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Interface Crack under Biaxial Load

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Abstract. *Exact solution for an interface crack with contact zone is analyzed. Oscillating and contact zone models has been obtained from this solution and procedure of transition from one model to another is demonstrated. Particularly relationship between real contact zone length and the length of crack faces overlapping zone is obtained. Energy release rate (ERR) for both models is given in the simple analytical form and slight difference between them for any contact zone length is found out. Quasi-invariance of ERR for any load and material properties is proved and simple way of ERR numerical determination is suggested.*

Notation

$\sigma_x, \sigma_y, \sigma_{xy}$	components of stress state
u, v	displacement components
b-c	crack length
b-a	contact zone length
k_1, k_2	stress intensity factors

Introduction

For composite materials interfacial and intergranular fracture are common and determine mainly the material's overall strength properties. That is why much attention has been devoted to interface crack problem. Starting from fundamental papers by Williams [1], Cherepanov [2], England [3], Erdogan [4], Rice and Sih [5] oscillating interface crack model has been initiated. This model was essentially developed in the recent works by Hutchinson et al.[6], Rice [7] and applied for numerical analysis by Tan and Gao [8], Yuuki and Cho [9], Raveendra and Banerjee [10]. Elastic-plastic material behavior at the interface crack tip has been taken into account by Shih and Asaro [11,12].

Contact model of interface crack was initially invented and investigated numerically by Comninou [13-15], Dundurs and Comninou [16]. Analytical treatment of this approach was made by Akinson [17], Simonov [18], Gautesen and Dundurs [19,20], Loboda [21] and with taking into account friction in contact area by Antipov [22].

In spite of great number of essential results on the problem in question some peculiarities of an interface crack deformation are not defined sufficiently. They are connected first of all with physically unreal crack faces inpenetration for oscillating model and its unaccessibility in the case of essential shear field. Undetermined still are the prospects of contact model application for numerical analysis.

In the paper [21] due to original approach to the contact model derivation useful quasi-invariant has been found. It gave possibility to simplify the contact model application for numerical interface crack problem solution. But the method and results of this paper can be used for demonstration of relationships of oscillating and contact model solutions and for assessment of the frames of each model applicability. With these problems the main aim of the paper is connected.

Contact zone solution for a crack between two dissimilar half-planes

An interface crack in dissimilar materials as shown in Fig.1 is considered. We assume $\sigma_y^\infty, \sigma_{x1}^\infty, \sigma_{x2}^\infty$ satisfy the continuity conditions [5], and crack surfaces are traction free for $x \in (a,b) = L_1$,and are in frictionless contact for $x \in (a,b) = L_2$. Position of the point a is arbitrary for a time. Without loss of generality, we can take

$$\gamma = \frac{\mu_1 + \mu_2 k_1}{\mu_2 + \mu_1 k_2} \geq 1$$

and $\tau_{xy}^\infty \leq 0$. In this case the longer contact zone will arise at the right crack tip and that is why only this zone we shall take into account. It is clear that oscillating singularity at the left crack tip will not influence to the stress -strain field at the right crack tip.

