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## On the Transition of Incipient Growth of Fatigue Crack from Stage I to Stage II under Combined Bending/Torsion

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*ABSTRACT:* Four kinds of fatigue tests, namely, cyclic torsion, cyclic bending, combined bending/torsion, and uniaxial push-pull loading, were carried out so as to examine the mechanics condition governing the transition from stage I to stage II crack growth on plain specimen. Furthermore, the effect of the grain size on the transient condition was also studied. Each fatigue test was interrupted intermittently and specimen surface was slightly removed to get information of the incipient crack growth from slip band on specimen surface and in the depth direction. The results showed that a critical mode II stress intensity factor range  $\Delta K_{II}$ , which changes depending on the grain size, controls the transition, irrespective of the level of stress amplitude and loading type. In each fatigue test, the stage I crack growth rate first increased and then retarded when growing the transient region. The stage II crack growth rate after transition was lower than that of the stage I crack growth rate. The reason is unclear within the scope of this study.

### Notation

a	half crack length measured on specimen surface
$d_m$	mean diameter of ferrite grain
N	number of stress cycles
$\sigma_a$	normal stress amplitude in push-pull loading and pure bending
ibid	normal stress component in combined bending/torsion loading
$\tau_a$	shear stress amplitude in pure torsion

ibid	shear stress component in combined bending/torsion loading
$\Delta K_I$	mode I stress intensity factor range for opening mode crack
$\Delta K_{II}$	mode II stress intensity factor range for in-plane shear crack
$\Delta K_{III}$	mode III stress intensity factor range for out-of-plane shear crack
$\Delta K_{eff}$	stress intensity factor range for mixed mode crack of mode I and mode II

## Introduction

It is well known that total fatigue life consists of crack initiation life and propagation life. In high cycle fatigue, the crack initiation life occupies the dominant part of the total fatigue life. Therefore, a quantitative analysis of crack initiation life is of importance so as to evaluate the total fatigue life. In this respect, Kitagawa et al (1) and Miller (2) carried out a pioneering work on so-called "small crack problem". They clarified that the small crack shows crack growth rate faster than long crack at the initial stage of fatigue process or for a lower value of  $\Delta K$  and that this difference of crack growth rate asymptotically diminishes with increasing  $\Delta K$ . This means that the prediction of fatigue life of unnotched or plain specimen can hardly be done only using the growth law of long crack.

To do fatigue life estimation of the unnotched specimen, more rigorous quantitative analysis should be done into the incipient fatigue crack growth. In this regard, it is widely known that the incipient crack is formed in slip band in maximum shear stress direction and called "stage I crack". This stage I crack grows in mode II or mode III manner at an initial stage of fatigue process (3). With increasing subsequent stress cycles, the stage I crack gradually change its growth path to the direction perpendicular to loading axis in push-pull loading. More rigorously, the stage II crack just after transition from the stage I crack is denoted as "stage II (a) crack". With subsequent stress cycles, the stage II (a) crack grows up to so-called "stage II (b) crack", which is referred to as long crack.

Obviously the propagation life of long crack is easily done using a crack growth law. The difficulty in predicting fatigue life is the crack propagation life until the stage II(a). Indeed, the crack growth at this stage is, in particular, strongly affected by the effect of micro

structure. In addition, owing to extensive studies of small cracks, the growth characteristics of small cracks, even under multiaxial/biaxial stressing, has been qualitatively clarified to a great extent so far(4). However, two matters, the fatigue life prediction and the governing mechanics condition of crack growth in the transient region until the incipience of the stage II(a) crack growth, are still unclear.

Except the case where the stage I crack initiates at inclusions or defects, the stage I crack is usually formed in slip bands. During formation process of slip band cracking, the stage I crack is affected by the mechanics condition as well as the microstructure. Previous studies concerned have mainly been done under uniaxial stressing. Furthermore, the characteristics of initiation and growth of the stage I crack has been dealt on the basis of the observation of the fatigue behavior in specimen surface. It is, however, easily suggested that the surface growth of the stage I growth is greatly affected by the three dimensional crack geometry in which multiaxial stress state is responsible for.

The main objective of the present study is to examine the transient behavior of incipient crack growth from stage I to stage II(a) under combined bending/torsion stressing. Especially a criterion which describes a mechanics condition of the transition of the initial crack growth is studied. Additionally the effect of the grain size on the transition condition of the incipient crack growth is also investigated.

## **Material and experimental procedures**

The material tested is a low carbon steel round bar with a chemical composition of 0.11C,0.23Si,0.44Mn,0.015P,0.015S(wt%). To change the grain size the material was differently heat treated. After heat treatment, the grain size was varied as 96,72,24  $\mu\text{m}$  designated as the materials A,B,C respectively. The static mechanical properties and the heat treatment conditions are listed in Table 1.

