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Manuel da FONTE *, Edgar GOMES ** and Manuel de FREITAS **

Stress Intensity Factors for Semi-elliptical Surface Cracks in Round Bars Subjected to Mode I (Bending) and Mode III (Torsion) Loading

* Escola Náutica, Dept. Máq. Marítimas, Paço de Arcos, 2780 Oeiras, Portugal

** Instituto Sup. Técnico, Dept. Engª Mecânica, Av. Rovisco Pais, 1096 Lisboa, Portugal

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ABSTRACT: Crack growth under mixed-mode loading of rotor shafts is the reason for many failures occurring under cyclic Mode I (ΔK_I) combined with steady Mode III (K_{III}). This study presents Stress Intensity Factors calculations of semi-elliptical surface cracks in round bars subjected to bending, torsion and bending/torsion using a three-dimensional finite element model. The configuration of the semi-ellipse follows the experimentally determined equation: $b=(2s)/\pi$, where b is the crack depth and s is the semi-arc crack length. Stress Intensity Factors were obtained for bending loading and compared with available literature results in order to validate the proposed model. For Mode III, Stress Intensity Factors are obtained with the full shaft geometry due to non-symmetrical torsion loading. When the analysis is performed with both bending and torsion, different Stress Intensity Factors were obtained at both points where the crack front intersects the shaft surface. This result explains clearly the rotation of the crack front experimentally observed whenever a steady torsion is superimposed to the reversed or rotary bending. Results are compared with experimental crack growth measurements.

Notation

b	crack depth, minor semi-ellipse axis
a	major semi-ellipse axis
r	Shaft radius
D	Shaft diameter
s	half crack length
B_s	bending stress
T_s	torsion stress
K_I	Stress Intensity Factor in Mode I
K_{II}	Stress Intensity Factor in Mode II
K_{III}	Stress Intensity Factor in Mode III

Introduction

Cylindrical components have many different applications in mechanical constructions such as shafts, bolts and screws or wires. Fatigue cracks in such components are dependent on its geometry and loading conditions. Surface cracks, either circumferential or semi-elliptical shape, can occur in these structural components and cause premature failures. These cracks occur with different types of loading: tension, bending, torsion or combined loads of tension/torsion or bending/torsion. In order to predict crack growth behavior, the linear elastic fracture mechanics can usefully be applied since the respective Stress Intensity Factor solutions are available for the geometry and loading.

Despite the practical importance of mixed-mode fatigue, the majority of fracture mechanics research has been devoted with Mode I crack growth. Consequently, Stress Intensity Factors solutions for a wide range of geometries in Mode I loading are reported in the literature. Concerning surface cracks in round bars, several solutions have been proposed: in Raju and Newman (1) Stress Intensity Factors for circumferential surface cracks in pipes and rods under tension and bending loads are presented. Shiratori et al (2) and Murakami et al (3) present some solutions for semi-elliptical surface cracks subjected to tension and bending. In Carpinteri et al (4) Stress Intensity Factors are presented for part-through cracks in round bars under cyclic combined axial and bending loading. For torsion loading of shafts with cracks, where Mode III Stress Intensity Factors is present, solutions are less common than for Mode I loading due to difficulties in obtaining analytical or numerical solutions. As consequence, in structural components under combined tension or bending with torsion, Mode (I+III), fracture mechanics analysis presents some difficulties. However, experimental studies on mixed-mode have been performed with simple geometries such as circumferentially notched round bars in cyclic bending and torsion using mainly servo-hydraulic fatigue testing machines, since the Stress Intensity Factor solutions are known (5-8).

For semi-elliptical surface cracks in round bars subjected to Mode III loading, no solutions are available for the Stress Intensity Factors. Furthermore, the aspect ratio b/a changes during cyclic loading which affects the accurate Stress Intensity Factors to provide a suitable fatigue life prediction. This fact results in difficulties for the analysis of mixed-mode fatigue crack growth in structural components where surface fatigue cracks occur under bending and steady torsion, such as power shafts. Experimental results are anyway available in Akhurst and Lindley (9) and Freitas et al (10-12), where fatigue crack growth tests were carried out in round bars subjected to bending and torsion stresses.