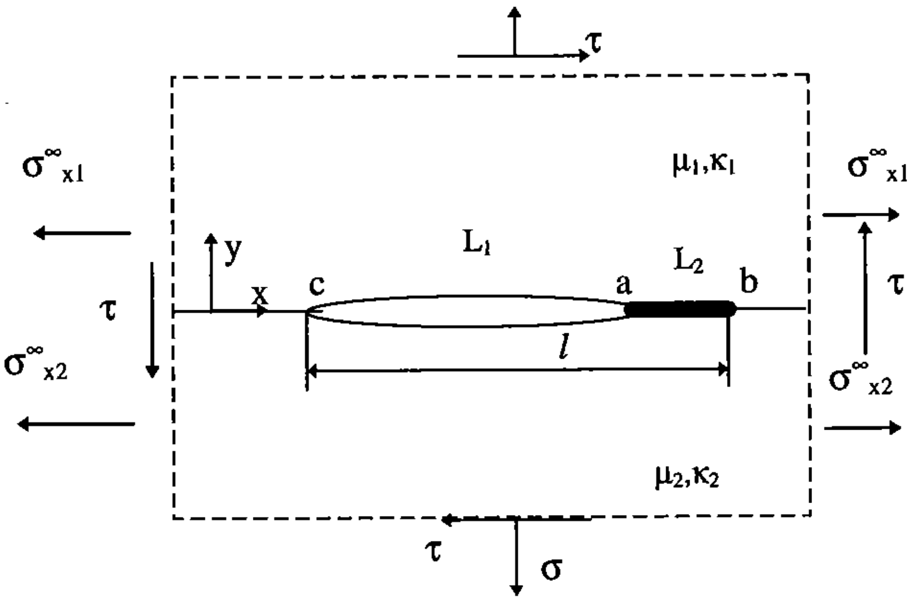

Fig.1. An interface crac with frictionless contact zone (a,b)

By using the method described in [21] the following formulas for stresses and displacement derivatives needed is subsequent analysis have been found (see for details in [21])

$$x \in L_2:\quad \sigma_y = \frac{P(x)}{\sqrt{(x-c)(b-x)}}\left[\frac{1-\gamma}{1+\gamma}\cosh\phi_0(x) + \sinh\phi_0(x)\right] +$$

$$\frac{Q(x)}{\sqrt{(x-c)(x-a)}}\left[\cosh\phi_0(x) + \frac{1-\gamma}{1+\gamma}\sinh\phi_0(x)\right]; \tag{1}$$

$$x > b:\quad \sigma_y - i\tau_{xy} = \frac{Q(x)\cos\phi(x)}{\sqrt{(x-c)(x-a)}} - \frac{P(x)\sin\phi(x)}{\sqrt{(x-c)(x-b)}} +$$

$$+ i\left[\frac{P(x)\cos\phi(x)}{\sqrt{(x-c)(x-b)}} + \frac{Q(x)\sin\phi(x)}{\sqrt{(x-c)(x-a)}}\right]; \tag{2}$$

$$x \in L_1:\ 2\mu[v'] = \frac{\mu_1 + k_1\mu_2}{\mu_2\sqrt{\gamma}}\left[\frac{P(x)\sin\phi^*(x)}{\sqrt{(x-c)(b-x)}} + \frac{Q(x)\cos\phi^*(x)}{\sqrt{(x-c)(a-x)}}\right], \qquad (3)$$

where

$$\phi(x) = 2e\ \ln\frac{\sqrt{(b-a)(x-c)}}{\sqrt{(b-c)(x-a)} + \sqrt{(a-c)(x-b)}}, \qquad e = \frac{1}{2\pi}\ln\gamma$$

$$\phi_0(x) = 2e\ \tan^{-1}\sqrt{\frac{(a-c)(b-x)}{(b-c)(x-a)}}$$

$$\varphi^*(x) = 2e\ln\frac{\sqrt{(b-a)(x-c)}}{\sqrt{(b-c)(a-x)} + \sqrt{(a-c)(b-x)}},$$

$$[f] = f^+ - f^-.$$

Polynomials P(x) and Q(x) were found from conditions at the infinity and can be written in the form

$$P(x) = C_1 x + C_2, \quad Q(x) = D_1 x + D_2,$$

$$D_1 = \sigma\cos\beta + \tau\sin\beta, \quad C_1 = \tau\cos\beta - \sigma\sin\beta$$

$$D_2 = \beta_1 C_1 - \frac{c+a}{2}D_1, \quad C_2 = -\frac{c+b}{2} - \beta_1 D_1$$

$$\beta = e\ \ln\frac{1-\sqrt{1-\lambda}}{1+\sqrt{1-\lambda}}, \quad \beta_1 = e\sqrt{(a-c)(b-c)}$$

$$\sigma = \sigma_y^\infty, \tau = -\tau_{xy}^\infty$$

parameter $\lambda=(b-a)/l$ describes- the relative contact zone length at the right crack tip.