The fatigue tests containing pure torsion, pure bending and combined torsion/bending were carried out in a Shimadzu torsional fatigue testing machine TB10, with a chucking device as shown in Fig. 1. In combined torsion/bending loading, a stress ratio  $\lambda$ , calculated as a

ratio of bending stress  $\sigma_a$  to torsional stress  $\tau_a$ , is selected as unity. A solid cylindrical specimen of 7mm in diameter, as shown in Fig. 2 was used in fatigue tests. Each fatigue test was conducted at a rate of 33.3Hz under fully reversed loading condition.

In order to clarify the transient behavior of the incipient fatigue crack growth from the stage I to stage II(a), each fatigue test was interrupted at a definite cycle ratio ranging from 2 to 12% under the loading of a definite stress amplitude. During interruption, each specimen surface was removed by 5  $\mu m$  by electric polishing. Then, the formation of persistent bands in specimen surface was observed using an optical microscope. In observing the stage II(a) crack growth, a plastic replication technique was applied.

As the transient behavior of the incipient crack growth under different loading mode is mainly observed on each specimen surface, it is rather suspicious whether the surface crack growing in the depth direction indicates the same crack growth characteristics as that of the surface crack extending in the specimen surface. In this regard, a lots of studies carried out so far are concerned with the change of the aspect ratio of small crack. Only a limited amount of papers concerning the stage I crack growth rate can be seen in the literature. Therefore, a supplementary study of examining the difference of the stage I crack growth behavior between specimen surface and the depth direction was also conducted on the same material under uniaxial push-pull loading condition.

## **Experimental results and discussions**

### **Criterion for the transient condition of the incipient crack growth from stage I to stage II ---Combined bending /torsion and cyclic pure torsion**

In cyclic pure torsion and combined bending/torsion fatigue, two kinds of stress amplitude level were selected. For the former and the latter, they are 160,190MPa and 130,150MPa, respectively. In each fatigue test, the crack length during transient region of the incipient

crack growth was measured. When the stage I crack shows no distinct transition to stage II(a) crack growth, that means, the crack shows no clear transition from initial shear mode growth to opening mode or mode I crack growth, the termination of the crack growth transition was tentatively defined as a moment of crack branching at which the direction of the stage I crack growth deviates from the co-axial direction of the original crack growth. In this study, all the subcracks growing in this manner until crack branching were measured and the average length of the stage I crack at the transition to the stage II(a) was counted in each fatigue test. Fig.3 indicates those values of the branched stage I crack collected in each specimen and plotted on normal distribution diagram. Obviously no definite trend can be seen in the figure. To take into account the difference of stress level and crack length the results shown in Fig. 3 are replotted using a fracture mechanics parameter, mode II stress intensity factor range,  $\Delta K_{II}$ , as shown in Fig. 4. It is evident in the figure that the trend of the  $\Delta K_{II}$  curves can be classified into three groups, depending on only a single parameter of grain size, irrespective of the loading type and the stress level. The value of  $\Delta K_{II}$  at a cumulative probability of 50% in Fig. 4 is further plotted against the grain size or mean diameter of ferrite grain  $d_m$  Fig. 5. The results under axial push-pull loading done for comparison is also shown in the figure. Clearly the  $\Delta K_{II}$  versus grain size curve shows a linear relationship, irrespective of the loading type.

### **Criterion for the transient condition of incipient crack growth from stage I to stage II ---Uniaxial push-pull loading**

To trace the change of the three dimensional shape of the stage I crack growth, the uniaxial push-pull fatigue tests were conducted by interrupting intermittently. At each interruption, specimen surface was slightly removed by electric polishing and the change of the crack shape was examined. This process was repeated until the stage I crack growing in the depth direction changes its growth path to the direction perpendicular to the loading axis. Then, the three dimensional growth rate and shape of the crack in the depth direction was finally obtained by composing each segment of the two dimensional crack morphology

