth results, using equation (6). The location of the experimental curve is dependent on applied strain range and can be determined by threshold conditions. Equation (6) allows us to analyse mechanical threshold conditions for Stage II. Rankine's criterion correlates with experimental results for the low strain range regime. The use of Tresca's criterion is more justified in the high strain range regime of loading. It is clear that it is necessary to modify these criteria and the parameters of equation (6) to describe threshold conditions for any possible strain range regime. The following procedure is proposed to establish the modified criterion and parameters of equation (6).

It is considered that two points will create a modified threshold criterion. The first point is determined by Rankine's criterion at the fatigue limit strain range $\Delta\gamma_f$. The second point is determined by the intersection of the Tresca's criterion threshold line with the experimental threshold in the high strain range regime of fig. 1. Thus, modified equations of Stage II fatigue short crack growth under torsion may be determined by the following threshold condition

$$a_{th}\Delta\gamma_{th}^{\beta_{eq}} = constant \qquad (8)$$

which gives a new modified constant and exponent for a medium carbon steel.

The "modified" parameter β_{eq} is found as follows:

i) taking the first fixed point at the fatigue limit, derive a_f from the Rankine equation (6),

ii) taking the second fixed point at $a = 7\mu m$, calculate the threshold strain range $\Delta\gamma$ from the Tresca equation (6),

iii)assuming a straight line of slope $-1/\beta_{eq}$ in fig. 1, evaluate β_{eq} from a line shown between these fixed points,

iv) evaluate B_{eq} from $B_{eq}\Delta\gamma_f^{\beta_{eq}}a_f = D_{th}$.

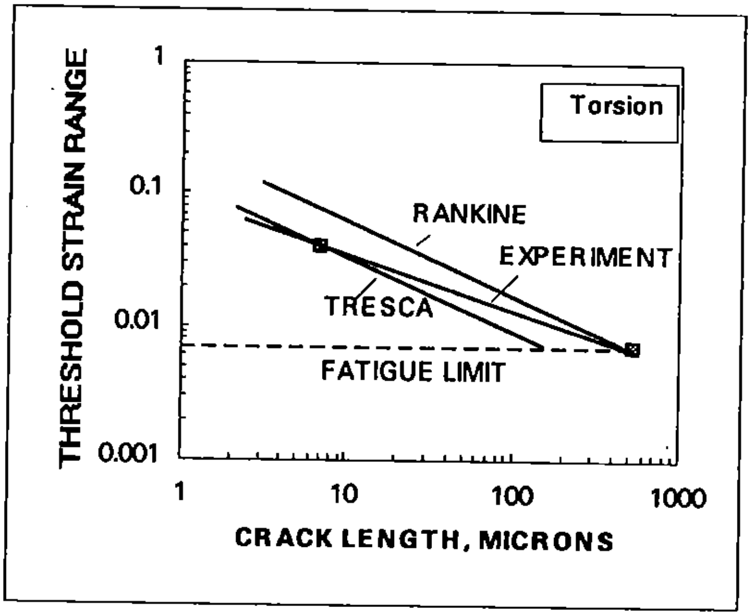

Fig.1 Threshold conditions for a medium carbon steel under torsion loading.

Table 1 shows excellent agreement with the experimental results because the critical value of 7 μm was selected for the torsional data in fig. 1.

Table 1. Constants and exponents of equations (5) and (6) and microstructural fatigue limit parameters for a medium carbon steel under torsion loading

Criterion	A_{eq}	α	B_{eq}	β_{eq}	$\Delta\gamma_f$	a_f $[\mu m]$
Tresca	1.03×10^5	4.5	1.63×10^{-1}	1.771	6.9×10^{-3}	154
Rankine	-	-	4.78×10^{-2}	1.771	6.9×10^{-3}	526
Experiment	8.66×10^3	4.02	1.64	2.483	6.9×10^{-3}	531
Modified	1.03×10^5	4.5	1.605	2.477	6.9×10^{-3}	526

The modified model presentation of Stage I and Stage II fatigue short crack propagation gives a very good correlation with experimental results for torsion loading (6).

Let us use the previous procedure with two selected boundary threshold points to establish a modified equation for Stage II short crack propagation under combined loading at strain ratio λ. Obviously, the fatigue limit condition for combined loading differs from the fatigue limit under torsion and push-pull. To estimate the fatigue limit condition under combined loading, an approach based on the Γ-plane (9) can be used. Fatigue limit strain range components for combined loading is given in table 2.

Table 2. Constants and exponents of equations (5) and (6) and microstructural fatigue limit parameters for a medium carbon steel under combined loading at λ=1.5

Criterion	A_{eq}	α	B_{eq}	β_{eq}	$\Delta\gamma_f$	a_f [μm]
Tresca	4.21×10^5	4.5	2.84×10^{-1}	1.771	4.6×10^{-3}	182
Rankine	-	-	2.09×10^{-1}	1.771	4.6×10^{-3}	247
Modified	4.21×10^5	4.5	5.14×10^{-1}	1.938	4.6×10^{-3}	247

The use of two selected boundary threshold points, transferred to combined axial and torsional loading, is illustrated in fig. 2. The modified threshold line described by equation (8) allows us to obtain the real constant B_{eq} and exponent β_{eq} of the crack growth equation (6) for Stage II (table 2). We have assumed that the critical length 7 μm used to obtain the second fixed point is a constant value, irrespective of the strain state λ.