Stress intensity factors (SIF)

$$k_1 = \lim_{x\to a+0}\sqrt{2(x-a)}\sigma_y(x,0), \quad k_2 = \lim_{x\to b+0}\sqrt{2(x-b)}\tau_{xy}(x,0) \qquad (4)$$

due to the last formulas have the following form

$$K_1 = \frac{\sqrt{2\gamma}}{\gamma+1}\sigma\sqrt{b-c}\left[\sqrt{1-\lambda}(\cos\beta + \delta\sin\beta) + 2e(\delta\cos\beta - \sin\beta)\right],$$

$$K_2 = -\sigma\sqrt{\frac{b-c}{2}}\left[\delta\cos\beta - \sin\beta - 2e\sqrt{1-\lambda}(\cos\beta + \delta\sin\beta)\right],$$

where $\delta = \dfrac{\tau}{\sigma}$ and $l=b-c$ is the crack length.

Obtained solution is valid for any values of parameter a from [c,b] . But this solution will be physically correct if the following additional conditions are satisfied

$$\sigma_y(x,0) \le 0 \text{ for } x \in L_2 \text{ and } [v(x,0)] \ge 0 \text{ for } x \in L_1, \tag{5}$$

(excluding zone of oscillation near left crack tip).To satisfy last inequalities we take $K_1 = 0$ that leads to the following equation [21]

$$e \ln \frac{1+r}{1-r} = \tan^{-1}\frac{2e}{r} + \tan^{-1}\frac{1}{\delta} + m\pi, \qquad r = \sqrt{1-\lambda} \tag{6}$$

with respect to λ . Analysis showed that for relations (5) validity we should take m=0 for $\delta \ge 0$ and m=1 for δ <0. More over for a small root of (6) due to assumptions $1+r \approx 2, \tan^{-1}(2e/r) \approx \tan^{-1}(2e)$, the following asymptotic formulacan be applied

$$\overline{\lambda_0} = 4\exp\left[-\frac{1}{\varepsilon}\left(\tan^{-1}(2e) + m\pi + \tan^{-1}\left(\frac{1}{\delta}\right)\right)\right].$$

Taking into account that both $\tan^{-1}\left(\dfrac{1}{\delta}\right)$ for $\delta \ge 0$ and $\pi + \tan^{-1}\left(\dfrac{1}{\delta}\right)$ for $\delta < 0$

are equal to $\pi/2 + \tan^{-1}(-\delta)$ last formula can be reduced to

$$\overline{\lambda_0} = 4\exp\{[-\tan^{-1}(2e) - (\psi + \pi/2)]/e\}, \tag{7}$$

where $\psi = \tan^{-1}(-\delta) = \tan^{-1}(\tau_{xy}^{\infty}/\sigma_y^{\infty})$. The roots of (6) obtained numerically and their asymptotic values $\overline{\lambda_0}$ for $\gamma = 3$ and various δ are given in the Table 1

It is clear from the Tables 1,2 that with good accuracy asymptotic formula (7) can be used for λ_0 determination in the range of $\lambda_0 \le 0.01$. It should be noted as well that results of λ_0 determination are in good agreement with correspondent results of the papers [20] (for example for $\gamma=3$ and $\delta \to \infty$ their value rounded to 3 digits is 0.329).

Table 1. Exact and asymptotic values of contact zone lenght for $\gamma=3$ and various shear fields

δ	λ_0	$\bar{\lambda}_0$
0	$0.7327*10^{-4}$	$0.7327*10^{-4}$
1	$0.6503*10^{-2}$	$0.6542*10^{-2}$
2.5	$0.6072*10^{-1}$	$0.6629*10^{-1}$
5	0.1502	0.1889
10	0.2285	0.3303
100	0.3182	0.5516
$\rightarrow \infty$	0.3291	0.5841

and for $\gamma=1.8$ they are given in the table 2.

Table 2.Exact and asymptotic values of contact zone length for $\gamma=1.8$ and various shear fields

δ	λ_0	$\bar{\lambda}_0$
0	$0.2825*10^{-7}$	$0.2825*10^{-7}$
1	$0.1252*10^{-3}$	$0.1251*10^{-3}$
2.5	$0.9256*10^{-2}$	$0.9482*10^{-2}$
5	$0.6121*10^{-1}$	$0.6714*10^{-1}$
10	0.1498	0.1908
100	0.2937	0.4977
$\rightarrow \infty$	0.3123	0.5539