measured in each surface layer. The fatigue tests were conducted at three different stress amplitudes of 200,220,240MPa. Fig. 6 shows the relationship between the crack inclination angle  $\theta$  and the depth measured from the surface during the stage I crack growth. The value of  $\theta$  was defined as an angle between the crack plane and the specimen surface. Evidently each crack initiated nearly in the direction of maximum shear grows in the coplanar direction, irrespective of the stress level, until the crack grows up to about  $80\ \mu\text{m}$ . Then, the crack quickly changes its growth path into the direction perpendicular to the loading axis. To derive the criterion for the transition of the crack growth from stage I to stage II, the growth characteristics of the crack shown in Fig. 6 is divided into three regions, namely, stage I growth ( $\theta=45^\circ$ ), mixed mode transient growth ( $45^\circ < \theta < 90^\circ$ ) and stage II(a) growth. Furthermore, the critical values of  $\Delta K_{II}$  and  $\Delta K_I$  at the growth mode transition of the stage I type to transient mixed mode type and further to the stage II(a) type respectively, were calculated. Fig. 7 shows the relationship between the critical values of  $\Delta K_{II}$  and  $\Delta K_I$  and the stress amplitude. It should be noted that the critical value for the onset of each crack growth mode transition is constant, irrespective of the stress amplitude.

### **The change of the stage I crack growth rate when the crack approach the transient region**

In pure bending, slip band and slip band cracking are generally formed in the direction of maximum shear stress in specimen surface. As the maximum shear stress plane forms a conical plane, the direction of the slip band cracking, when observed on specimen surface, theoretically changes arbitrarily, depending on the intersecting angle between the conical plane and the specimen surface. According to the results of the observation of the slip band cracking in pure bending, the slip band-initiated subcracks were mostly directed perpendicularly to the loading axis.

The stage I crack growths on specimen surface and in the depth direction were mode III and mode II respectively. Assuming the aspect ratio of the crack unity, the value of  $\Delta K_{II}$

is almost equal to  $\Delta K_{III}$ , then the surface crack growth can be characterized by  $\Delta K_{II}$  and is compared with that in pure torsion and combined bending/torsion stressing.

Fig.8 shows the changes of the growth rate of the stage I crack when the crack approaches the transient region under pure torsion and combined bending/torsion loading. The hatched columns in the figure indicates the transient region of each grain size. It is to be noted, that, ignoring the scatter of the data, increase of the stage I crack growth rate shows saturation when the crack approaches the transient region, irrespective of the loading condition and the level of stress amplitude. On the contrary, Fig. 9 shows the results for pure bending. In this case, although the crack growth rate is characterized by a parameter  $\Delta K_{II}$ , the parameter  $\Delta K_{III}$ , instead of  $\Delta K_{II}$ , can be also applied, as described above. Then, from the results of both figures, it is, as a whole, to be said that the stage I growth rate before reaching the transient region is affected by the grain size as well as the level of the stress amplitude. That is, increasing grain size and stress amplitude resulted in acceleration of the stage I growth rate.

### **Characteristic feature of the growth rate of the crack before and after passing through the transient region**

As described in the foregoing, the stage I crack first initiated in the direction of maximum shear stress in every stressing condition. However, after transition, the stage II(a) crack grew in the direction of maximum shear stress in pure torsion, while it grew in the direction of principal stress in combined bending/torsion stressing. Taking into account these characteristics of the crack growth directions, the crack growth rates during transition were characterized using  $\Delta K_{II}$  and  $\Delta K_{eff}$  for pure torsion and combined bending/torsion respectively, The value of  $\Delta K_{eff}$  proposed by Tanaka(5) is expressed as follows:

$$\Delta K_{eff} = \left( \Delta K_I^4 + 8\Delta K_{II}^4 \right)^{1/4}$$

In pure bending, the stage I and stage II crack growth modes were mode III type and mode I type respectively, within the scope of the observation results of the surface crack growth.

So, the incipient crack growth in the depth direction is evaluated by assuming that the aspect ratio of the crack is 1 and the  $\Delta K$  value for the inner crack growth  $\Delta K_{II}$  is almost equal to the  $\Delta K_{III}$  for the surface crack growth. Then, the transient crack growth was characterized in terms of  $\Delta K_{eff}$ , as being done in combined bending/torsion.

Figs.10,11,12 show the crack growth rate  $da/dN$  versus  $\Delta K_{II}$  (pure torsion),  $da/dN$  versus  $\Delta K_{eff}$  (combined, bending/torsion) and  $da/dN$  versus  $\Delta K_{eff}$  (pure bending) respectively. The hatched columns shown in each figure show the critical value of  $\Delta K_{II}$  and  $\Delta K_{eff}$  in each grain size material in each type loading. The stage I crack growth rate was evaluated as a mean value of the slip band-formed subcracks. On the contrary, the stage II crack growth rate was evaluated from the growth rate of a single leading crack. It should be noted, as a whole, in each figure that the stage I crack growth rate first increases as it approaches the transient region. Then, the growth rate in the transient region gradually decreases. Observing the change of the crack growth rate before and after passing through the transient region, it is to be noticed that the stage I crack growth rate is faster than the stage II(a) crack growth rate. This experimental evidence suggests that the retardation of the stage I crack growth rate, when growing in the transient region may be induced by the effect of some factor which changes depending on the critical  $\Delta K_{II}$  value, as shown in Fig.4. During working of this mechanism, the stage II(a) crack, which is formed as a leading crack among the stage I subcracks, gradually governs the fatigue process. The transition of the crack growth stage is then completed. To clarify the crack growth retarding factor further study is needed, including careful fractographic observation in each crack growth stage.