Stress Intensity Factor Calculations

A three-dimensional finite-element analysis was used to obtain the Mode I, Mode III and mixed-mode (I+III) Stress Intensity Factors calculations along the crack front for a semi-elliptical surface crack in a shaft subjected to bending, torsion and bending/torsion simultaneously. The elliptical shape of the crack front was adopted with a constant arc crack length/depth ratio, according to the experimental equation $b=(2s)/\pi$, obtained in previous studies (10), as shown in Fig. 1.

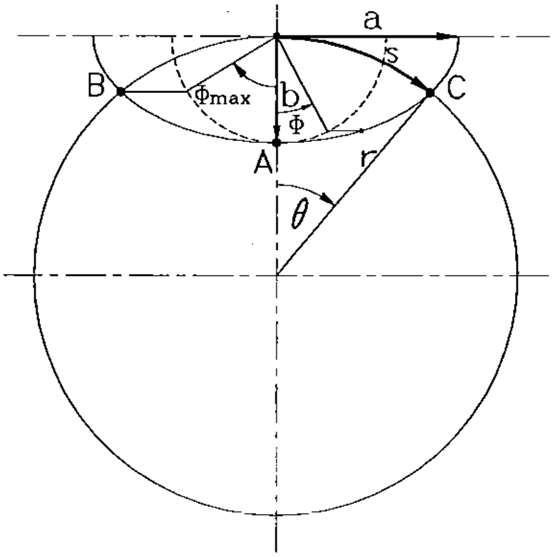

Fig. 1. Geometry aspects of semi-elliptical surface cracks with important parameters.

In this study a numerical analysis is carried out using three-dimensional finite element methods through the commercial COSMOS/M (version 1.75A) program, for the Stress Intensity Factors calculations of semi-elliptical surface cracks in round bars subjected to bending and torsion. As a first step, Stress Intensity Factors are computed for Mode I and compared with available results (1-4) in order to validate the proposed model; finally, stress Intensity Factors due to the torsion (Mode III) and the mixed-mode (I+III) loading are computed.

Symmetry conditions could only be used to obtain the Stress Intensity Factors in bending, as performed in previous studies of Raju and Newman (1) or a quarter of the geometry as made by Carpinteri (4). In the present case, the aim of the study is to obtain Stress Intensity Factors for the present geometry subjected to torsion and bending/torsion. Due to the existence of the torsion loading, the symmetry conditions are not obvious and it was necessary to model all the geometry of the shaft with the semi-elliptical crack placed at the mid-distance of the applied loading as shown in Fig.2 and Fig. 3. This geometry was performed for a shaft of 80 mm diameter with 120 mm of total length. The three-dimensional finite elements analysis is carried out by employing 20-node isoparametric solid elements. The stress square-root singularity is modeled by shifting the finite element mid-side nodes near the crack front to quarter-point position (13,14). Elastic material properties were assumed to be equal to E=207000 MPa and 0.3 Poisson's ratio. The present finite element mesh was developed with a total of 3000 elements and 14000 nodes approximately, depending on the considered ellipses case.

In order to have a comparison between the proposed model and the available results reported in the literature, a first series of tests were performed subjecting the crack geometry to pure bending load. Several solutions are available in Raju and Newman (1), Shiratori (2), Murakami (3) and Carpinteri (4) where the Stress Intensity Factors were obtained by different techniques (finite elements, body forces) and used different parameters for the measurement of the crack, either crack depth (b) or arc crack length (s). When comparing the different results available with the present ones, an uniformity in the K factors must be present.