Contact zone and oscillating solution

It is important that the solutions (1)-(3) of the previous section remain applicable for any contact zone length and even for $a=b$. Particularly for $a=b$ and $x>b$ $\sigma_y - i\tau_{xy}$ can be written in the form

$$\sigma_y - i\tau_{xy} = \frac{1}{\sqrt{(x-c)(x-b)}}[Q(x)\cos\phi(x) - P(x)\sin\phi(x) + i(P(x)\cos\phi(x) + Q(x)\sin\phi(x))],$$

where

$$Q(x) = (x - \frac{c+b}{2})(\sigma\cos\beta + \tau\sin\beta) + \beta_1(-\sigma\sin\beta + \tau\cos\beta),$$

$$P(x) = (x - \frac{c+b}{2})(-\sigma\sin\beta + \tau\cos\beta) - \beta_1(\sigma\cos\beta + \tau\sin\beta).$$

Taking into account that

$$\beta - \phi(x) = 2e\ln\frac{\sqrt{(b-c)(x-a)} + \sqrt{(a-c)(x-b)}}{\sqrt{(x-c}(\sqrt{b-c} + \sqrt{a-c})}$$

for a=b we arrive $\beta - \phi(x) = e\ln\frac{x-b}{x-c}$ and

$$\sigma_y - i\tau_{xy} = \frac{1}{\sqrt{(x-c)(x-b)}}([(x - \frac{c+b}{2})(\sigma\cos\omega + \tau\sin\omega) + e(b-c)(-\sigma\sin\omega + \tau\cos\omega)] +$$

$$+ i[(x - \frac{c+b}{2})(-\sigma\sin\omega + \tau\cos\omega) - e(b-c)(\sigma\cos\omega + \tau\sin\omega)]) \qquad (8)$$

where

$$\omega = e\ln\frac{x-b}{x-c}.$$

In the similar way formula (3) for a=b can be reduced to the form

$$2\mu_1[v'] = -\frac{\mu_1 + k_1\mu_2}{\mu_2\sqrt{\gamma}}\frac{1}{\sqrt{(x-c)(b-x)}}[(x - \frac{c+b}{2})(\sigma\cos\omega^* + \tau\sin\omega^*) + \qquad (9)$$

$$e(b-c)(-\sigma\sin\omega^* + \tau\cos\omega^*)]$$

where $\omega^* = e\ln\frac{b-x}{x-c}$. Formulas (8),(9) present well known oscillating solution for interface crack. Particularly for c=-b formulas (8),(9) coincide with the previously reported formulas of [11].

Due to applicability of solution (1)-(3) both for contact ($\lambda=\lambda_0$) and oscillating ($\lambda=0$) models we'll use this solution now for demonstration of the process of these models derivation.

In the Fig.2 the values of $\sigma_{yy}(x,0)/\sigma$ for $x\in L_2$, k=2.8, τ=0, b= -c=1 and for various values of λ are shown. It is clear from these results that for $2\lambda=10^{-3}$ longer part of contact zone ($\approx$0.7(b-a)) are in tension. Decreasing of λ leads to the relative length of compressed zone increasing (for $2\lambda=10^{-4}$ approximately 0.9(b-a) are compressed). For any $\lambda\leq\lambda_0$ we have σ_y (a,0)=0. Fig. 3 and 4 illustrate the procedure jf the second inequality of (5) satisfaction.