## Conclusions

To understand a mechanics condition governing the transient growth of the incipient crack from stage I to stage II, fatigue tests were carried out on the low carbon steel in high cycle fatigue region under different loading type, namely, uniaxial push pull, pure torsion, combined bending/torsion and pure bending. The effect of the grain size was also examined as one of the influential factors on the transient crack growth behavior. The results obtained are summarized as follows:



(1) The criterion for the transition from stage I to stage II crack growth was that a critical value of  $\Delta K_{II}$ , which increased with increasing grain size, irrespective of the loading type and cyclic stress level, governed the transient behavior.

(2) The stage I crack growth first showed faster growth rate and its rate was gradually decelerated as it passed through the transient region. On the other hand, the stage II crack, which first showed lower growth rate than that of the crack growth in transient region, gradually showed accelerated crack growth. At this moment, the reason why the stage I crack growth was decelerated and the working of the stage II crack growth began to start, is unclear.

## References

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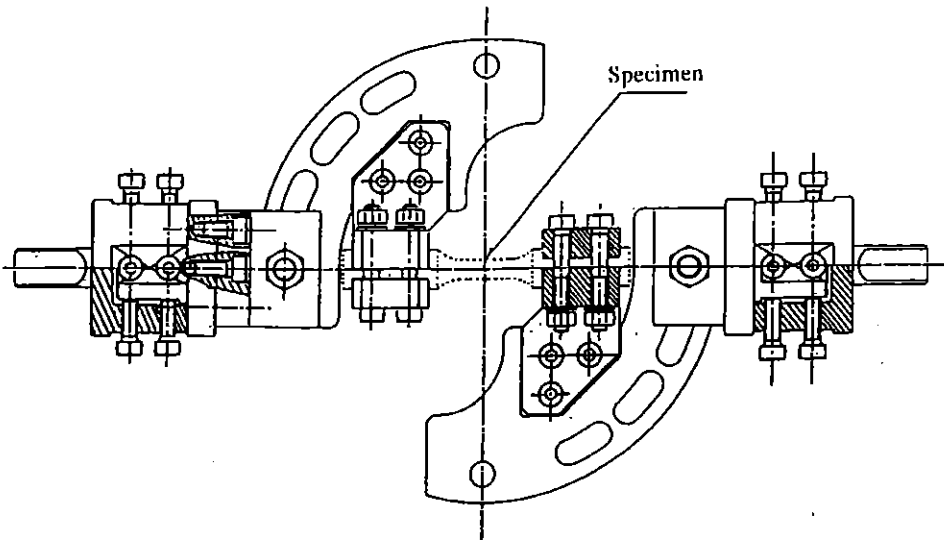
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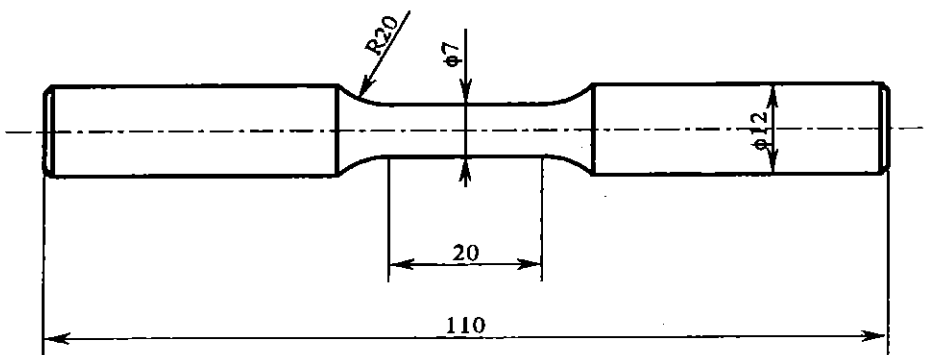
**Table 1. Mechanical properties of the material tested**