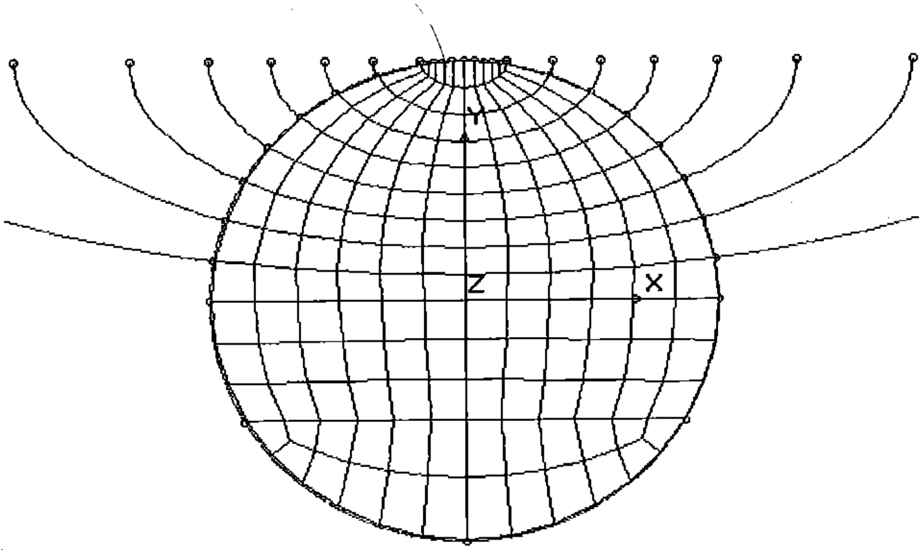

Fig. 2. Shaft surface crack with all semi-elliptical cracks and finite element mesh.

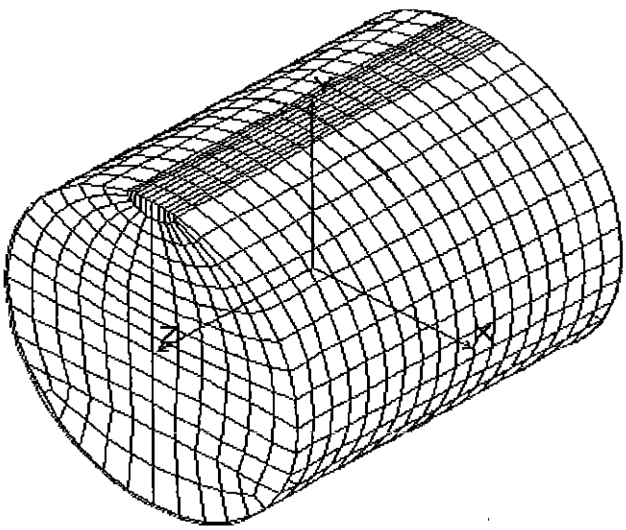

Fig. 3. Finite element mesh for the entire round bar.

The arc crack length (2s) was chosen because it is the experimentally determined parameter in fatigue crack growth tests and according to Shiratori et al (2). Therefore the Mode I Stress Intensity Factor K_I for any point along the surface crack is obtained by the equation:

$$K_I = Y_I B_s \sqrt{\pi s} \tag{1}$$

where B_S is the remote applied bending stress, s is the half arc crack length and Y_I is the boundary correction factor for Mode I which is a function of the elliptical crack shape (b/a), shaft diameter D and of the position points along the crack front determined by the parametric angle ϕ, Fig. 1.

Assuming that the crack has always an elliptical shape, the points at the crack front at the deepest crack depth (A) and where the crack intersects the outer side of the shaft surface (B and C) are the most important ones. The comparison will be only presented at these particular points.

As referred before, there is no available results for the Stress Intensity Factors of semi-elliptical surface cracks in round bars subjected to torsion stress. Therefore any comparison will not be presented. Stress Intensity Factors K_{III} will be presented using the same geometric parameters that were used for Mode I (bending) and is obtained by:

$$K_{III} = Y_{III} T_S \sqrt{\pi s} \qquad (2)$$

where T_S is the remote applied torsion stress in the shaft, s is the half crack length and Y_{III} is the boundary correction factor for Mode III which is a function of the shape of the elliptical crack (b/a), shaft diameter D and of the position along the crack front determined by the parametric angle ϕ. Once again, it is assumed that the crack has always an elliptical shape and points A, B, C are the most important parameters and only these ones will be presented.

A further study was performed in order to obtain the Stress Intensity Factors along the crack front for the case where both remote bending and remote torsion stresses were applied. According to linear elastic fracture mechanics, this mixed loading will present mixed-mode Stress Intensity Factors at the crack front and will be a linear function of the previous single loading cases. So the equations for the Stress Intensity Factors in Mode I and in Mode III will be obviously the same as before, respectively equations (1) and (2).