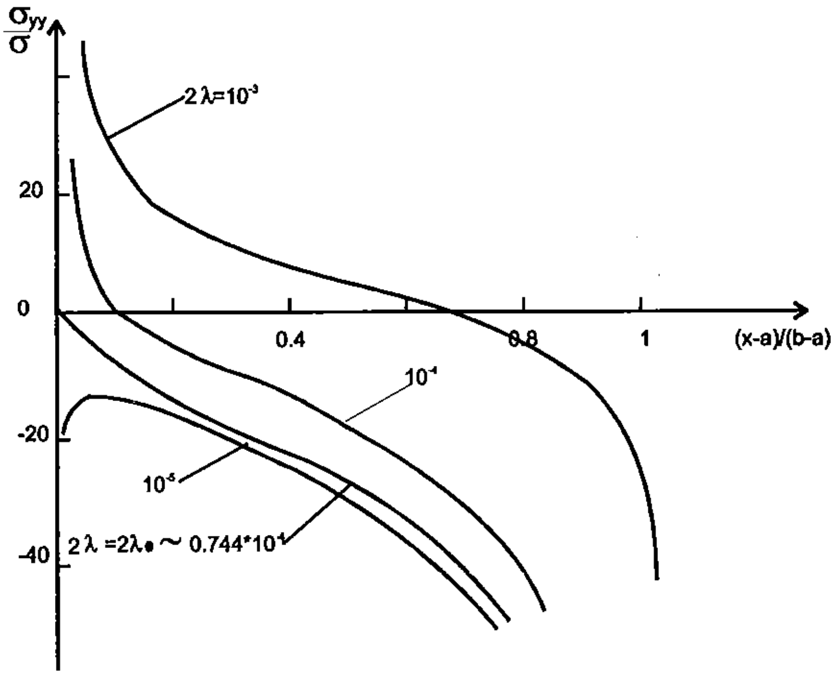

Fig.2. Normal stress in the contact zone for various its lenghts.

Particularly in the Fig.3 the diagrams of

$$[\bar{v}(x)] = [v(x,0)] / (\sigma \frac{\mu_1 + \mu_2 k_1}{2\mu_1\mu_2\sqrt{\gamma}})$$

in the left neighborhood of the right crack tip are given for various λ and k=2.8,τ=0, b= -c=1. One can clearly see that for $\lambda\geq\lambda_0$ (for $2\lambda=3*10^{-4}, 2*10^{-4}, 10^{-4}$) crack is opened and the second inequality of (5) is valid. Particularly for $\lambda>\lambda_0$ $[\bar{v}'(x)]\to\infty$ when $x\to a-0$, but for $\lambda=\lambda_0$ the jump of $[\bar{v}'(a)]=0$. Next for $\lambda<\lambda_0$ overlapping of crack tips appears and decreasing of λ leads to the increasing of overlapping zone length and its amplitude. Finally dashed line corresponds to the classical (oscillating) solution (9). Similar effects can be seen from the Fig.4 where diagrams of

$$[\bar{v}(x)] = [v(x,0)] / [\sqrt{\sigma^2 + \tau^2}\,\frac{\mu_1 + \mu_2 k_1}{2\mu_1\mu_2\sqrt{\gamma}}]$$

for various λ,k=2.8, δ=2, and b= -c=1 are displayed. In this case λ_0=0.0351 e.i. contact macro zone arises. Crack faces overlapping amplitude in this case is more essential than in Fig.3 and they are not negligible small in this case with respect to max[v(x,0)] in [c,a]. It can be explicitly seen from the Fig.2-4 that both inequalities (5) are satisfied only for $\lambda=\lambda_0$. It is interesting to note that crack faces overlapping zones are larger than real contact zone length. This difference is the largest for pure oscillating solutions (dashed lines). In this case analytical relationship between asymptotic contact zone length λ_0l and crack faces overlapping zone length