Heat treatment	Metallurgical parameters		Mechanical properties						$\tau_w/\sigma_w$	Series
	$d_m$ $\mu\text{m}$	HvF	$\sigma_y$ MPa	$\sigma_B$ MPa	$\sigma_T$ MPa	$\psi$ %	$\phi$ %	E MPa		
1573K10.8Ks→ 73.2Ks to 963K→ F.C.	96	106	230	374	273	29	66	217	0.75	A
1473K7.2Ks F.C.	72	110	235	390	290	15	61	225	0.71	B
1473K7.2Ks F.C.→ 1173K3.6Ks F.C.	24	107	241	361	249	13	72	227	0.68	C

$d_m$ ; mean diameter of ferrite grain



**Fig.1 A schematic view of the chucking device used for biaxial bending/torsion.**



**Fig.2 Specimen geometry used for the fatigue tests except uniaxial push-pull loading.**

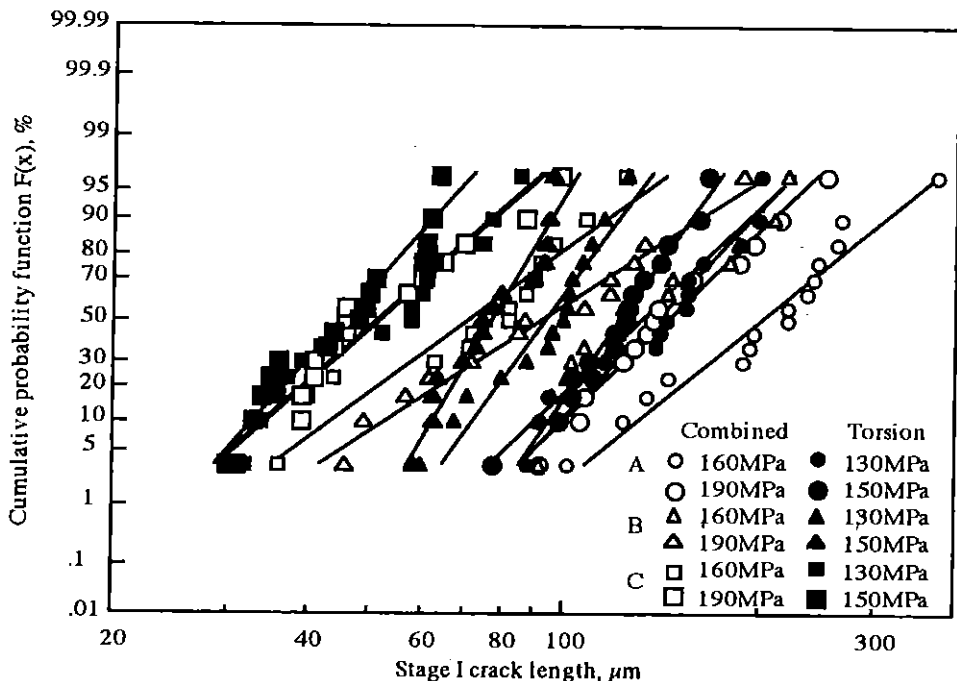


Fig.3 Crack length of the branched stage I subcracks plotted on the normal distribution diagram.

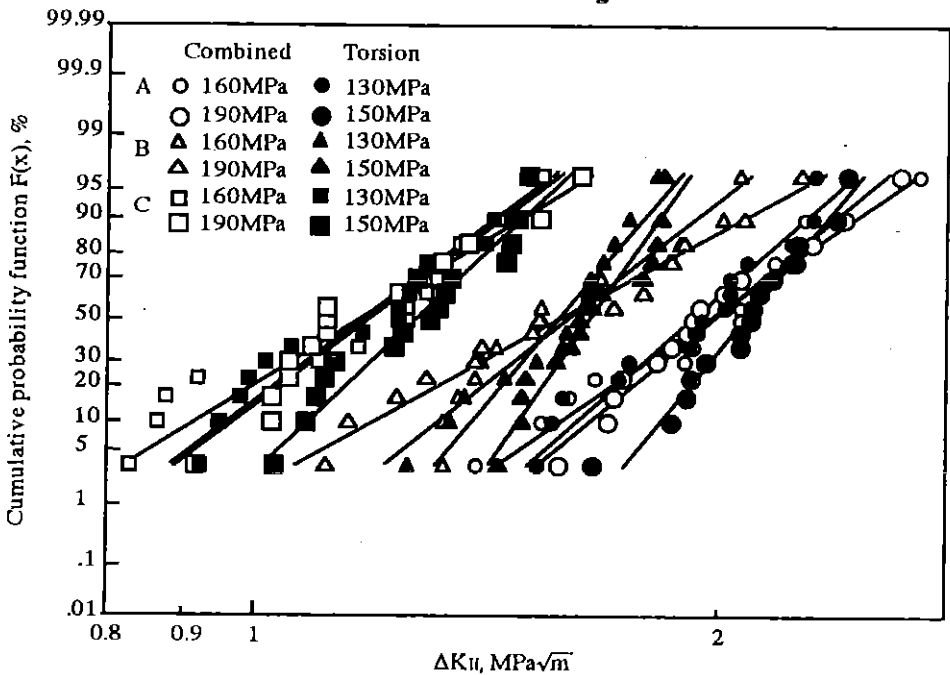


Fig.4 The values of  $\Delta K_{II}$  at the branching of the stage I subcracks plotted on the normal distribution diagram.