Results and Discussion

Bending case

Stress intensity factors were calculated for the geometry shown in Fig. 2 and Fig. 3 subjected to a remote bending stress. Due to the particular mesh geometry performed, a total number of 8 semi-elliptical surface cracks were designed with a constant ratio between the arc crack length 2s and the crack depth b, i.e. with $\theta = 10$ degrees, see Fig. 1.

For an applied remote stress, Bs=200 MPa, and for the semi-ellipse number 3, the results of the three Stress Intensity Factors (K_I, K_{II}, K_{III}) for the three points (A, B, C) along the crack front (maximum crack depth, point A; points B and C, where the crack intersects the shaft surface are shown in Table I, bending case).

At points B and C identical Stress Intensity Factors were obtained which confirms the symmetry of the present geometry when subjected to a remote bending stress. Mode II Stress Intensity Factors are of minor importance since these values are one order minor than those of Mode I and Mode III Stress Intensity Factors. For the point at maximum crack depth (A), Mode III Stress Intensity Factor is zero which means that if Mode II is neglected this point is the only one subjected to pure Mode I in bending loading stress.

In order to check the accuracy of the present results, the boundary correction factor Y_I defined for the bending stress, was calculated for the eight elliptical cracks and compared with the same correction factors available in the literature. It is to note that some of the literature results were recalculated in order to be also a function of the half arc crack length. Fig. 4. shows the comparison between the results reported by the referred authors (1-4) and the present model, at the deepest point (A) of the crack front.

A very good agreement is observed between them, despite that they were calculated according to different approaches, as mentioned before. For the points where the crack intersects the surface, B and C, the same comparison is shown in Fig. 5 and the same good agreement between the results is observed.

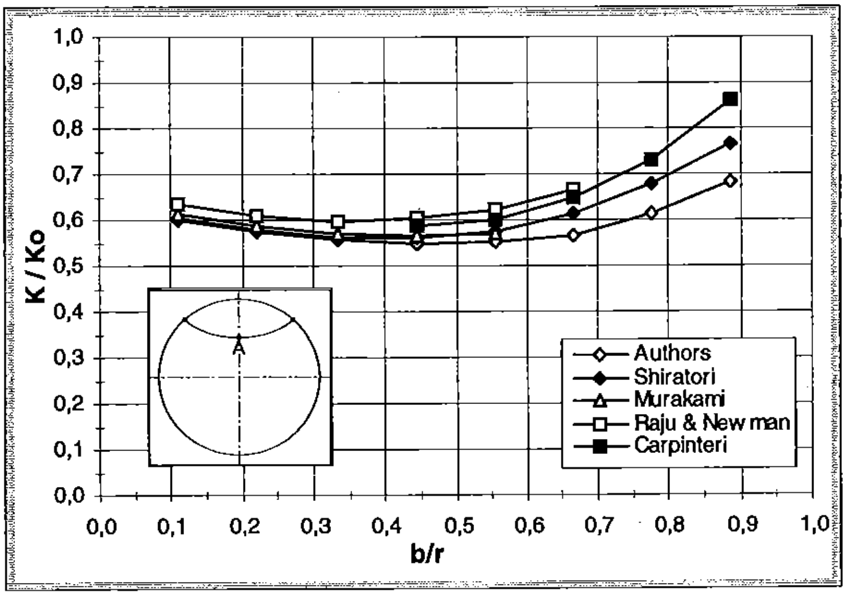

Fig. 4. Dimensionless Stress Intensity Factors Y_I at deepest point A for pure bending.