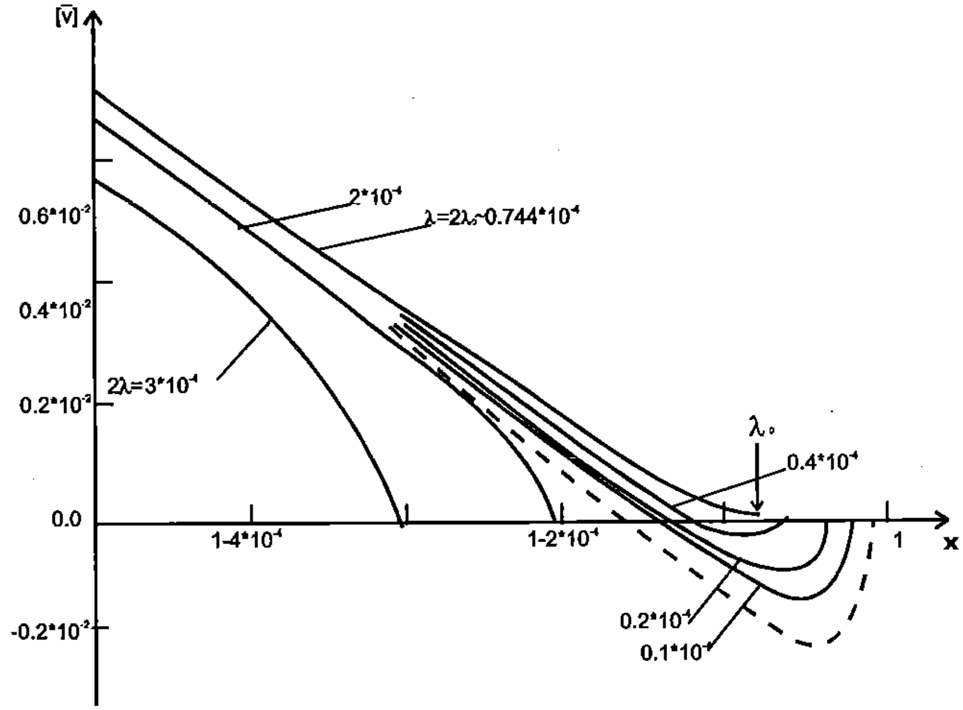

Fig. 3. Displacement jump at x≤a for pure tension field and various contact zone lenghts

$$r_c = l\exp[-(\psi+\pi/2)/e] \tag{10}$$

obtained by Rice [7] is following

$$\bar{\lambda}_0\, l = 4\exp[-\tan^{-1}(2e)]r_c \approx 4e^{-2}r_c \approx 0.5413 r_c. \tag{11}$$

But for large contact zone length according to Table 2 and Fig.4 their difference is more essential, because λ_0 is usually less than $\bar{\lambda}_0$.

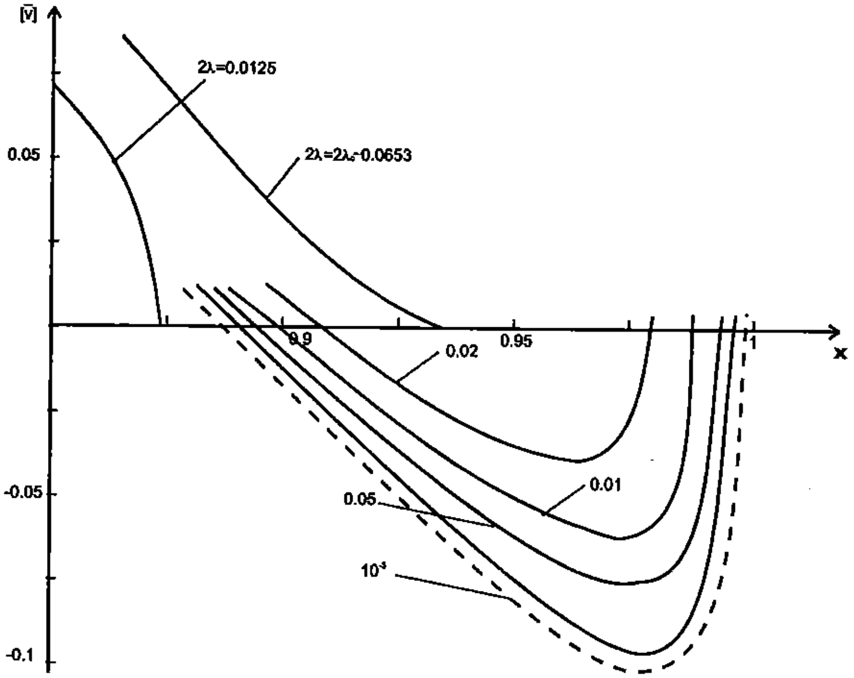

Fig. 4. Displacement jump at x≤a for pure tension-shear field and various contact zone lenghts.