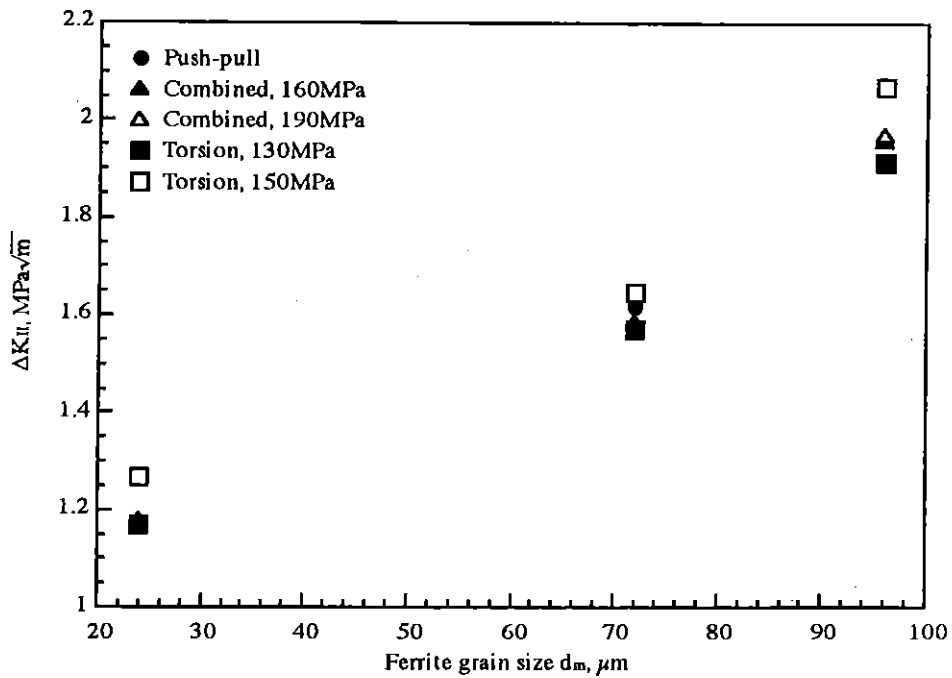


Fig.5 Relationship between  $\Delta K_{II}$  and mean ferrite grain size  $d_m$  where  $\Delta K_{II}$  is evaluated as a value at 50% cumulative probability in Fig.4.

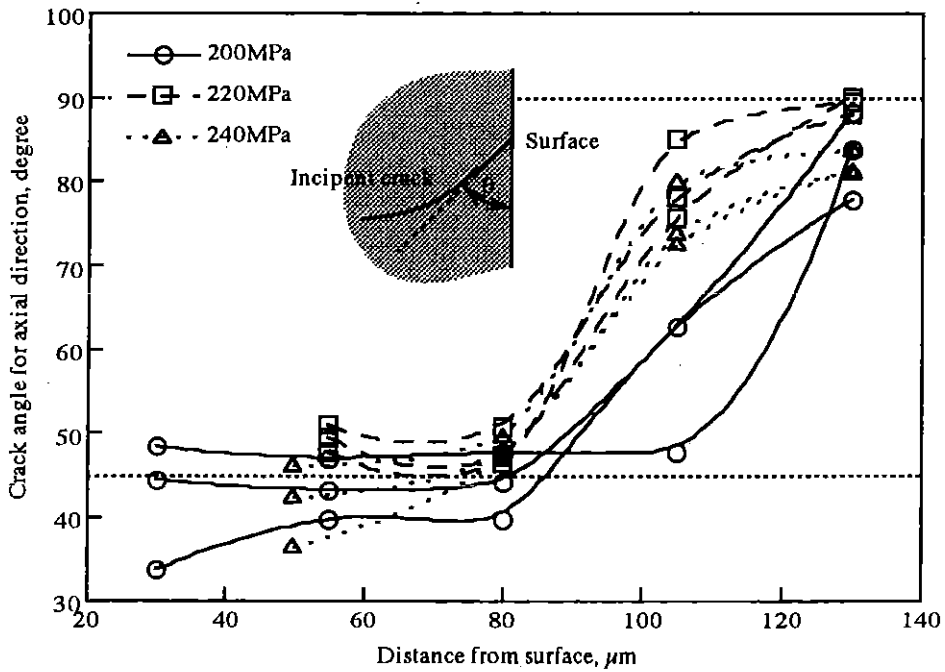


Fig.6 Change of inclination angle  $\theta$  between the stage I crack plane and surface in depth direction under push-pull loading.