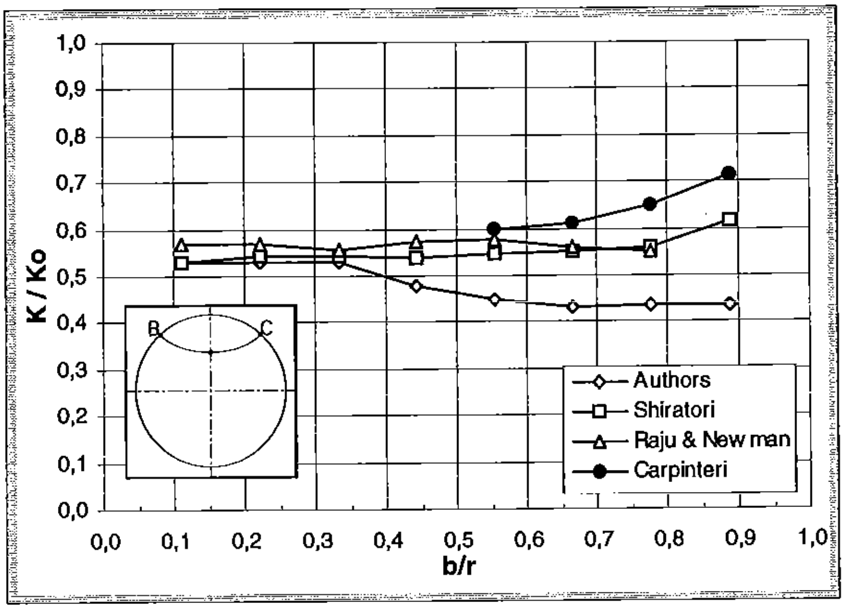

Fig. 5. Dimensionless Stress Intensity Factors Y_I at points B and C for pure bending.

Torsion case

Stress intensity factors K_I, K_{II} and K_{III}, where calculated for an applied remote torsion stress, according to the same procedure used for the bending stress, for the eight semi-elliptical surface cracks. In order to compare the results of bending and torsion, the torsion stresses Ts=200 MPa and Ts=100 MPa were used for the applied remote torsion stresses. Results are presented in Table I (torsion case) for the semi-elliptical surface crack number three. It is to note that at the point of maximum crack depth (A) a pure state of Mode III (K_I=K_{II}=0) is observed with a similar behavior as it was obtained for Mode I in bending stress loading. The numerical K_I value obtained is positive for the points B and C but for the K_{II} and K_{III} symmetrical numerical values of the Stress Intensity Factors are shown (points were the crack intersects the surface) which confirms the lack of symmetry conditions for the case of the torsion loading stress. Anyway the numerical values are symmetrical for Mode II and Mode III which confirms the symmetry of the geometry used in the model. For this case no comparisons can be performed since as far as we know, there is no available calculations in the literature.

Bending and torsion case

Finally, the same cracked geometry (shaft with a semi-elliptical surface crack) was subjected to both bending and torsion stresses. Stress intensity factors K_I, K_{II} and K_{III}, were calculated according to the same procedure as in the previous cases. Table I shows the results of the Stress Intensity Factors obtained for the semi-elliptical crack number 3 and points A, B and C, respectively for bending stress, torsion stress and bending and torsion stress. As expected, a linear elastic behavior is observed, but since the Mode I opening is always positive, careful must be taken when analyzing the results.

Table I. Stress Intensity Factors K_I, K_{II} and K_{III} for semi-elliptical surface crack n°3

ELLIPSE 3	[MPa $\sqrt{m}$]			[MPa $\sqrt{m}$]			[MPa $\sqrt{m}$]		
POINTS	B	A	C	B	A	C	B	A	C
Mode I; Mode III:									
BENDING (200MPa)	27.20	**28.93**	27.20	0,32	**-0,79**	0,32	-9.26	**0.00**	-9.26
TORSION (200 MPa)	10.87	**0.00**	10.87	-36.75	**0.00**	36.75	-16.49	**-35.48**	16.49
Mode (I+III):									
BEND + TORS (100)	32.63	**28.93**	21.75	-18.05	**-0,79**	18.69	-17.51	**-17.74**	1.015
BEND + TORS (200)	38.07	**28.93**	16.33	-36,43	**-0,79**	37.07	-25.75	**-35.48**	7.23

At point A (maximum crack depth) the contribution to K_I is given only by the bending stress case as well as the contribution to K_{III} is obtained only by the torsion stress while K_{II} remains nearly zero. At points B and C a different behavior is observed. The numerical K_{II} and K_{III} values are the algebraic sum of the respective numerical values obtained for both individual load cases which results in non-symmetrical numerical K_{II} and K_{III} values for both points in the crack front. It should be expected a similar behavior for the K_I, so the final numerical value of K_I is affected by the K_I value obtained from torsion loading, but with different results. For the case of torsion stress, K_I is obviously positive and symmetric at these points but in presence of a bending stress the real behavior is affected and non-symmetric numerical K_I values are obtained.