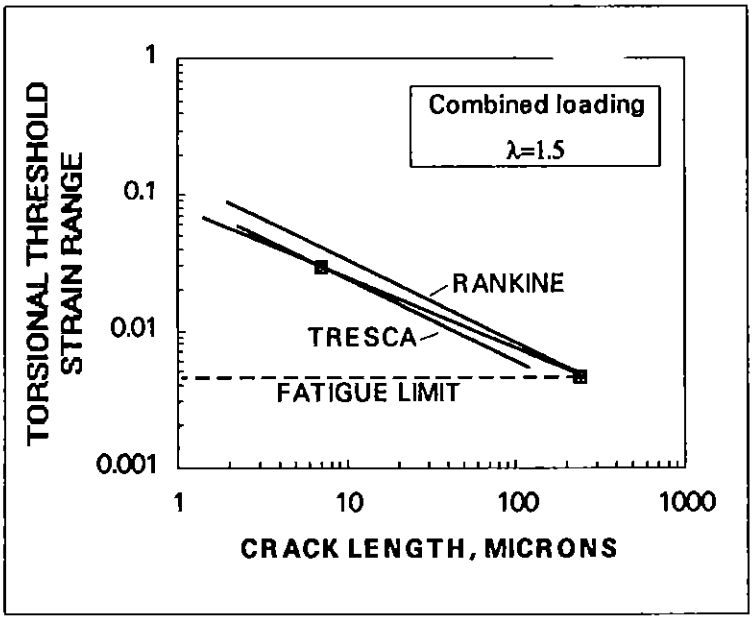

Fig.2 Threshold conditions for a medium carbon steel under combined loading at λ=1.5.

A modified threshold condition and an equation for Stage II short crack growth under combined loading have been derived from Tresca's and Rankine's equivalent strain criteria and two selected boundary threshold points, which are fitted to empirical torsion fatigue threshold results. It is clear that the parameters and exponents of short crack growth equations are dependent on microstructure as well as on type of loading.

The dependence on microstructure is illustrated by the maximum non-propagating fatigue crack lengths a_f (or distances between strong microstructural barriers), which have been derived from equations for Stage II with growth rate equal to zero under fatigue limit strain ranges (table 3).

Obviously, the change of size of non-propagating cracks for various loading conditions is connected with a change of crack growth directions between major microstructural barriers. The high value for a_f in torsion reflects the anisotropy of the material, with greater microstructural dimensions for cracks growing along the longitudinal axis of the bar.

Table 3. The influence of strain ratio λ on constants and exponent of Stage II short crack growth equations and non-propagating crack length.

	λ	B_{eq}	β	a_f [μm]	$\Delta\gamma_f$	$\Delta\varepsilon_f$
Uniaxial	0	0.296	1.771	213	0	0.0041
Combined	1.5	0.514	1.938	247	0.0046	0.0031
Pure shear	∞	1.605	2.477	526	0.0069	0

Conclusions

A model of microstructurally and physically short crack growth, with the threshold criterion for combined tension/torsion loading, has been proposed to analyse the short crack propagation in a medium carbon steel. The following conclusions can be drawn from the present study:

(1) Tresca's equivalent strain criterion for the high strain range regime and Rankine's criterion for fatigue limit strain range were employed for an analysis of Stage II crack

growth. As a result, the threshold conditions and equations for Stage II physically short crack growth under combined loading were derived from the equation for push-pull loading. The parameters and exponents of short crack growth equations are dependent on microstructure as well as on type of loading.

(2) The maximum non-propagating physically short fatigue crack length is dependent on distance between major microstructural barriers and loading conditions.

References

(1) FORSYTH P.J.E., (1961), A two stage process of fatigue crack growth, Proc. Crack Propagation Symposium, pp. 76-94.

(2) CARBONELL E.Perez and BROWN M.W., (1985), A study of short crack growth in torsional low cycle fatigue for a medium carbon steel, Fatigue Fracture Engng Mater. Structures, vol. 9, pp. 15-33.

(3) HOBSON P.D., BROWN M.W. and de los RIOS E.R., (1986), Two phases of short crack in a medium carbon steel, The Behaviour of Short Fatigue Cracks (EGF), (Edited by K.J. Miller and E.R. de los Rios), pp. 441-459.

(4) BROWN M.W., MILLER K.J., FERNANDO U.S., YATES J.R. and SUKER D.K., (1994), Aspects of multiaxial fatigue crack propagation, Fourth International Conference on Biaxial/Multiaxial Fatigue, vol. I, pp. 3-16.

(5) BROWN M.W. and BUCKTHORPE D.E., (1989), A crack propagation based effective strain criterion, Biaxial and Multiaxial Fatigue (EDF 3), (Edited by M.W. Brown and K.J. Miller), pp. 499-510.

(6) MATVIENKO Yu.G., BROWN M.W. and MILLER K.J., (1997), A model of fatigue short crack propagation under combined loading, Fatigue Fracture Engng Mater. Structures, (to be published).

(7) YATES J.R. and GRABOWSKI L., (1990), Fatigue life assessment using a short crack growth model, Fatigue' 90, vol. 4, pp. 2369-2376.

(8) ZHANG W., (1991), Short fatigue crack behaviour under different loading systems, Ph.D. Thesis, University of Sheffield, UK.

(9) BROWN M.W. and MILLER K.J., (1973), A theory for fatigue under multiaxial stress-strain conditions, Proc. Instn Mech. Engrs, vol. 187, pp. 745-755.

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