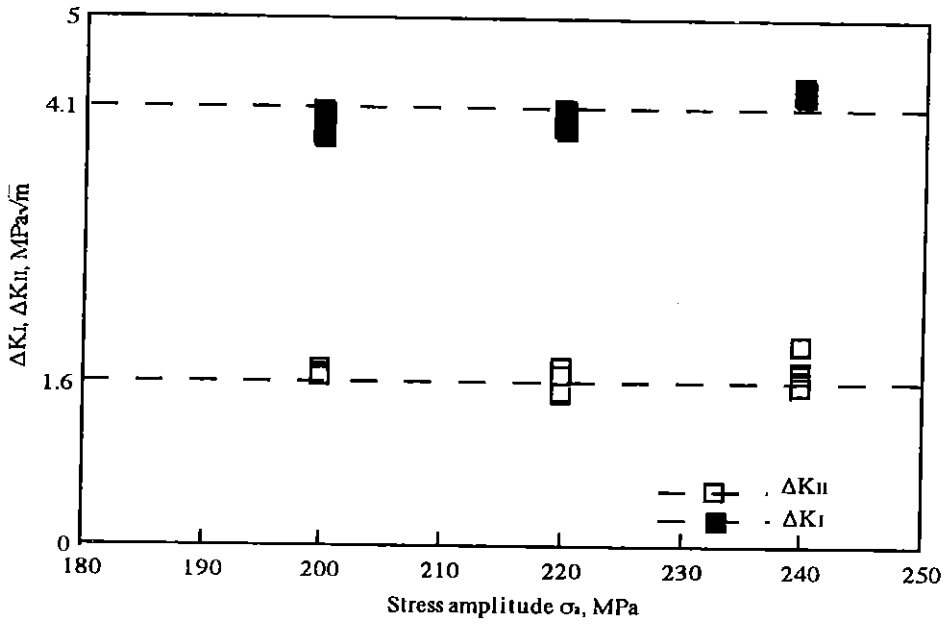


Fig.7 The values of  $\Delta K_I, \Delta K_{II}$  at the branching of the stage I crack growth in push-pull loading.

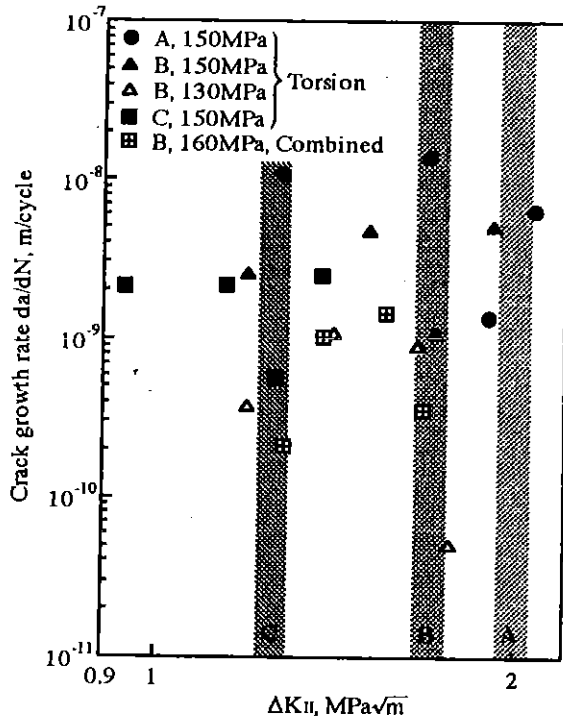


Fig.8 Relationship between the stage I crack growth rate and  $\Delta K_{II}$  when the crack approaches the transient region under pure torsion and combined bending/torsion.