Then, when performing fatigue crack growth tests in reversed bending subjected to static torsion it should be expected that different crack growths rates should be observed in points B and C, resulting from different ΔK_I at each point B and C. At maximum crack depth, point A, ΔK_I results only from the Stress Intensity Factor obtained from the remote bending stress; therefore it is not affected by the static torsion stress. In a previous work (11,12), the authors carried out fatigue crack growth tests in cylindrical specimens of Ck45K steel, pre-cracked and subjected to reversed bending with a static torsion. Results in Fig. 6. show the expected behavior: two crack growth rates at each side of the symmetry axis (fast and slow crack growth) for Bs=200 MPa and Ts=140 MPa. In Fig. 7 a fracture surface of a specimen subjected to reversed bending with steady torsion is shown, which confirms the obtained results and the non-symmetric crack growth.

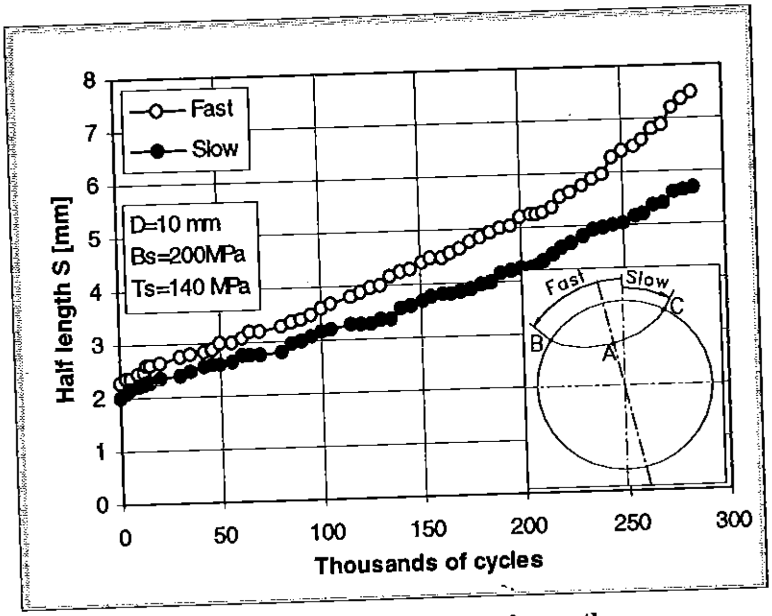

Fig. 6. Mixed-mode (I+III) crack growth

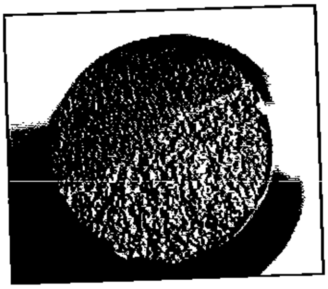

Fig. 7. Crack front surface of a specimen subjected to mixed-mode ($\Delta K_I + K_{III}$)

Conclusions

Stress Intensity Factors for semi-elliptical surface cracks in shafts have been obtained by three-dimensional finite element analysis. The shafts were subjected to either remote bending or to a remote torsion, as well as to both remote bending and torsion loading. The semi-elliptical crack shape was such that there was a constant ratio between the arc crack length and the crack depth. Stress intensity factors were obtained for several crack depths and arc crack lengths. For the bending stress loading, results were compared with other ones available in the literature but achieved by other methods and show a very good agreement. For the remote applied torsion new results were obtained and showed that at the point of maximum crack depth (A) a pure Mode III exists while at the points where the crack intersects the surface, a mixed-mode state is observed. For the bending case plus torsion loading, results show that at the points where the crack intersects the surface, the Mode I Stress Intensity Factor is different at each side of the crack front (B and C). This fact helps to clarify experimental results obtained in fatigue crack growth tests when two different crack growth rates are observed to each side of the shaft symmetry axis.

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