The energy release rates for contact zone and oscillatory models

For the crack shown in Fig.1 the ERR can be computed as the virtual work integral

$$G = \lim_{\Delta l \to 0}\left[\frac{1}{2\Delta l}\int_a^{a+\Delta l} \bar{\sigma}_y(x)\bar{v}(x+\Delta l)dx + \frac{1}{2\Delta l}\int_b^{b+\Delta b}\bar{\sigma}_{xy}(x)\bar{u}(x+\Delta l)dx\right], \tag{12}$$

where $\bar{\sigma}_y(x) = \sigma_y(x,0)$ for $x \to a+0$,

$$\bar{v}(x)=v(x,0) \text{ for } x \to a-0,$$
$$\bar{\sigma}_{xy}(x)=\sigma_{xy}(x,o) \text{ for } x \to b+0,$$
$$\bar{u}(x)=u(x,0) \text{ for } x \to b-0.$$

Using asymptotic expressions for stress and displacement derivative fields near singular points z=a+io and z=b+io [21]the following formulas has been found

$$\bar{\sigma}_y(x)=\frac{K_1}{\sqrt{2(x-a)}}, \qquad \bar{v}(x)=\frac{k_1+1}{4\mu_1}\sqrt{2(a-x)}K_1,$$
$$\bar{\sigma}_{xy}(x)=\frac{K_2}{\sqrt{2(x-b)}}, \qquad \bar{u}(x)=\frac{(k_1+\gamma)\sqrt{2(b-x)}}{2\mu_1(1+\gamma)}K_2.$$

Substitution of the last formulas into (12) and evaluation of integrals leads to the following result

$$G(\lambda)=\frac{\pi q}{4}(\alpha K_1^2+K_2^2), \tag{13}$$

where

$$\alpha=(\gamma+1)^2/(4\gamma), \qquad q=\frac{(\mu_1+\mu_2 k_1)(\mu_2+\mu_1 k_2)}{\mu_1\mu_2(\mu_1+\mu_2+\mu_2 k_1+\mu_1 k_2)},$$

which is the same as J(λ) in [21].

By using of expressions (4) for SIF K_1 and K_2 the formula (13) can be presented in the form

$$G(\lambda)=\frac{\pi q}{8}(1+4e^2)l(\sigma^2+\tau^2-\lambda(\sigma\cos\beta+\tau\sin\beta)^2). \tag{14}$$

Taking into account that for $\lambda=\lambda_0$

$$\tan\beta_0=\frac{r_0+2e\delta}{2e-\delta r_0}, r_0=\sqrt{1-\lambda_0} \tag{15}$$

after trigonometric simplification we arrive

$$G(\lambda_0)=\frac{\pi q}{4}K_{20}^2=\frac{\pi q}{8}(1+4e^2)l(\sigma^2+\tau^2)\left(1-\frac{4e^2\lambda_0}{1+4e^2-\lambda_0}\right). \tag{16}$$

where K_{20} is the value of K_2 for $\lambda=\lambda_0$. It worth to note that values of K_{20} were previously obtained in numerical manner in [15] and analytically but in more complicated form in [20]. Comparison of

$$K_{20}/(\tau\sqrt{0.5l}) = \sqrt{(1+4e^2)\left(1+\frac{\sigma^2}{\tau^2}\right)\left(1-\frac{4e^2\lambda_0^2}{1+4e^2-\lambda_0}\right)} \quad (17)$$

with previously reported results [20] showed that for γ=3 these values in [20] are 1.032, 1.071 and 1.138 for σ/τ=0, 0.2, 0.4 respectively, whereas formula (17) gives 1.0317, 1.0703 and 1.1369. Expression (16) is the ERR for contact zone model in terms of remote traction-shear field.

Now we consider ERR for oscillating model. Using for this purpose formula

$$G_{os} = \frac{Q_1^2 + Q_2^2}{4\cosh^2(\pi e)}\left[\frac{1-\nu_1}{\mu_1} + \frac{1-\nu_2}{\mu_2}\right]$$

from [11] and taking into account that

$$Q_1 + iQ_2 = [(\sigma - 2\tau e) + i(\tau + 2\sigma e)]\sqrt{\pi l/2},$$

we obtain

$$G_{os} = \frac{\pi q}{8}(1+4e^2)l(\sigma^2 + \tau^2). \quad (18)$$

It is clear that G_{os} could be found by assuming λ_0=0 in (16), i.e. G_{os}=G(0).