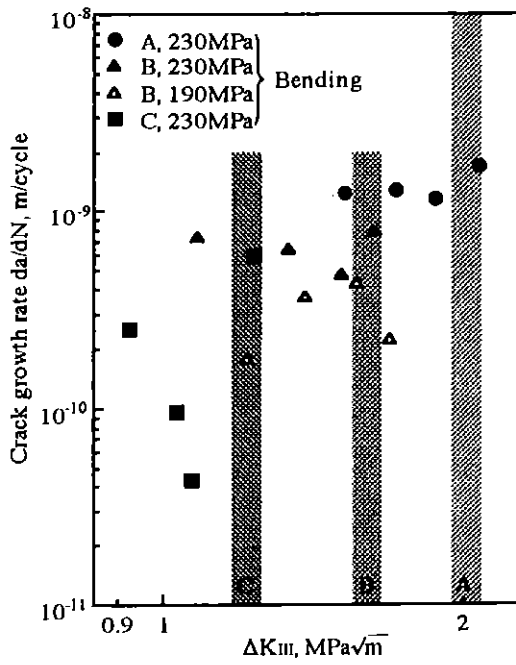


Fig.9 Relationship between the stage I crack growth rate and  $\Delta K_{III}$  when the crack approaches the transient region under pure bending.

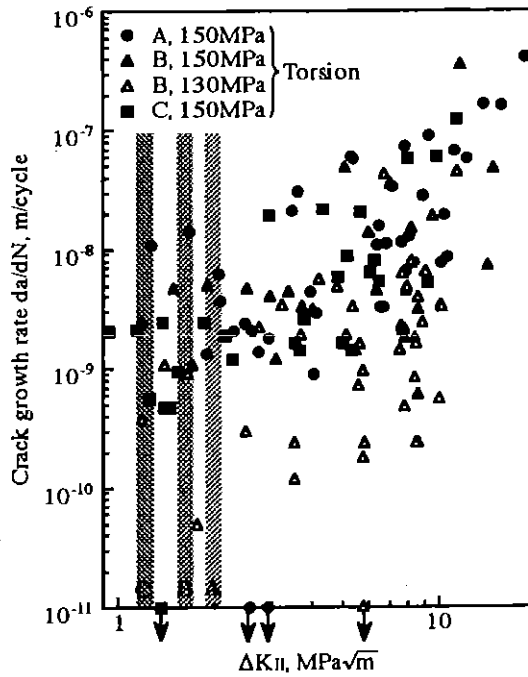


Fig.10 Change of the crack growth rate before and after passing through the transient region under cyclic pure torsion.

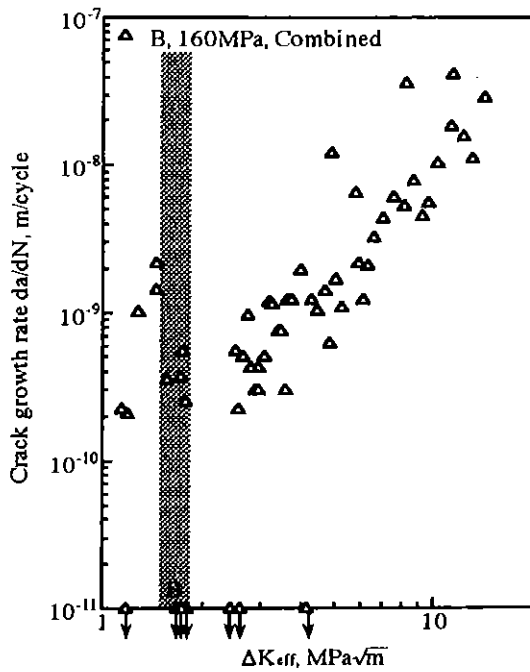


Fig.11 Change of the crack growth rate before and after passing through the transient region under combined bedding/torsion.

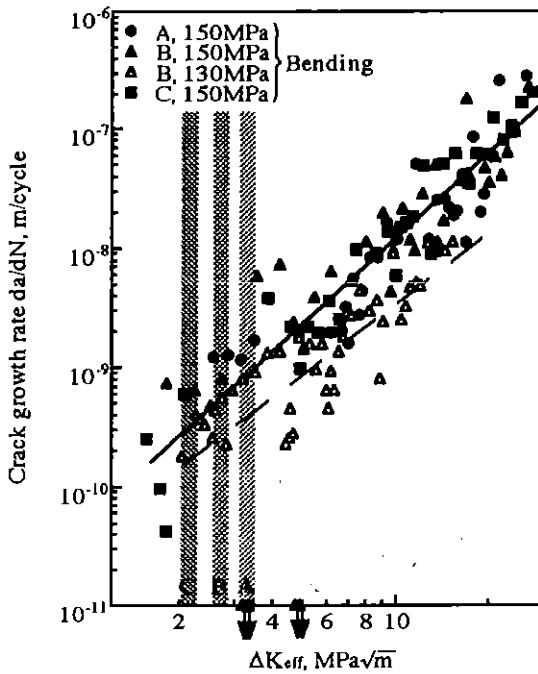


Fig.12 Change of the crack growth rate before and after passing through the transient region under cyclic pure bending.