It worth to compare the values of ERR obtained by using two models. Relative difference between these values can bedetermined by

$$\delta G = \frac{G_{os} - G(\lambda_0)}{G_{os}} = \frac{4e^2\lambda_0}{1+4e^2-\lambda_0} \quad (19)$$

For only tension field (τ=0, λ_0=O(10^{-4})) the value|δG| is negligible small of order 10^{-6} for any material combinations. But it is interesting that this difference is rather small even for essential contact zone length. For example for extrimal situation of $\delta\to\infty$ and γ=3 we have: $\lambda_0\approx$0.3291, e= 0.1748 and consequently $\delta G\approx$0.0507. It means that even in this case the difference in ERR determination by using two models is not very essential. Since for practically real material combinations the values e$\leq$ 0.1 (Rice [5]), δG is extremely small for any δ and both contact zone model and oscillating model can be used for ERR determination.

Oscillating model in spite of its simplicity is not explicitly convenient for numerical fracture parameters determination due to complex behavior of stresses and displacement at the crack tip. On the other hand contact model utilization in its conventional form requires λ_0 definition and only after that K_{20} or $G(\lambda_0)$determination. This way leads to the complex nonlinear problem which can not be easily solved for a finite size body containing an interface crack. Essential simplification of this procedure can be attained by using quasi-invariance of ERR $G(\lambda)$ with respect to λ [21]. According to (14)

$$\delta G(\lambda)=\frac{G(\lambda)-G(\lambda_0)}{G(\lambda_0)}=\frac{\lambda_0(\sigma\cos\beta_0+\tau\sin\beta_0)^2-\lambda(\sigma\cos\beta+\tau\sin\beta)^2}{\sigma^2+\tau^2-\lambda_0(\sigma\cos\beta_0+\tau\sin\beta_0)^2} \quad (20)$$

Using Teylor's series for $(\sigma\cos\beta+\tau\sin\beta)^2$ at the point λ_0 and taking into account that due to (15)

$$\sigma\cos\beta_0+\tau\sin\beta_0=\frac{2e\sigma\sqrt{1+\delta^2}}{\sqrt{1-\lambda_0+4e^2}},$$

$$-\sigma\sin\beta_0+\tau\cos\beta_0=-\frac{\sigma\sqrt{1-\lambda_0}\sqrt{1+\delta^2}}{\sqrt{1-\lambda_0+4e^2}},$$

we arrive

$$\delta G(\lambda)=\frac{1-2\lambda_0+4e^2}{\lambda_0(1-\lambda_0)^2(1+4e^2)}e^2(\lambda-\lambda_0)^2+o|\lambda-\lambda_0|^3 \quad (21)$$

(we notice that coefficient at the $(\lambda-\lambda_0)$is equal to zero).

For small λ_0 ($\lambda_0<\lambda<<1$) formula (21) is not convenient. In this case directly from (20) due to obvious inequality $(\sigma\cos\beta+\tau\sin\beta)^2\leq\sigma^2+\tau^2$ we obtain

$$0\leq-\delta G(\lambda)\leq\frac{\lambda}{1-\lambda_0}-\lambda_0\frac{\lambda+4e^2}{(1+4e^2)(1-\lambda_0)}\leq\frac{\lambda}{1-\lambda_0} \quad (22)$$

It follows from (21),(22) that

$$|\delta G(\lambda)|=\begin{cases}O(\lambda), & \textit{for small } \lambda_0<\lambda<<1,\\ O\left[(\lambda-\lambda_0)^2\right], & \textit{for remaining } \lambda_0\end{cases} \quad (23)$$

Formula (23) declare the quasi-invariance of $G(\lambda)$ in some vicinity $|\lambda-\lambda_0|<\varepsilon$ of λ_0. This phenomena eliminate necessity in precise λ_0 definition for $G(\lambda)$ determination,but permit to find $G(\lambda)$ for any λ from ε -vicinity of λ_0. After that assuming $G(\lambda_0)\approx G(\lambda)$

we'll make error (mistake) of order ε^2 (ε for small). For the most practically important weak shear field (small λ_0) we may directly take λ=0,01.In this case the error in $G(\lambda_0)$ determination by means of $G(\lambda)$ calculation will not exceed 1% . For essential value of $\lambda_0 \geq 0.02$ the needed ε-vicinity can be found by means of iterative solution of problem in question for $\lambda \neq \lambda_0$ with control of value K_1.

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